Form of the virtual term

The generalization to massive partons is the following. We define $\mathcal{I}_l$ and $\mathcal{I}_m$ to be the massless and massive sunbsets of $\mathcal{I}$. We also define

$$\beta_{ij} = \sqrt{1 - \frac{k_i^2 k_j^2}{(k_i \cdot k_j)^2}}\,.$$

We have:

$$\begin{aligned}
\mathcal{V}_b \;=\; & \mathcal{N}\frac{\alpha_s}{2\pi}\Bigg[-\sum_{i\in\mathcal{I}_l}\left(\frac{1}{\epsilon^2}C_{f_i} + \frac{1}{\epsilon}\gamma_{f_i}\right)\mathcal{B} \\
& -\frac{1}{\epsilon}\sum_{j\in\mathcal{I}_m} C_{f_j}\mathcal{B} \\
& +\frac{2}{\epsilon}\sum_{i,j\in\mathcal{I}_l.i>j} \log\frac{2k_i\cdot k_j}{Q^2}\;\mathcal{B}_{ij} \\
& +\frac{2}{\epsilon}\sum_{i\in I_l, j\in\mathcal{I}_m}\left(\log\frac{2k_i\cdot k_j}{Q^2} - \frac{1}{2}\log\frac{m_j^2}{Q^2}\right)\mathcal{B}_{ij} \\
& +\frac{1}{\epsilon}\sum_{i,j\in\mathcal{I}_m, i>j}\frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}\;\mathcal{B}_{ij} \;+\; \mathcal{V}_{\rm fin}\Bigg],
\end{aligned} \tag{1}$$

where the first term is as in the massless case; the second term arises from massive lines emitting and reabsorbing a gluon; the third line is as in the massless case. The fourth line is from a light-parton heavy-parton interference. The fift line is from heavy-heavy interference.

These formulae can be inferred from appendix A of the BOX paper. We begin by considering the massless case. The divergent part of the soft contribution is given in this case by

$$R_s = \mathcal{N}\frac{\alpha_s}{2\pi}\sum_{i\neq j}\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\log\frac{Q^2}{s\xi_c^2} - \log 2k_i\cdot k_j + \log 2k_i^0 + \log 2k_j^0\right)\right]\mathcal{B}_{ij}, \tag{2}$$

where we have used eqs. A.31, A.32 and A.33 of the BOX paper (remembering also eq. A.12, A.13 and A.14). This yields for the divergent part of the soft contribution

$$R_s = \mathcal{N}\frac{\alpha_s}{2\pi}\left[\frac{1}{\epsilon^2}\sum_i C_{f_i}\mathcal{B} - \frac{1}{\epsilon}\sum_{i\neq j}\log\frac{2k_i\cdot k_j}{Q^2}\mathcal{B}_{ij} + \frac{1}{\epsilon}\sum_i C_{f_i}\left(\log\frac{Q^2}{s\xi_c^2} + 2\log\frac{2k_i^0}{Q}\right)\mathcal{B}\right]. \tag{3}$$

These, combined with the soft-collinear divergent remnants, must cancel the divergences of the virtual term. Thus, the collinear divergent remnants must have the form

$$C_s = \mathcal{N}\frac{\alpha_s}{2\pi}\left\{\frac{1}{\epsilon}\sum_i\left[-C_{f_i}\left(\log\frac{Q^2}{s\xi_c^2} + 2\log\frac{2k_i^0}{E}\right) + \gamma_{f_i}\right]\mathcal{B}\right\}. \tag{4}$$

Now we extend this to the massive case. We have

$$\begin{aligned}
R_s \;=\; & \mathcal{N}\frac{\alpha_s}{2\pi}\Bigg[\sum_{i\neq j\in\mathcal{I}_l}\frac{1}{\epsilon}\left(\frac{1}{\epsilon} - \log\frac{2k_i\cdot k_j}{Q^2} + \log\frac{Q^2}{s\xi_c^2} + 2\log\frac{2k_i^0}{Q}\right)\mathcal{B}_{ij} \\
& +2\sum_{i\in\mathcal{I}_l, j\in\mathcal{I}_m}\frac{1}{\epsilon}\left(\frac{1}{2\epsilon} - \log\frac{2k_i\cdot k_j}{Q^2} + \log\frac{Q^2}{s\xi_c^2} + \log\frac{2k_i^0}{Q} + \log\frac{m_j}{Q}\right)\mathcal{B}_{ij} \\
& -\sum_{i\neq j\in\mathcal{I}_m}\frac{1}{\epsilon}\frac{1}{2\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}\;\mathcal{B}_{ij} \\
& +\sum_{i\in\mathcal{I}_m}\frac{1}{\epsilon}C_{f_i}\mathcal{B}\Bigg].
\end{aligned} \tag{5}$$

The first line is again from A.32 and A.33. The second line is from A.26, A.27. The third line is from A.39 and A.41. The last line is from A.54, remembering eq. A.11. The collinear remnants must have the form

$$C_s = \mathcal{N}\frac{\alpha_s}{2\pi}\left\{\frac{1}{\epsilon}\sum_{i\in\mathcal{I}_l}\left[-C_{f_i}\left(\log\frac{Q^2}{s\xi_c^2}+2\log\frac{2k_i^0}{E}\right)+\gamma_{f_i}\right]\mathcal{B}\right\}. \tag{6}$$

Summing up (5) and (6) we get exactly the opposite of the divergent part of eq. (1).