Calculation of $\mathcal{V}_{\rm fin}$ in the FKS framework: an example using POWHEG-BOX scales

February 23, 2011

Abstract

We show how to calculate the $\mathcal{V}_{\rm fin}$ term needed by the POWHEG-BOX program, starting from a generic expression for the virtual term $\mathcal{V}_b$. We show this in the case of DY processes.

We start from FNO, eq.7.190, which is taken from the original paper by Altarelli, Ellis and Martinelli:

$$\mathcal{V}_{b,\,q\bar{q}} = \left(\frac{4\pi\mu_{\rm R}^2}{s}\right)^{\epsilon} \frac{\Gamma(1+\epsilon)\,\Gamma(1-\epsilon)^2}{\Gamma(1-2\epsilon)} \frac{\alpha_{\rm S}}{2\pi} C_{\rm F} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right] \mathcal{B}_{q\bar{q}}, \tag{1}$$

Here $\mu_{\rm R}$ stands for the renormalization scale, and $s = 2(k_\oplus \cdot k_\ominus)$. To extract $\mathcal{V}_{\rm fin}$ from eq. (1), we need to equal the expression in (1) to the following one (FNO, eq.2.92):

$$\mathcal{V}_{\rm b} = \mathcal{N} \frac{\alpha_{\rm S}}{2\pi} \left[-\sum_{i\in\mathcal{I}} \left(\frac{1}{\epsilon^2} C_{f_i} + \frac{1}{\epsilon}\gamma_{f_i}\right)\mathcal{B} + \frac{1}{\epsilon} \sum_{\substack{i,j\in\mathcal{I}\\ i\neq j}} \log\frac{2k_i\cdot k_j}{Q^2}\,\mathcal{B}_{ij} + \mathcal{V}_{\rm fin}\right] \tag{2}$$

where

$$\mathcal{N} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu_{\rm R}^2}{Q^2}\right)^{\epsilon}. \tag{3}$$

For the case at hand, equation (2) reduces to

$$\mathcal{V}_{b,\,q\bar{q}} = \left(\frac{4\pi\mu_{\rm R}^2}{Q^2}\right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \frac{\alpha_{\rm S}}{2\pi} \left[-\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{2}{\epsilon}\log\frac{s}{Q^2}\right) C_{\rm F}\mathcal{B}_{q\bar{q}} + \mathcal{V}_{{\rm fin},\,q\bar{q}}\right]. \tag{4}$$

where we used the fact that the only colored partons involved are the two incoming quarks, and we also have that $\mathcal{B}^{q\bar{q}}_{\oplus\ominus} = C_{\rm F}\mathcal{B}_{q\bar{q}}$.

We notice that Q^2 is here an *arbitrary* scale. In the FKS approach, the soft-virtual term is obtained summing to $\mathcal{V}_{\rm fin}$ a quantity that is Q^2 dependent (see FNO, eq. 2.99, 2.100 and 2.101). In the POWHEG-BOX code, this operation is authomatized and it is performed in a file that the user is not supposed to edit. Therefore, an explicit choice for this scale was needed. Since we choosed

$$Q^2 = \mu_{\rm R}^2\,, \tag{5}$$

it is important to use the same value for Q^2 in eq. (4), in order to extract a consistent expression for $\mathcal{V}_{\rm fin}$.

Therefore we evaluate the expression in (4) with $Q^2 = \mu_{\rm R}^2$, and equal it to the r.h.s. of eq. (1), obtaining:

$$(4\pi)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left[-\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{2}{\epsilon}\log\frac{s}{\mu_{\rm R}^2}\right)\mathcal{B}_{q\bar{q}} + \frac{\mathcal{V}_{{\rm fin},\,q\bar{q}}}{C_{\rm F}}\right] = \left(\frac{4\pi\mu_{\rm R}^2}{s}\right)^\epsilon \frac{\Gamma(1+\epsilon)\,\Gamma(1-\epsilon)^2}{\Gamma(1-2\epsilon)} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right]\mathcal{B}_{q\bar{q}}\,. \quad (6)$$

By using the fact that

$$\frac{\Gamma(1+\epsilon)\,\Gamma(1-\epsilon)^2}{\Gamma(1-2\epsilon)} = \frac{1}{\Gamma(1-\epsilon)} + \mathcal{O}\left(\epsilon^3\right), \quad (7)$$

we have

$$\frac{\mathcal{V}_{{\rm fin},\,q\bar{q}}}{C_{\rm F}} = \left[\left(\frac{\mu_{\rm R}^2}{s}\right)^\epsilon \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right) + \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{2}{\epsilon}\log\frac{s}{\mu_{\rm R}^2}\right)\right]\mathcal{B}_{q\bar{q}}\,, \quad (8)$$

that reduces to

$$\mathcal{V}_{{\rm fin},\,q\bar{q}} = \mathcal{B}_{q\bar{q}}\left[\pi^2 - 8 - 3\log\frac{\mu_{\rm R}^2}{s} - \log^2\frac{\mu_{\rm R}^2}{s}\right] C_{\rm F} \quad (9)$$

after expanding $(\mu_{\rm R}^2/s)^\epsilon$ as usual. The expression in eq. (9) is the one needed by the POWHEG-BOX program, and it has to be coded inside the subroutine `setvirtual`.