Collective flow
and
flow fluctuations at RHIC

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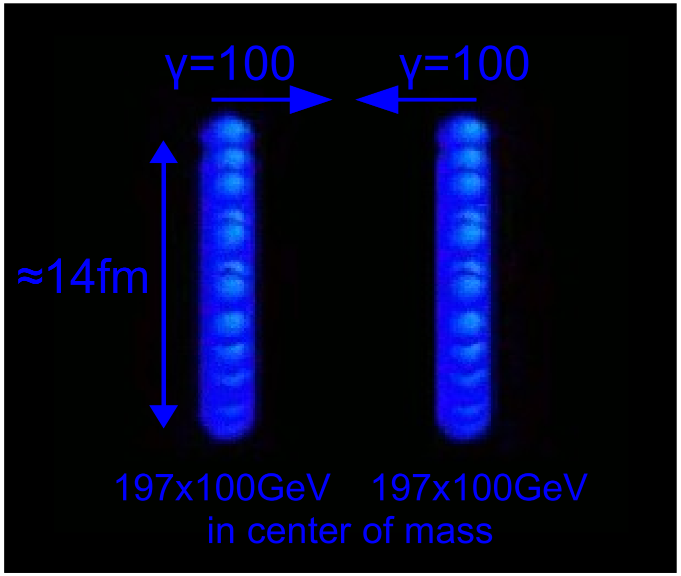
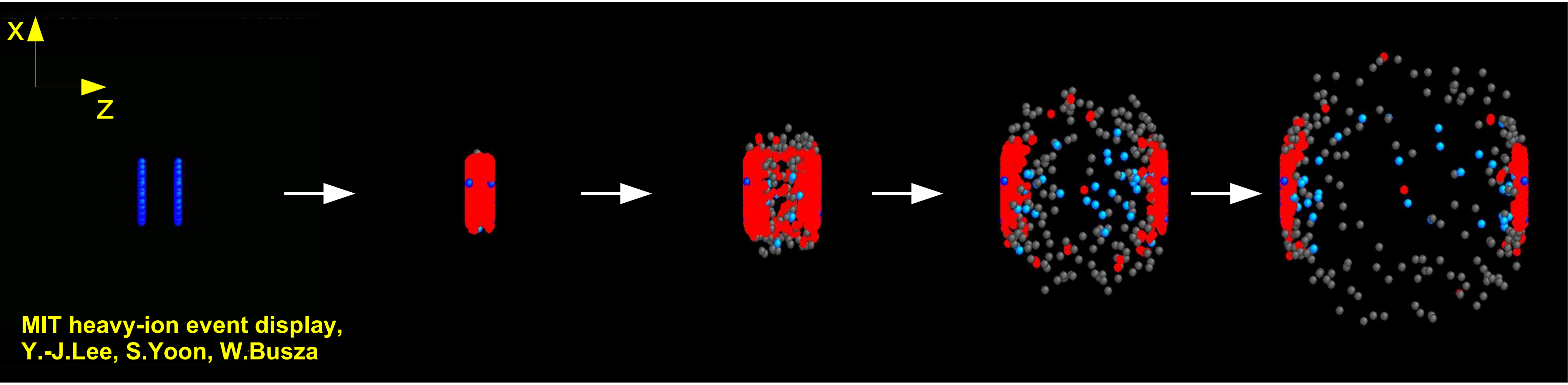
LNS lunch seminar
May 8th, 2007

Au+Au @ 200 GeV at RHIC





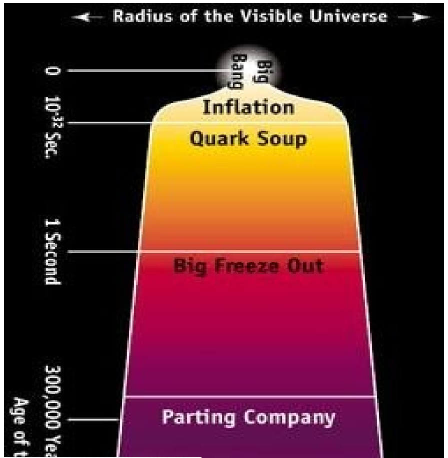
MIT heavy-ion event display,
Y.-J.Lee, S.Yoon, W.Busza

Why do we do this?

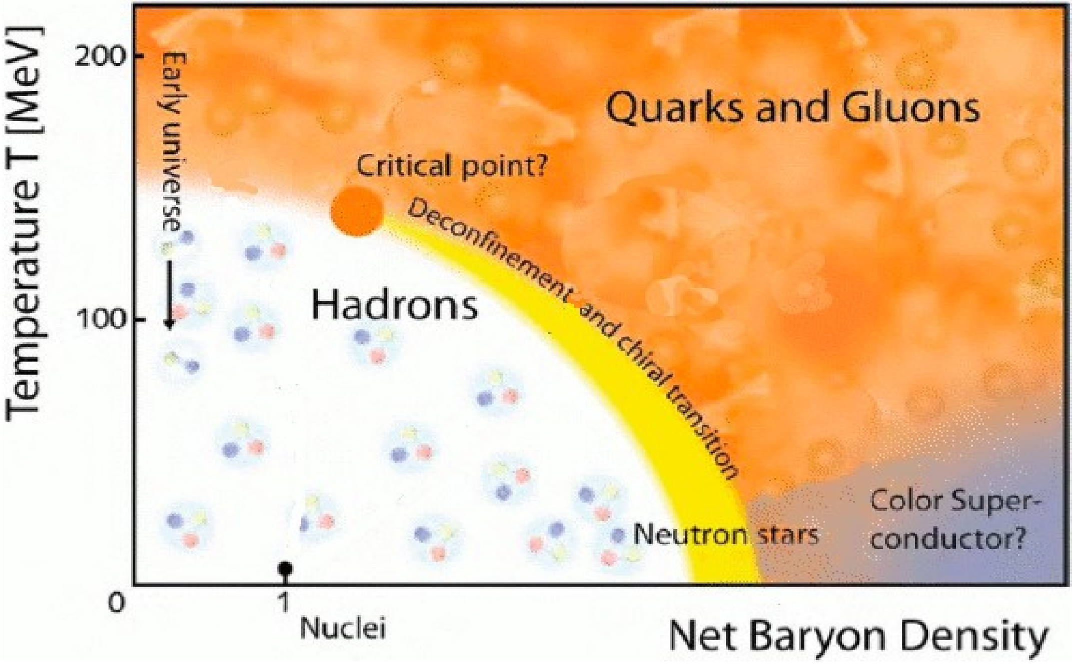
In high energy physics we have concentrated on experiments, in which we distribute a higher and higher amount of energy into a region with smaller and smaller dimensions. In order to study the question of “vacuum”, we must turn to a different direction; we should investigate some “bulk” phenomena by distributing high energy over a relatively large volume.

T.D.Lee, Rev.Mod.Phys. 47(1975)267

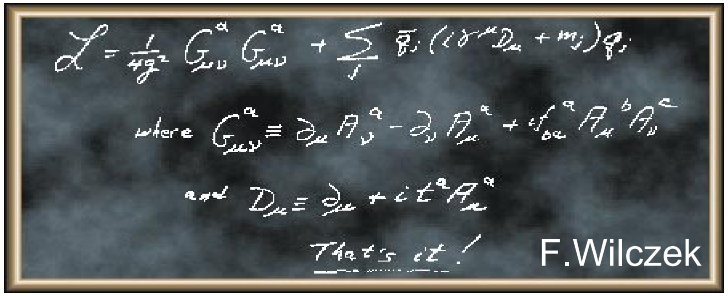
Quark-hadron phase transition in the primordial universe



QCD phase diagram



Confinement + chiral symmetry breaking



$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

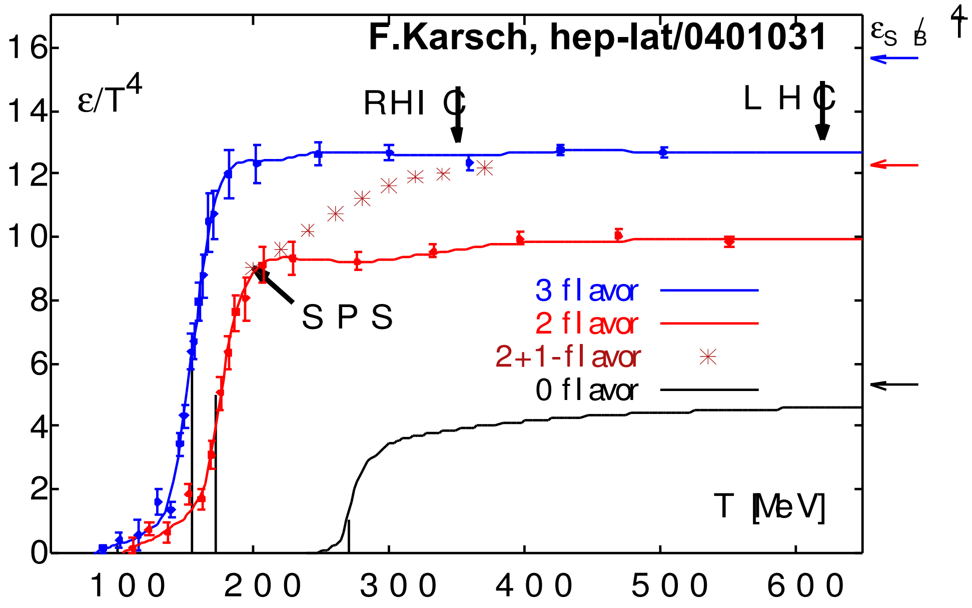
That's it!

F.Wilczek

QCD “thermodynamics”

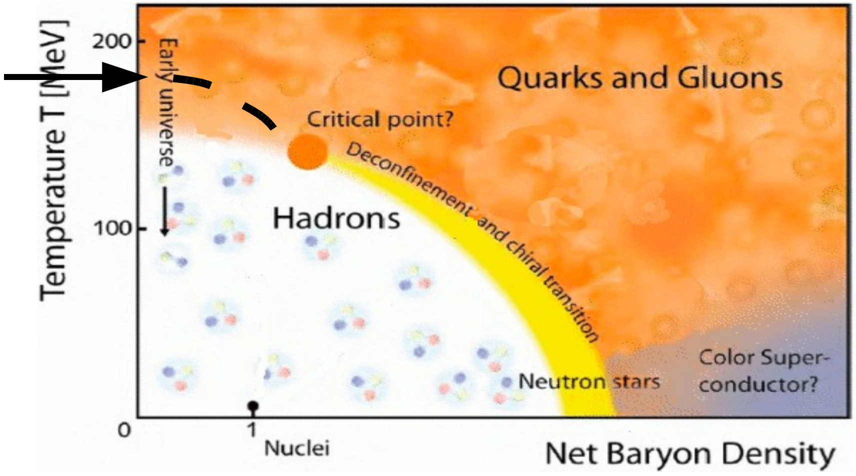
TFLOPS super computer

$$Z=\int\prod dU\,e^{-S_G}$$



- LQCD simulations at n_b=0 (μ=0)

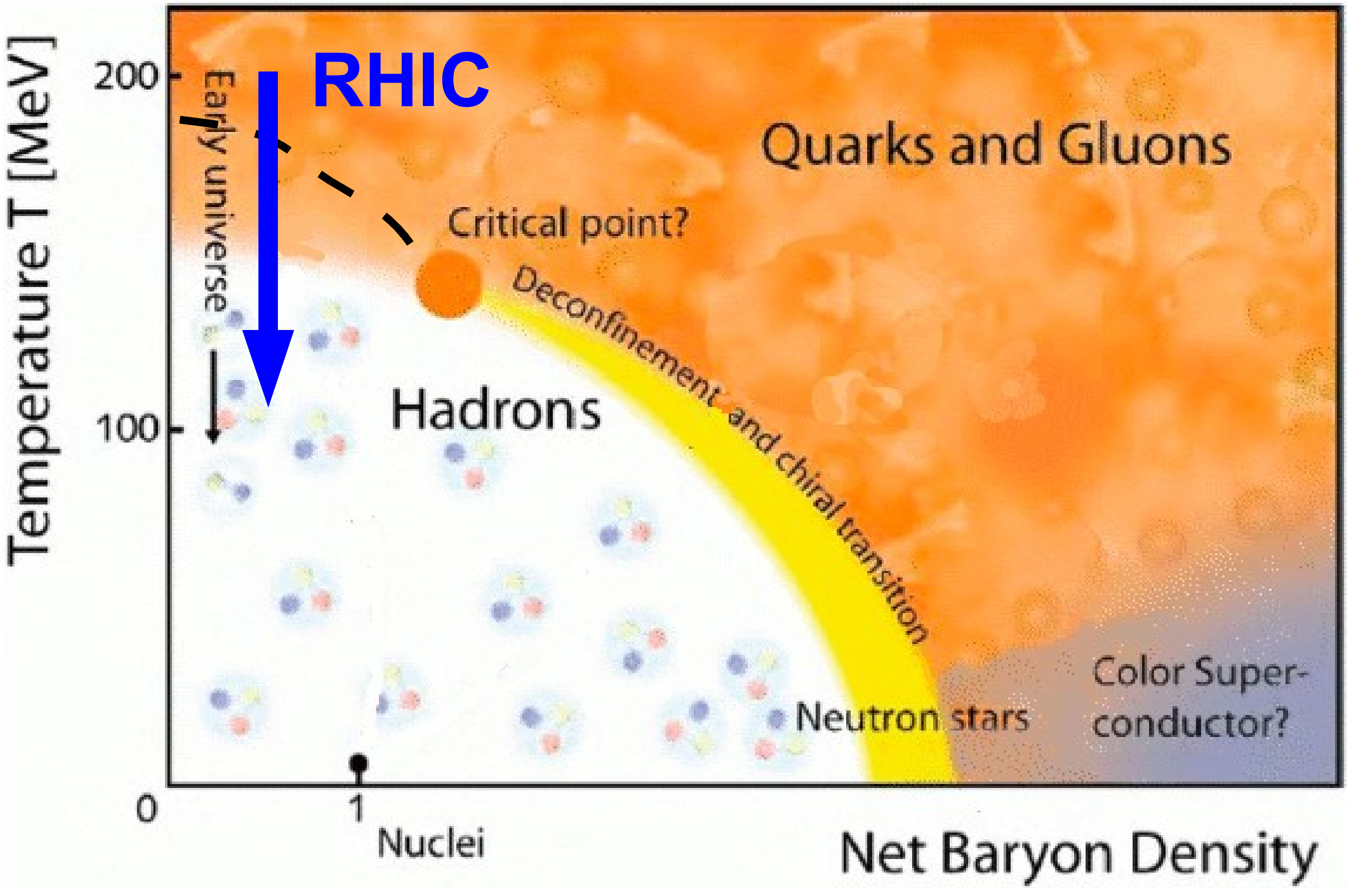
T_{crit}≈170MeV
ε_{crit}≈0.7GeV/fm^3



- Indications for cross-over with critical point for $n_b \geq 0$ ($\mu_b \geq 0$)

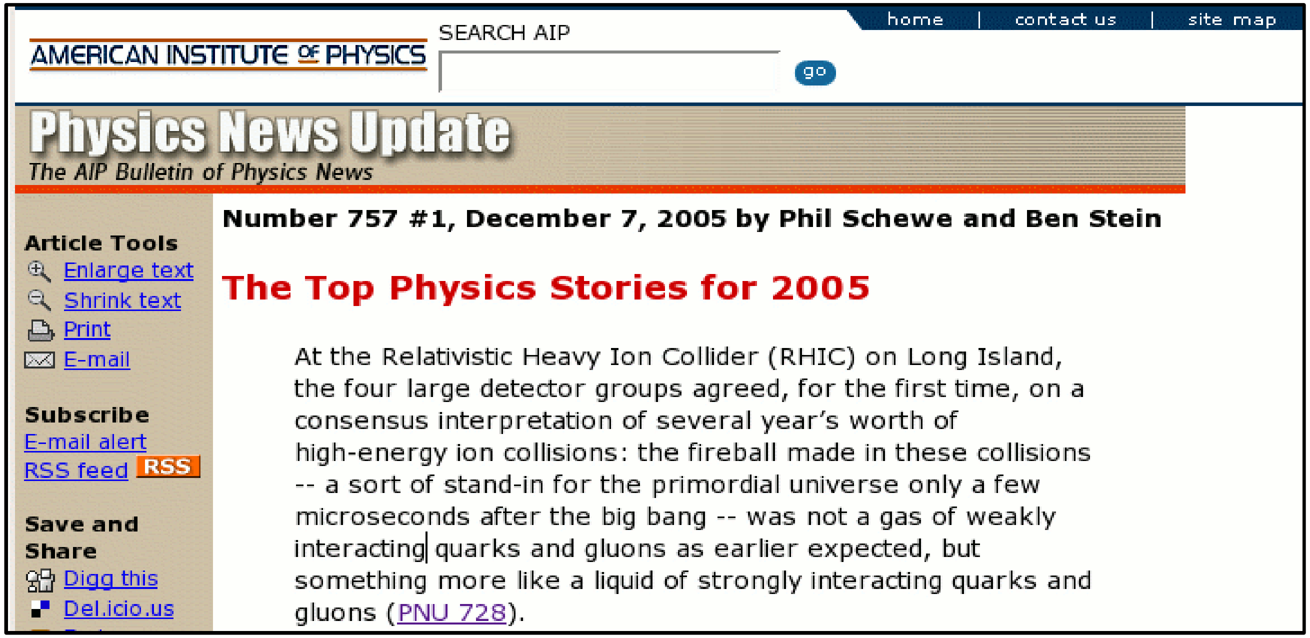
(quantitative results depend on lattice parameters and chiral + continuum limit extrapolations)

QCD matter at high temperature



RHIC events (at mid-rapidity) are net-baryon free ($\bar{p}/p\approx0.8$):
RHIC explores cross-over region of QCD phase diagram

AIP Top Physics Story, Dec 2005



Number 757 #1, December 7, 2005 by Phil Schewe and Ben Stein

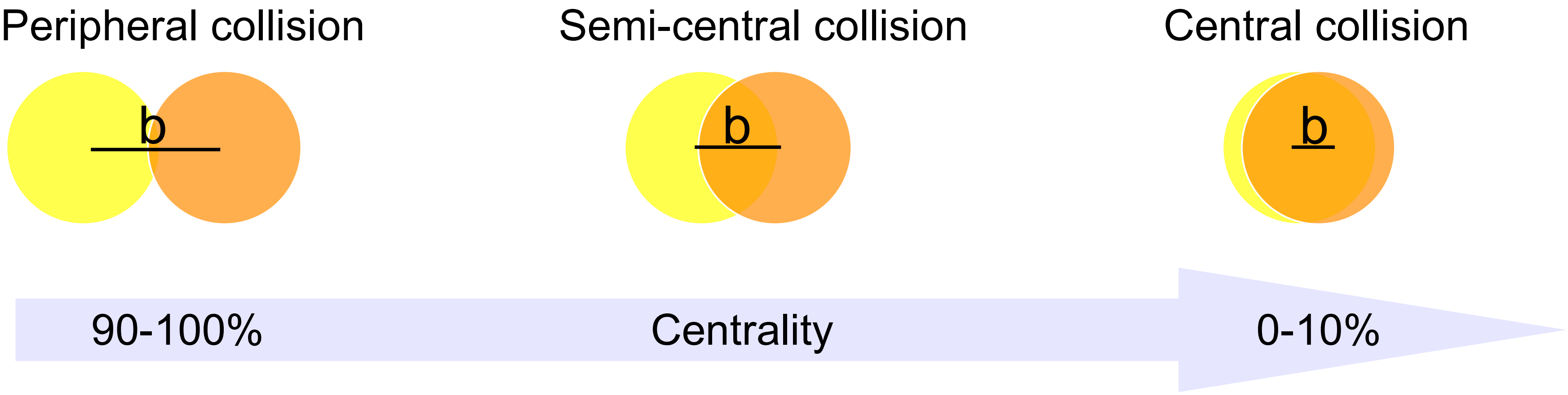
The Top Physics Stories for 2005

At the Relativistic Heavy Ion Collider (RHIC) on Long Island, the four large detector groups agreed, for the first time, on a consensus interpretation of several year's worth of high-energy ion collisions: the fireball made in these collisions -- a sort of stand-in for the primordial universe only a few microseconds after the big bang -- was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons (PNU 728).

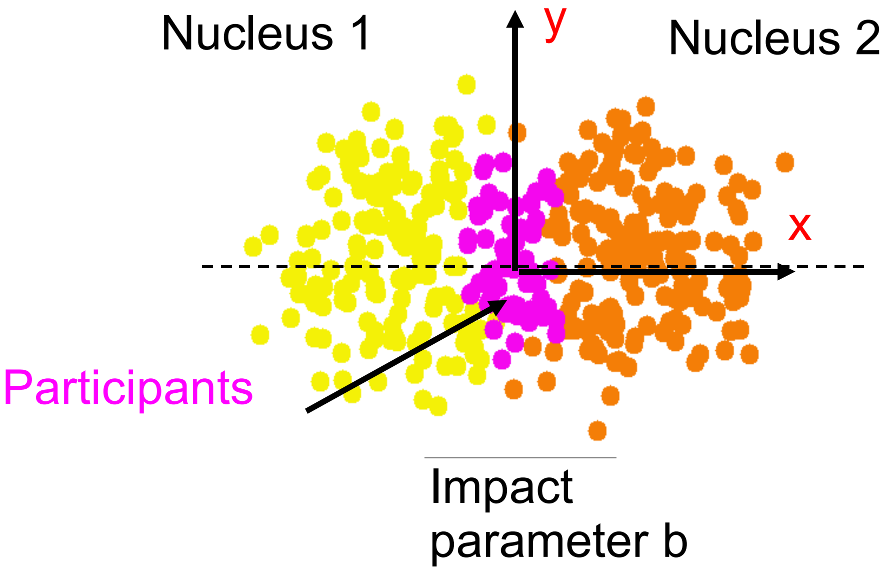
"... the fireball made in these [heavy-ion] collisions ... was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid..."

http://www.aip.org/pnu/2005/split/757-1.html
RHIC whitepapers: NPA 757 (2005)1-283

Heavy-ion jargon: Collision centrality

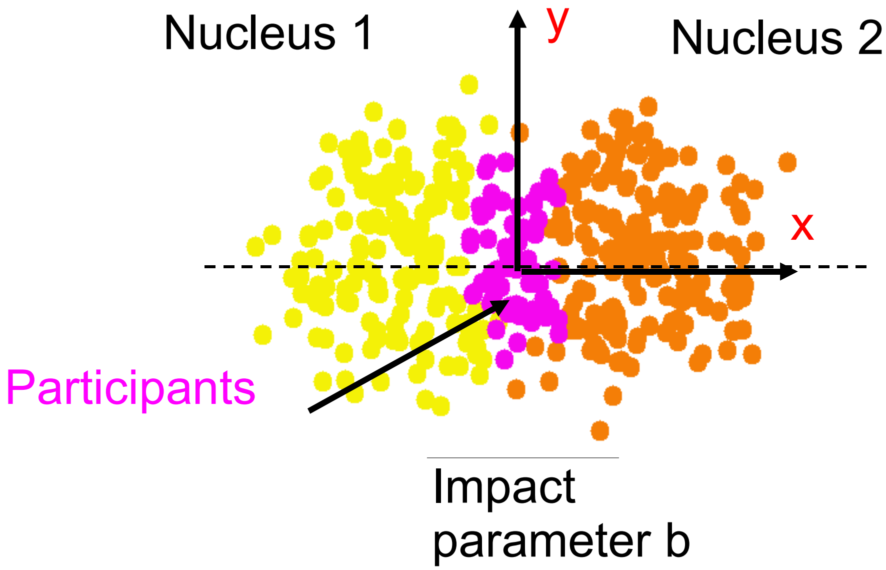


- Centrality classes
  - Impact parameter (<b>)
  - #Participants (<N_{part}>)
  - #NN-collisions (<N_{coll}>)
- Relate to data via Glauber MC based detector simulations

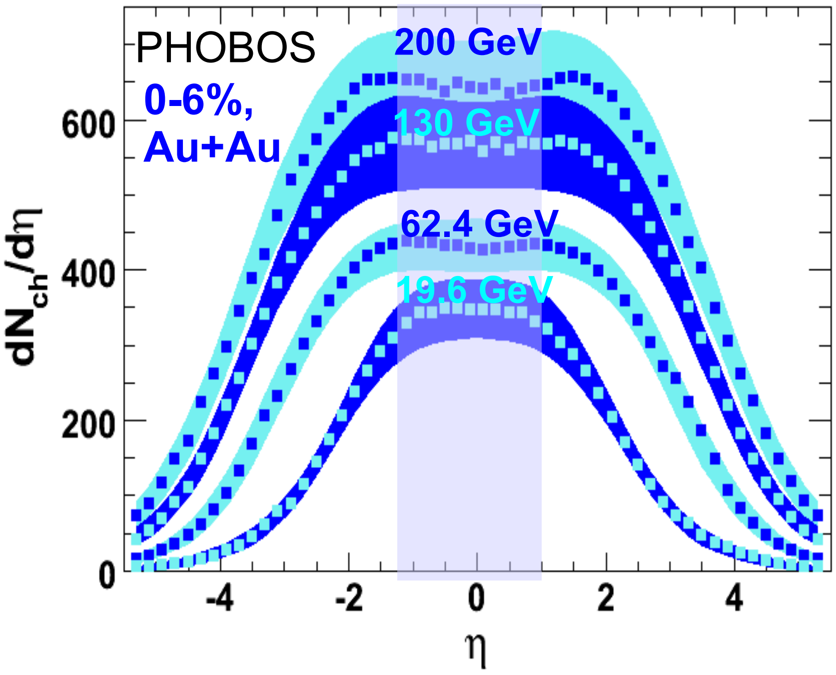


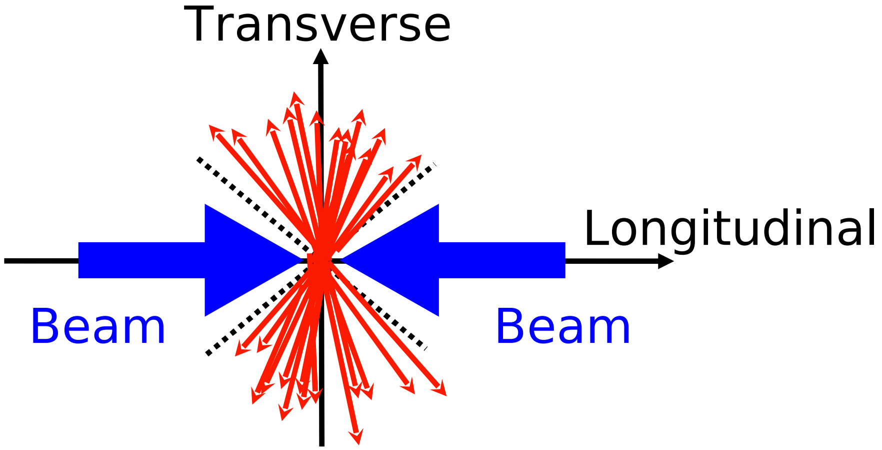
Heavy-ion jargon: Glauber MC

- Setup nuclei
  - Radial distribution of nucleons (in nucleus) drawn from Wood-Saxon distribution
  - Isotropic angular distribution
  - Separate by b (with dN/db~b)
- Simulate collision
  - Assume: Nucleons travel on straight-line paths and interact inelastically when $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}<\sqrt{\sigma_{inel}^{NN}/\pi}$
  - #Participants (N_{part}~A)
    - Nucleons that interact at least once
  - #NN-collisions (N_{coll}~$A^{4/3}$)
    - Total number of collisions suffered by the nucleons of one of the nuclei
- Repeat to gather arbitrary many events



Energy density reached at RHIC





Use “energy flow” from longitudinal (=beam) to transverse direction to estimate energy/volume

$$\frac{1000 \text{ particles x } 0.5 \text{ GeV/particle}}{\pi \text{ x } (7 \text{ fm})^2 * 1 \text{ fm}} \approx 3\text{GeV/fm}^3$$

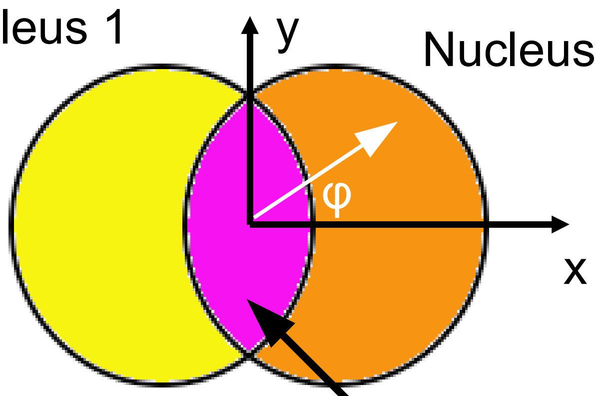
Much larger than $\varepsilon_{crit} \approx 0.7 \text{GeV/fm}^3$

But what about equilibrium?

PHOBOS WhitePaper: NPA, 757 (2005) 28

How do we prove that we make “matter”?

Non-central collision in the transverse plane

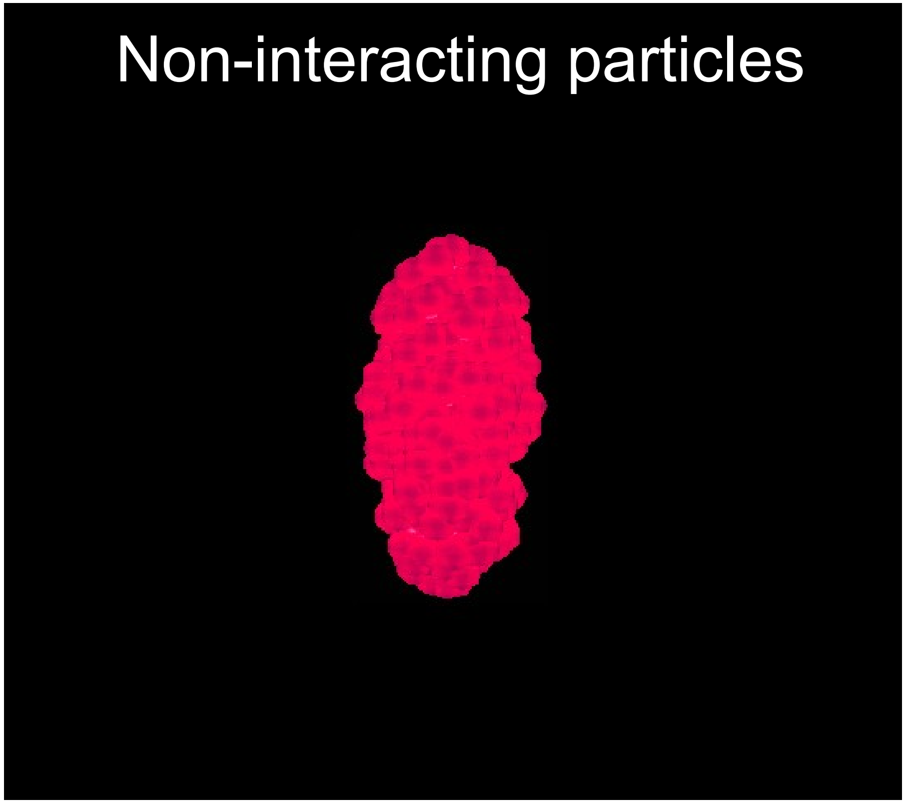


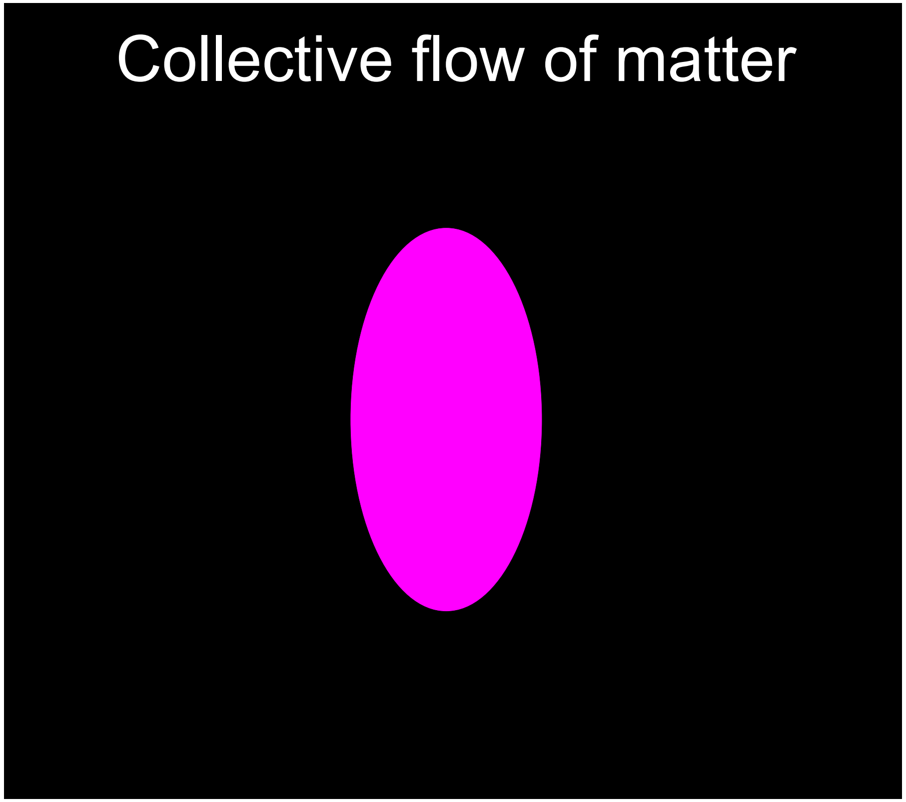
Overlap (participant) region is asymmetric in azimuthal angle

Define initial state spatial eccentricity:

$$\epsilon = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2}$$

How do we prove that we make “matter”?





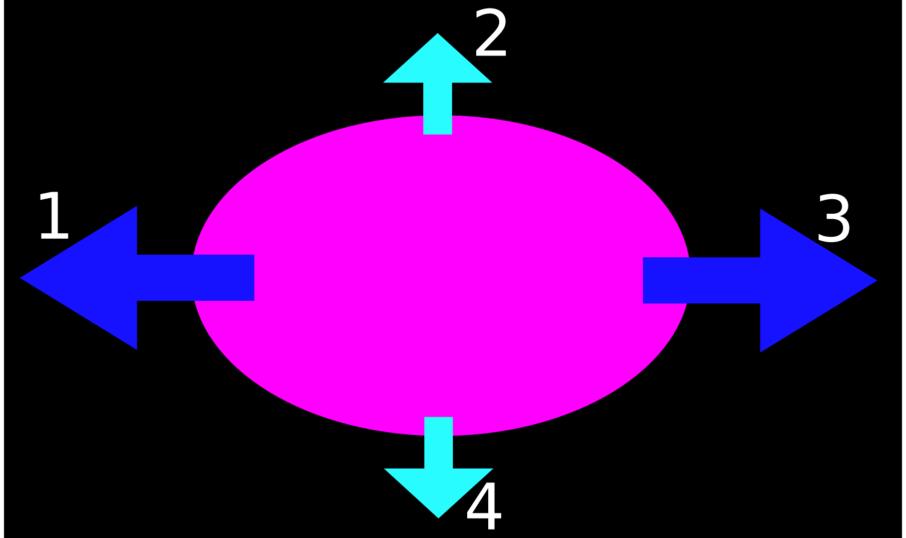
What happens to the shape (eccentricity) information during the expansion?

How do we prove that we make “matter”?

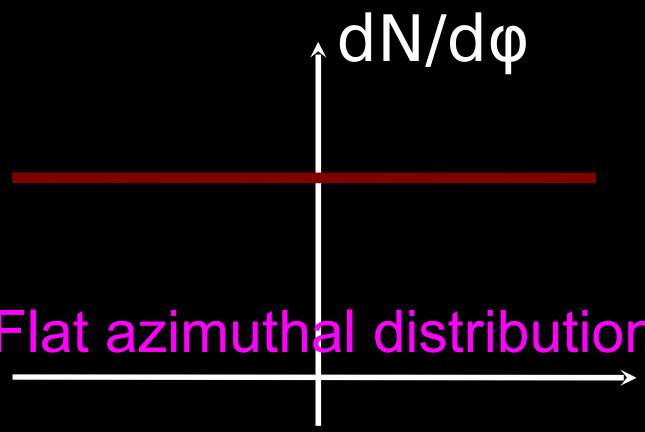
Non-interacting particles

Eccentricity information is not transfered to momentum space

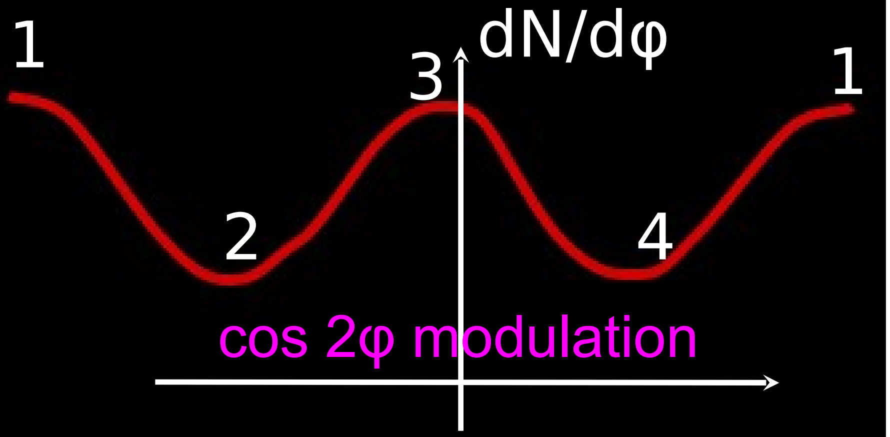
Collective flow of matter



Eccentricity information does get transfered into momentum space



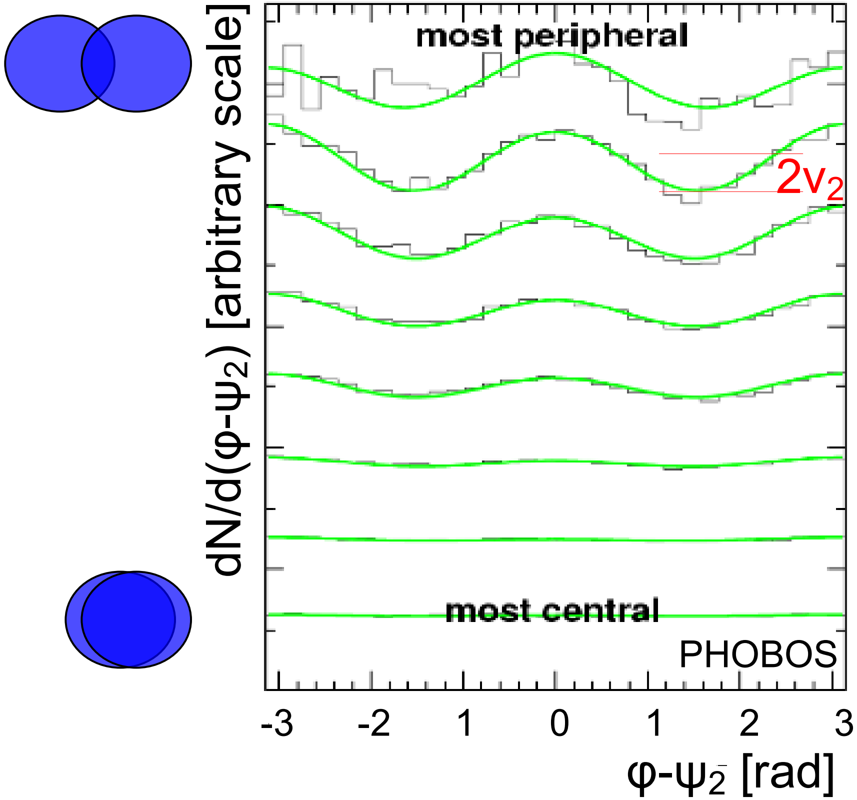
Flat azimuthal distribution



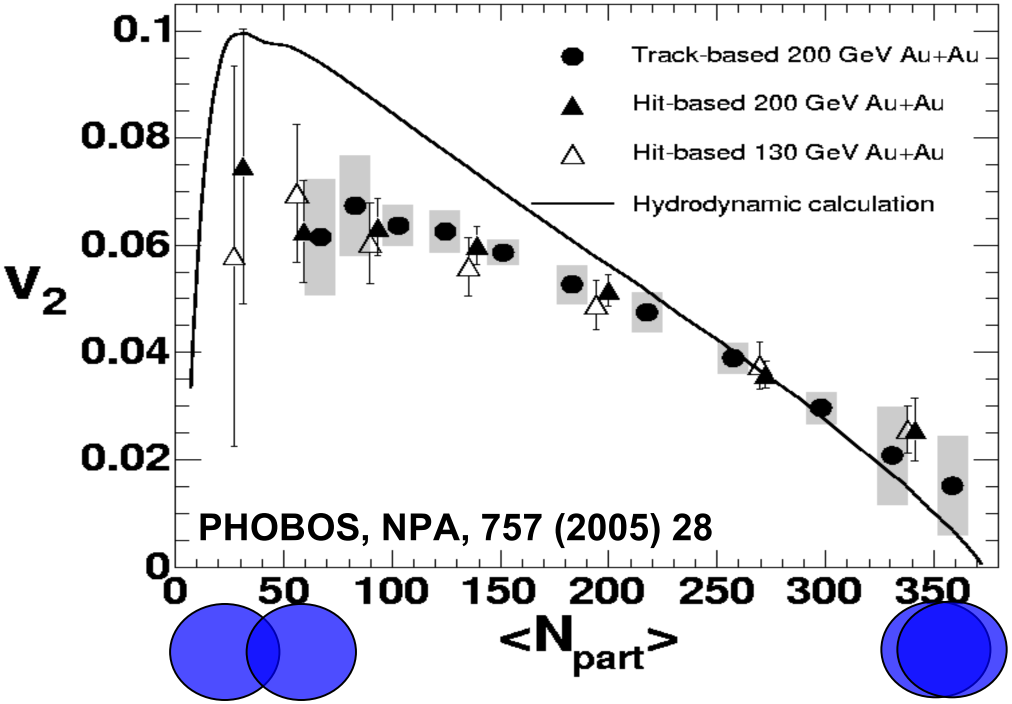
cos 2φ modulation

How do we prove that we make “matter”?

$$dN/d(\phi-\Psi_2) \propto 1+v_2\cos(2\phi-2\Psi_2)$$



Elliptic flow



Initial anisotropy in coordinate space is translated into momentum space: Interactions are present!

But what about equilibrium?

“...something more like a liquid...”

Ideal relativistic hydrodynamics

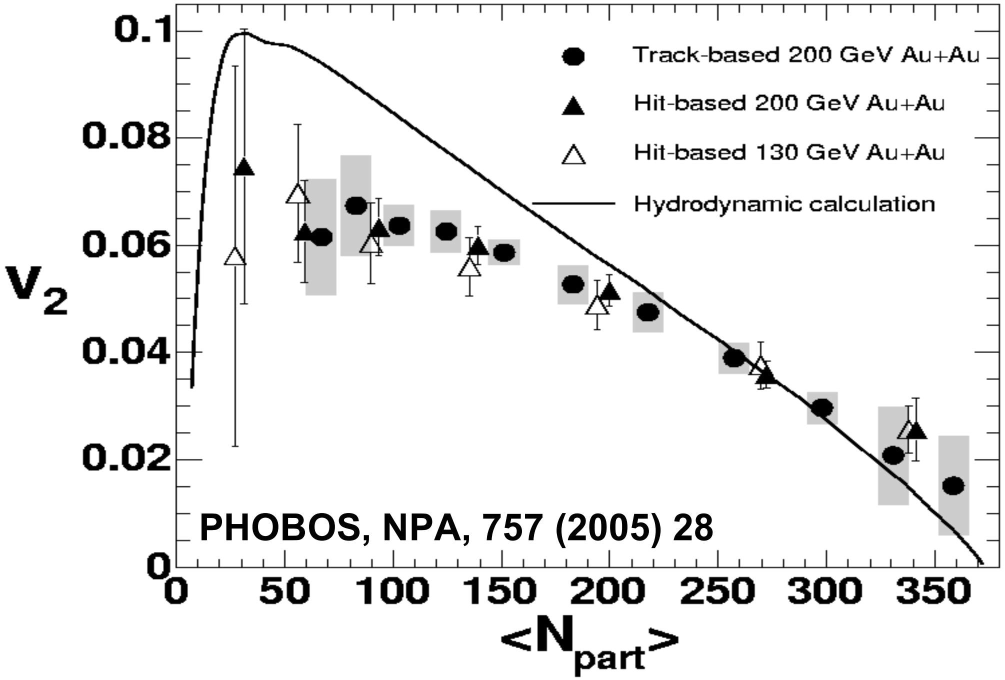
$$T^{\mu\nu}=(e+p)u^{\mu}u^{\nu}-p\,g^{\mu\nu}$$

$$\delta_{\mu}T^{\mu\nu}=0$$

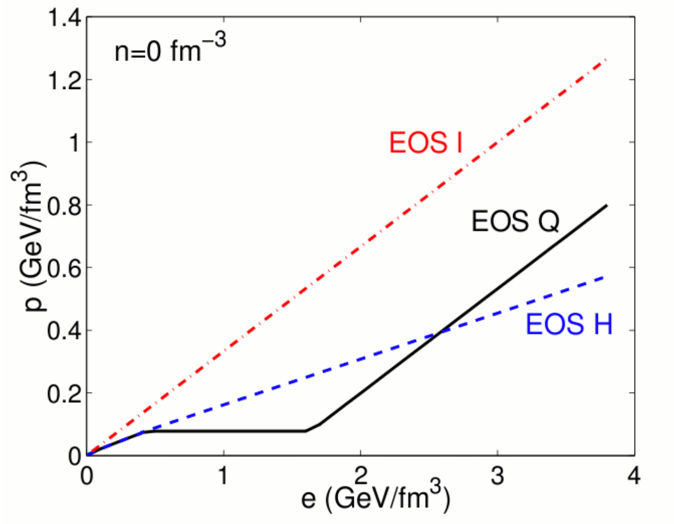
$$\delta_{\mu}N_{i}^{\mu}=0,\ i=B,S,\ldots$$

$N_{B}^{\mu}=n_{b}u^{\mu}$ Conserve net-baryon density

$p=p(e,n)$ Specify EoS



PHOBOS, NPA, 757 (2005) 28



Assumption: Shortly after the initial collision (<1-2fm/c) a system in local equilibrium with very small mean free path and shear viscosity is created.

For the first time in history of HI collisions: Mid-central data reach hydro-prediction!!!

Outline for remainder of the talk

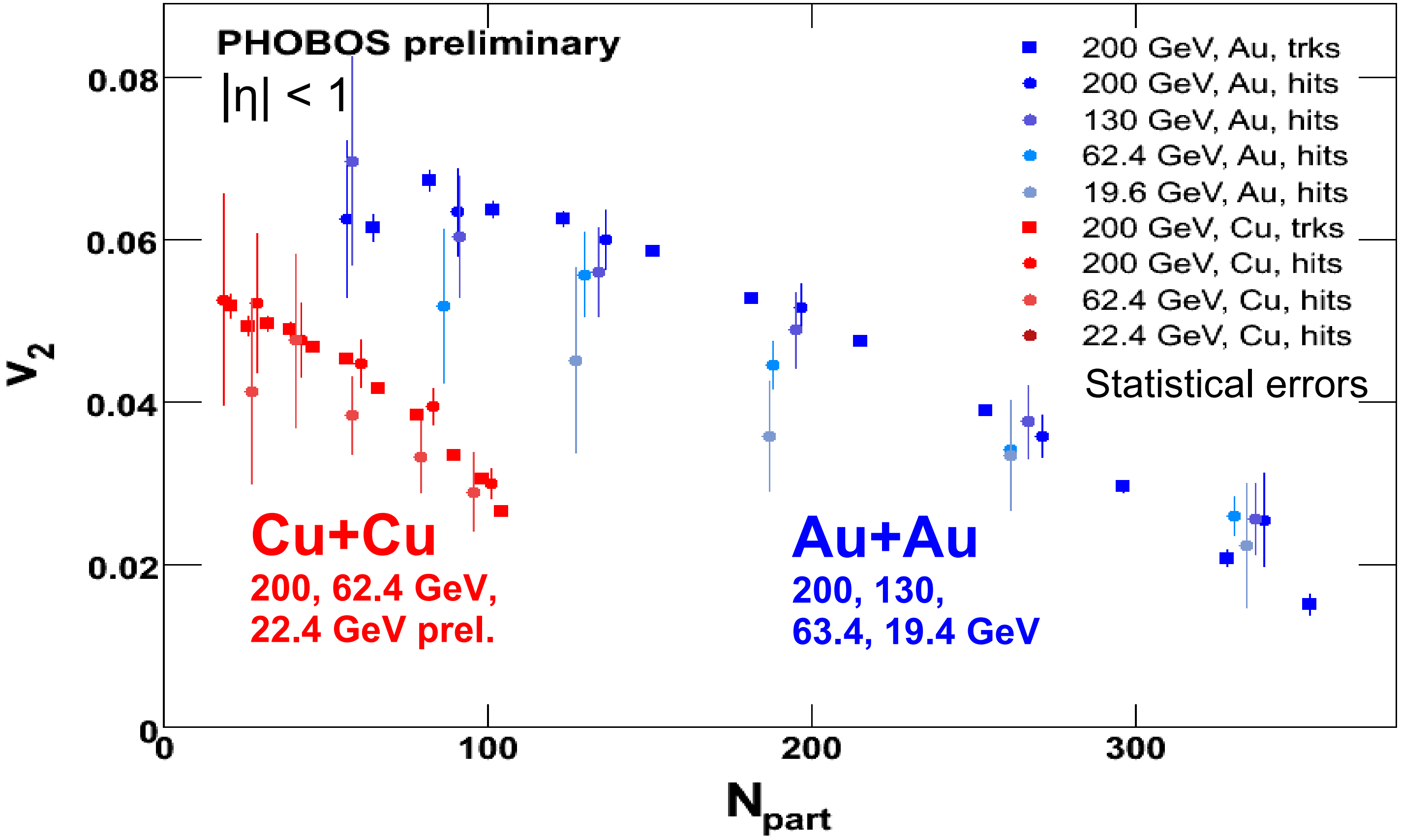
Initial conditions →
Equation of state →
Freeze-out conditions →

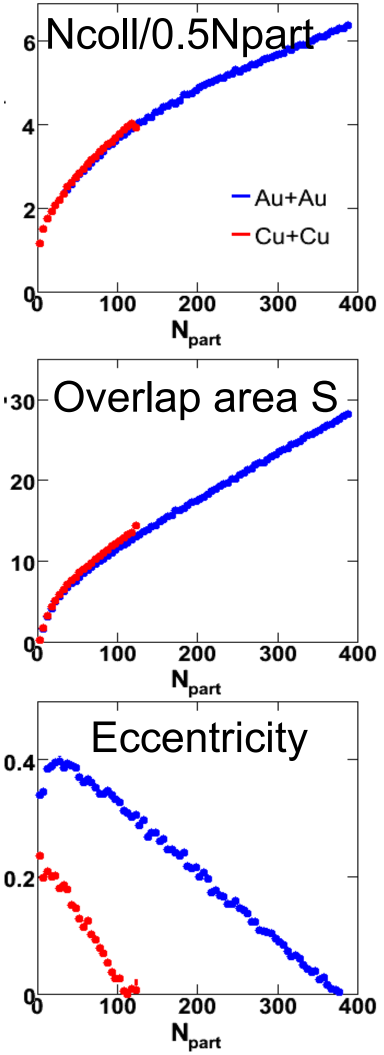
Hydro model →

Particle distributions/ observables to be checked experimentally

- Propose initial state eccentricity fluctuations to explain system comparison between Cu+Cu and Au+Au
- Predict eccentricity driven flow fluctuations
- Flow fluctuations exist with predicted magnitude

Elliptic flow and collision geometry (2)



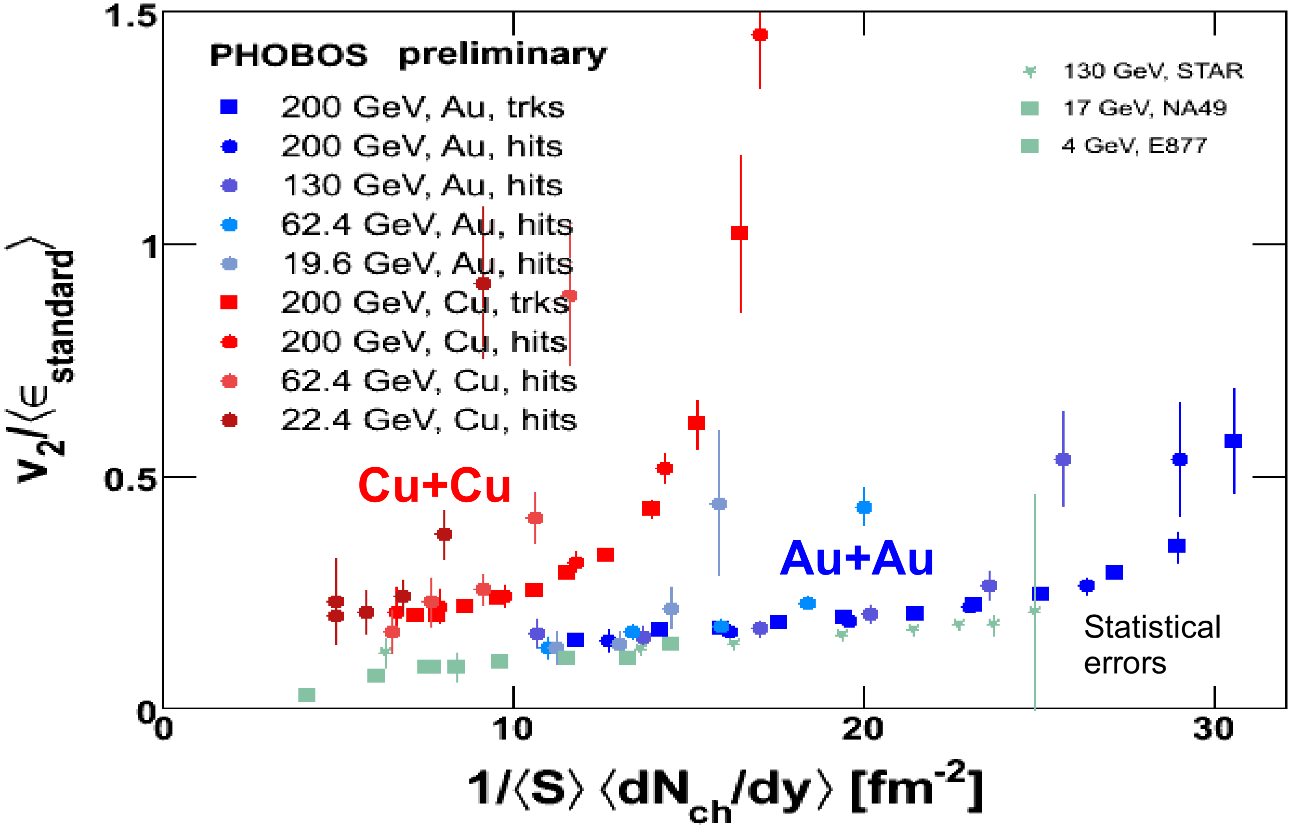


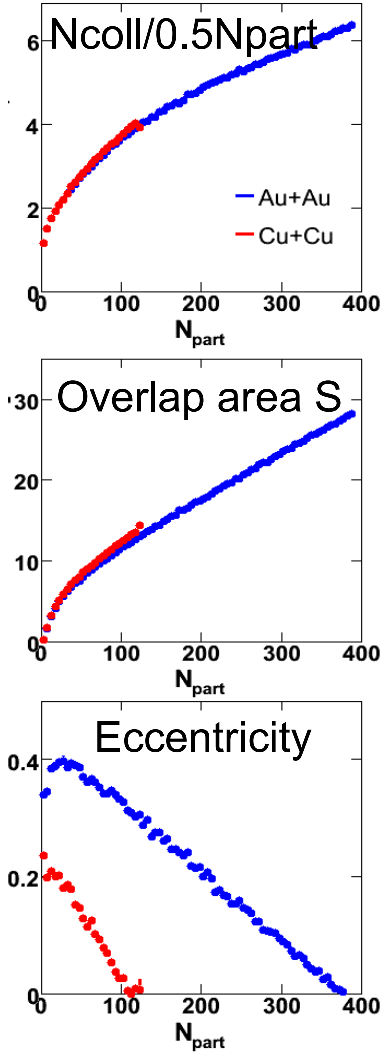
Generally expect: $v_2/\varepsilon = f(n,R)$, where n are part.density R trans.size at time when flow develops. When mean free path much smaller than R, $v_2/\varepsilon = f(n)$ only.

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (PRL in press)
Cu+Cu, 22.4 GeV: prel. QM06

Heiselberg, Levy, PRC 59 2716, (1999)
Voloshin, Poskanzer, PLB 474 27 (2000)
STAR, PRC 66 034904 (2002)

Elliptic flow and collision geometry (2)





No scaling between Cu+Cu and Au+Au

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
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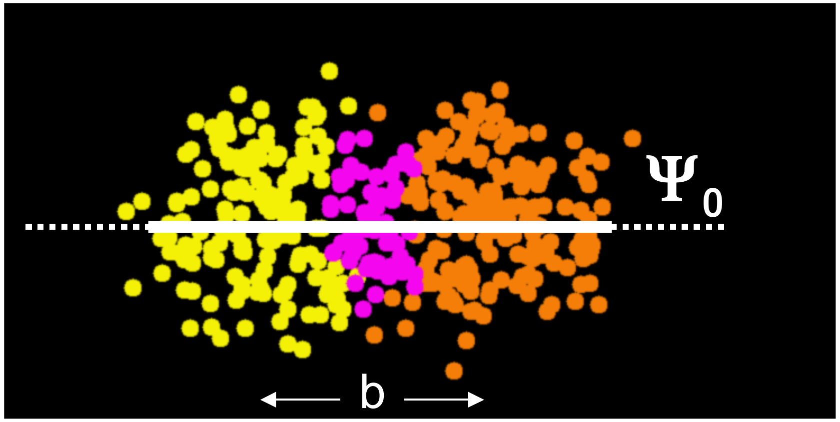
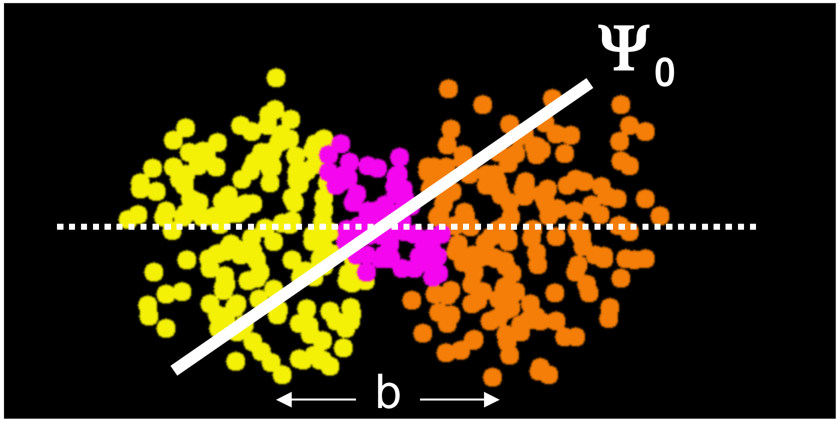
Participant eccentricity

The spatial distribution of
the interaction points of participating nucleons
for the same b will vary from event-to-event





Thus, the relevant eccentricity for
elliptic flow should vary event-by-event

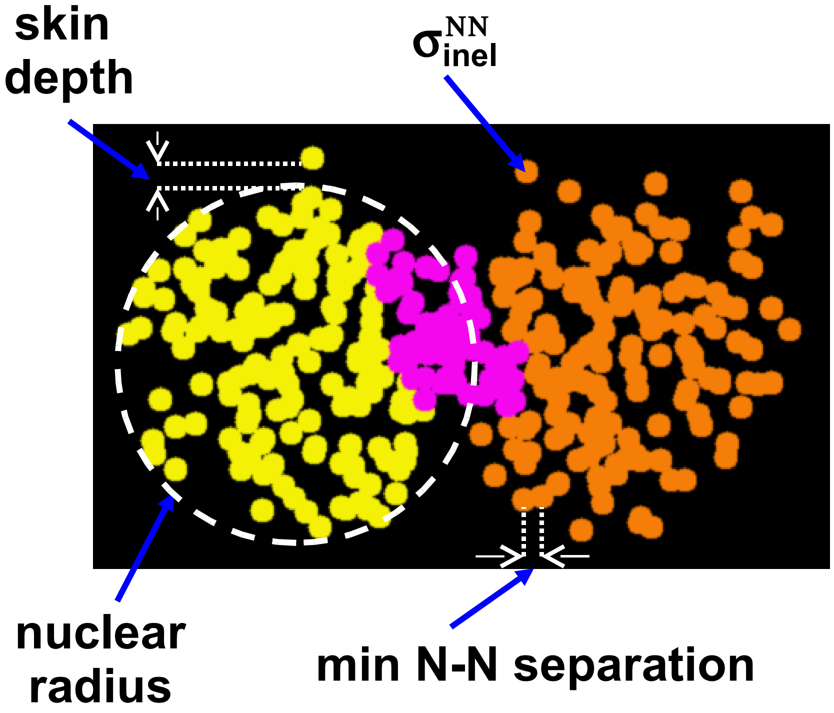
$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\boxed{\sigma_{xy}^2}}}{\sigma_y^2 + \sigma_x^2}$$

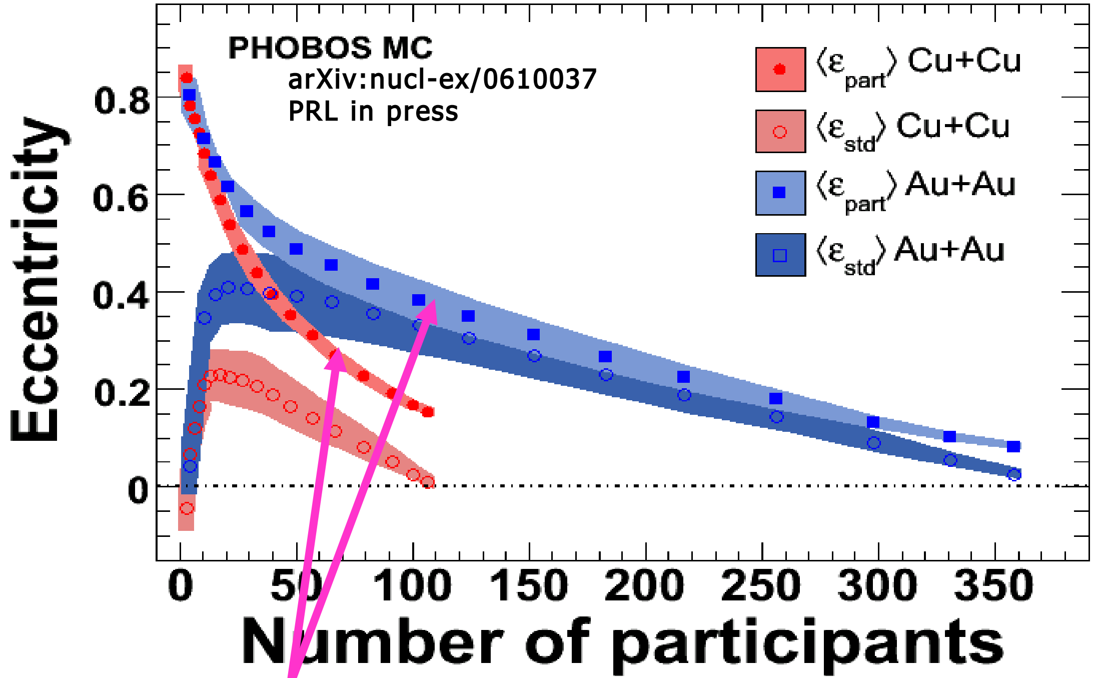
$$(0 < \epsilon_{part} \leq 1)$$

Introduced at QM05, nucl-ex/0610037 (PRL in press)

Comparison of eccentricity definitions

Studied variations to obtain 90% CL bands on calculation



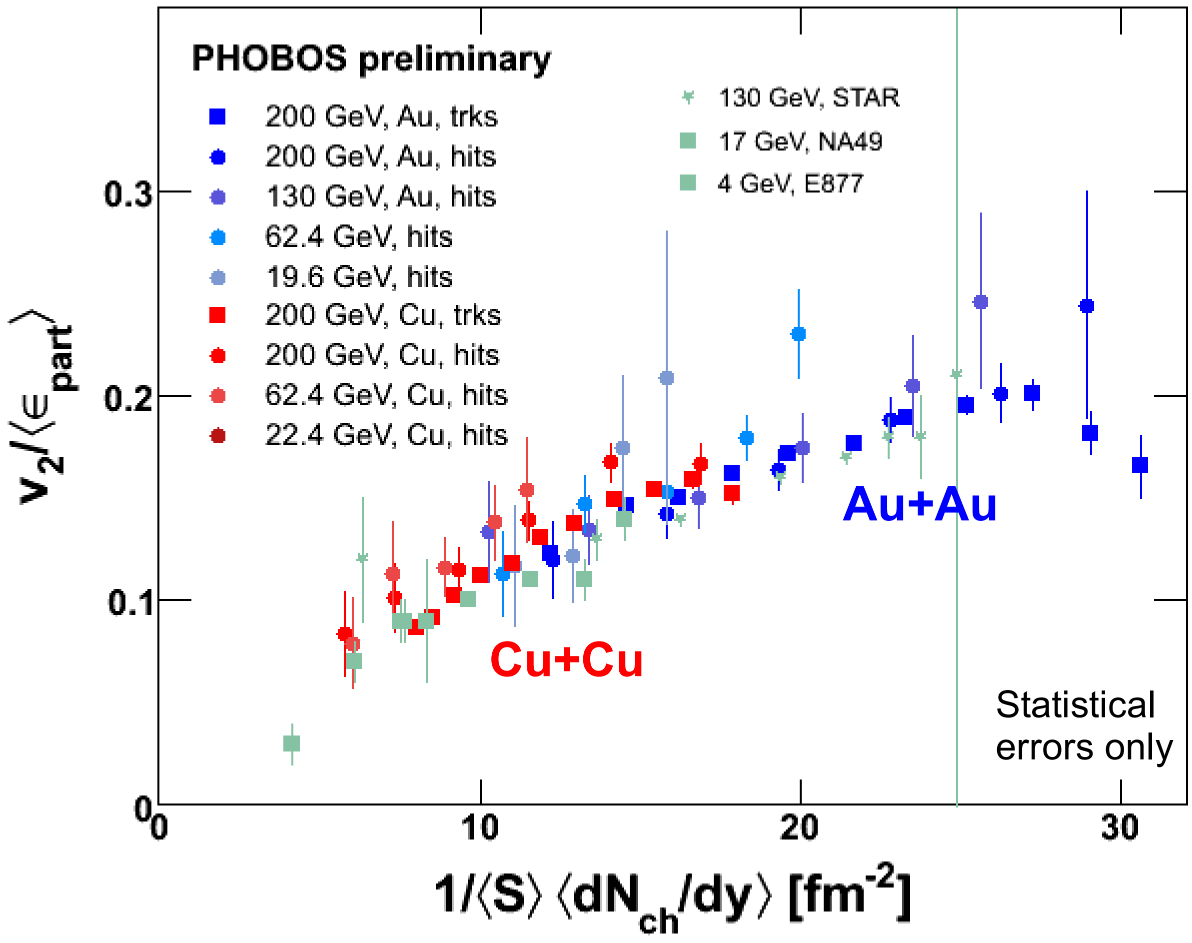


"Participant" eccentricity event-by-event calculation

Important for smaller systems

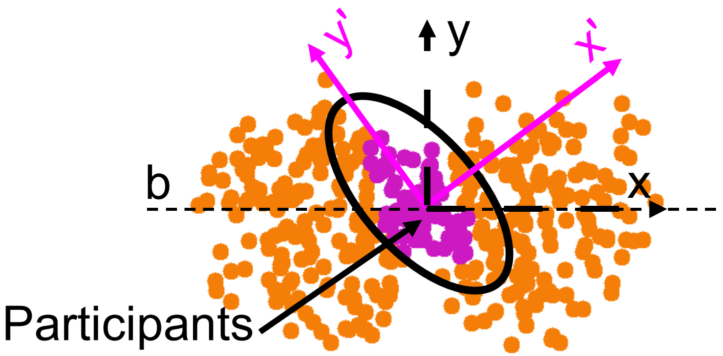
Introduced at QM05, nucl-ex/0610037 (PRL in press)

Elliptic flow and collision geometry (3)

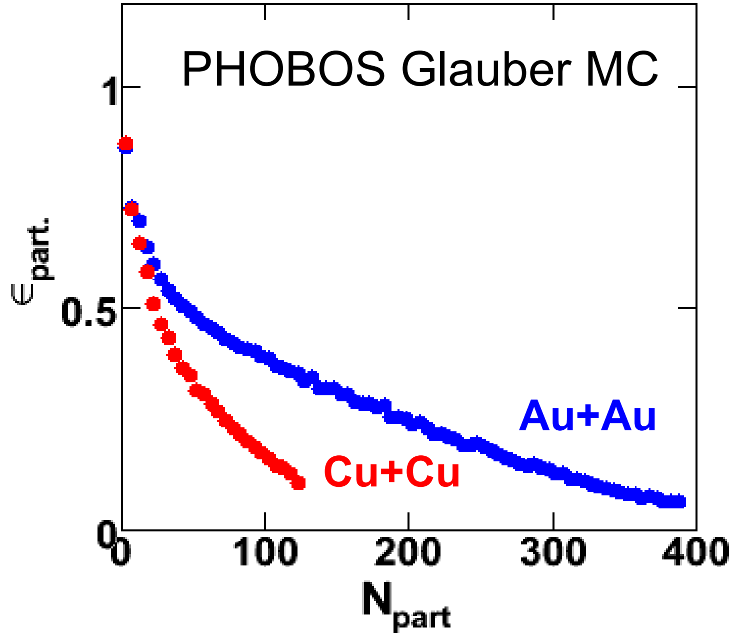


Scaling between Cu+Cu and Au+Au

Participant Eccentricity



$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2-\sigma_x^2)^2+4\boxed{\sigma_{xy}^2}}}{\sigma_y^2+\sigma_x^2}$$

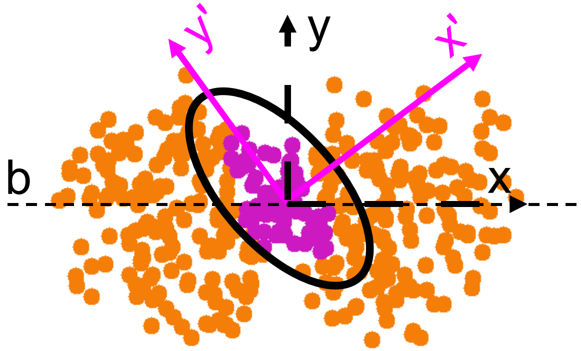


Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (PRL in press)
Cu+Cu, 22.4 GeV: prel. QM06

Eccentricity driven elliptic flow fluctuations?

Elliptic flow seems to be developed event-by-event with respect to the orientation of the overlap region

$$v_2 \sim \epsilon_{part}$$

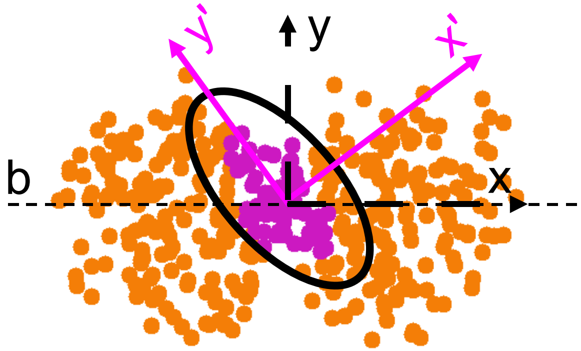


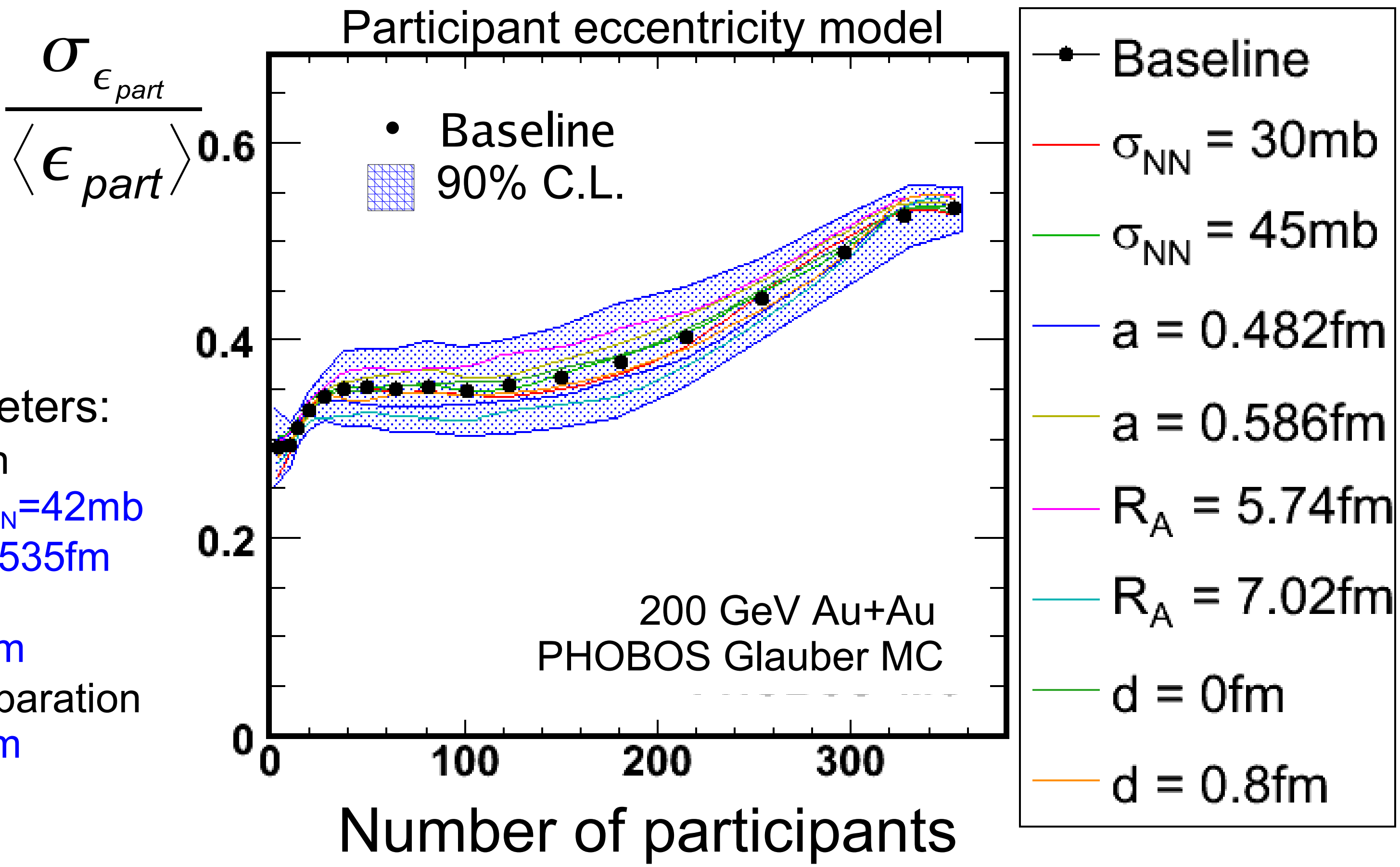
$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$

Expected relative elliptic flow fluctuations

Elliptic flow is developed event-by-event with respect to the overlap region

$$V_2 \sim \epsilon_{part}$$



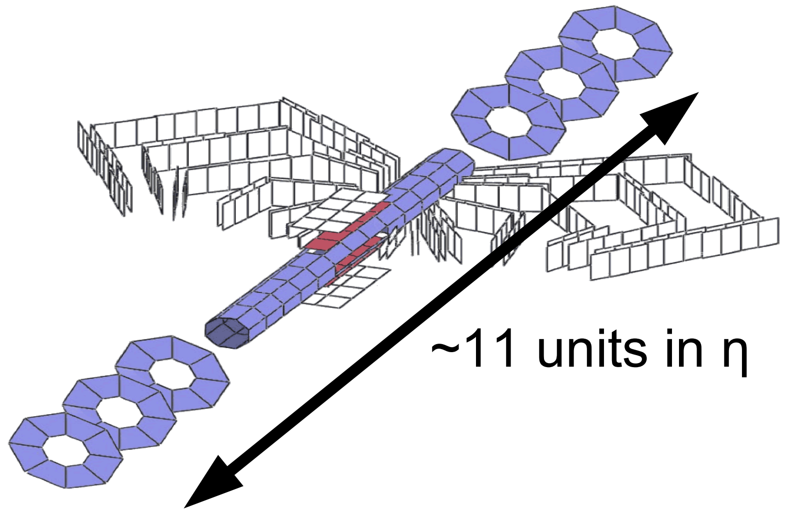


Baseline parameters:

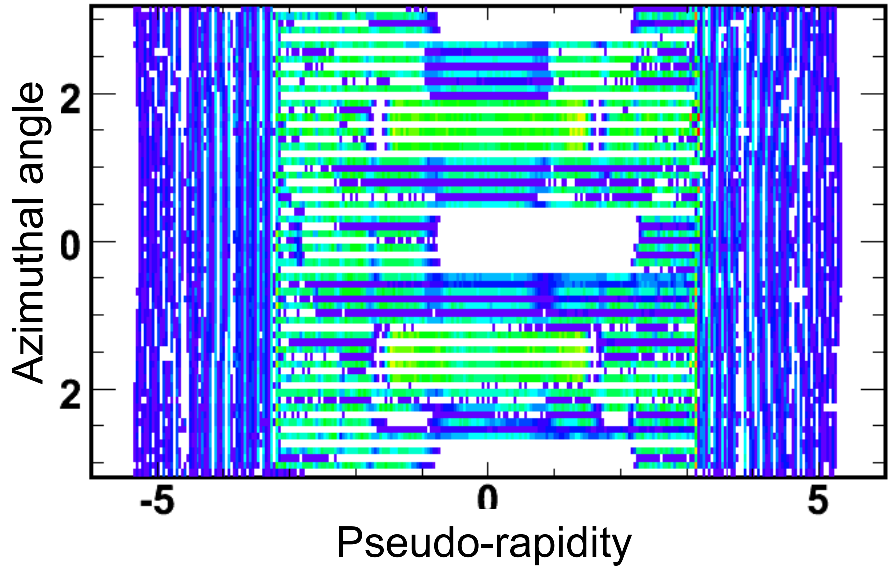
- Nucleon-nucleon cross section: σ_{NN}=42mb
- Skin depth: a=0.535fm
- Wood-saxon radius: R_A=6.38fm
- Inter-nucleon separation distance: d=0.4fm

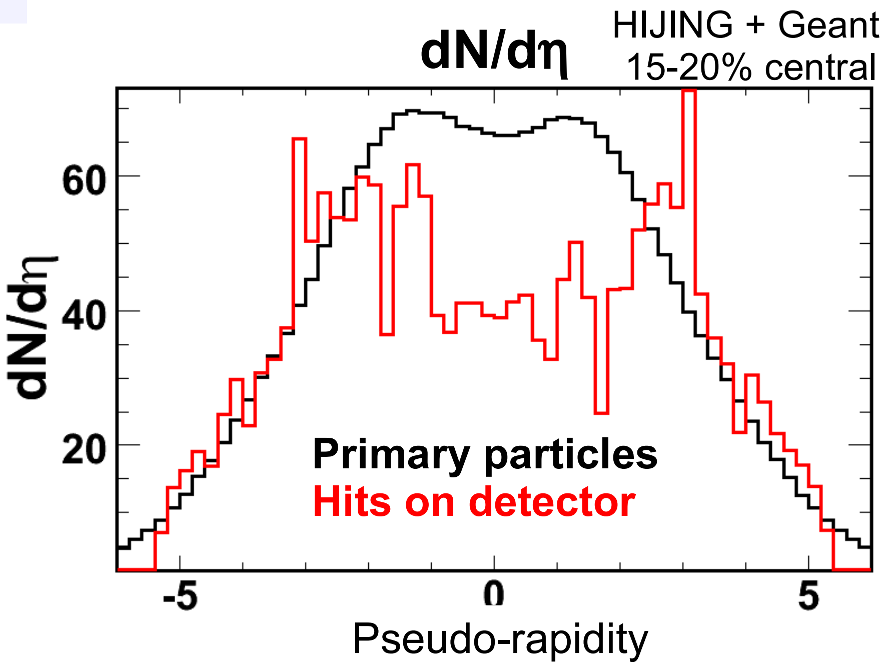
Challenges of event-by-event determination of v_2^{obs}

- PHOBOS Multiplicity Array
  - $-5.4<\eta<5.4$ coverage
  - Holes and granularity differences
- Usage of all available information
  in event to determine event-by-event a single value for v_2^{obs}



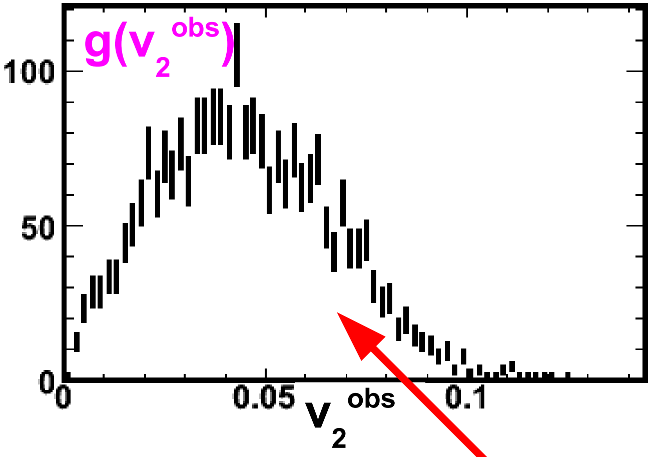
Hit Distribution



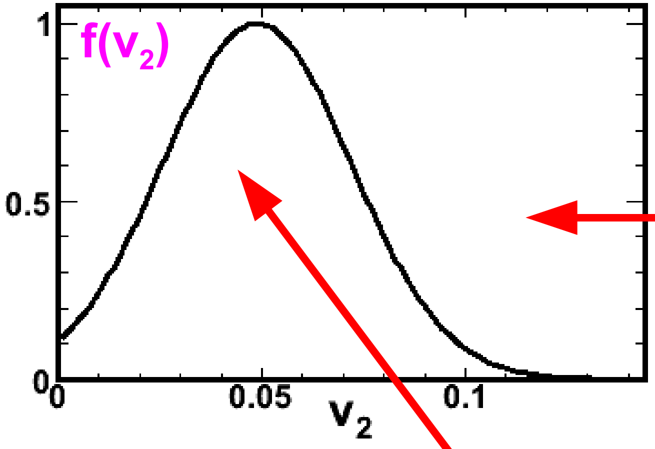


Measuring elliptic flow fluctuations

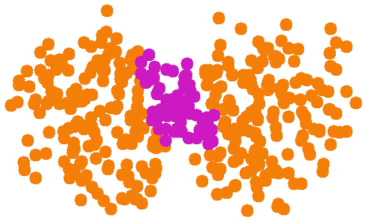
Observed v_2 distribution



True v_2 distribution



Source of v_2 fluctuation

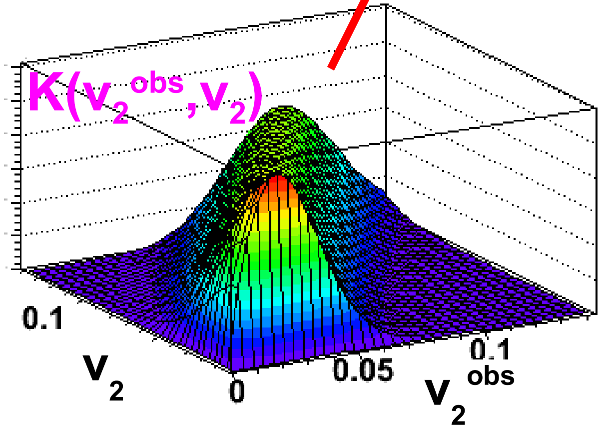


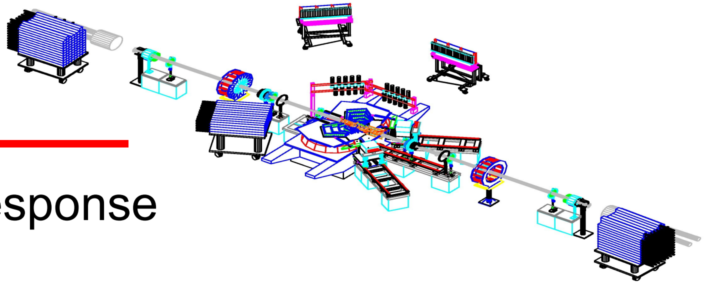
$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2) dv_2$$

Kernel

- Detector and acceptance effects
- Finite-number fluctuations
- Multiplicity fluctuations

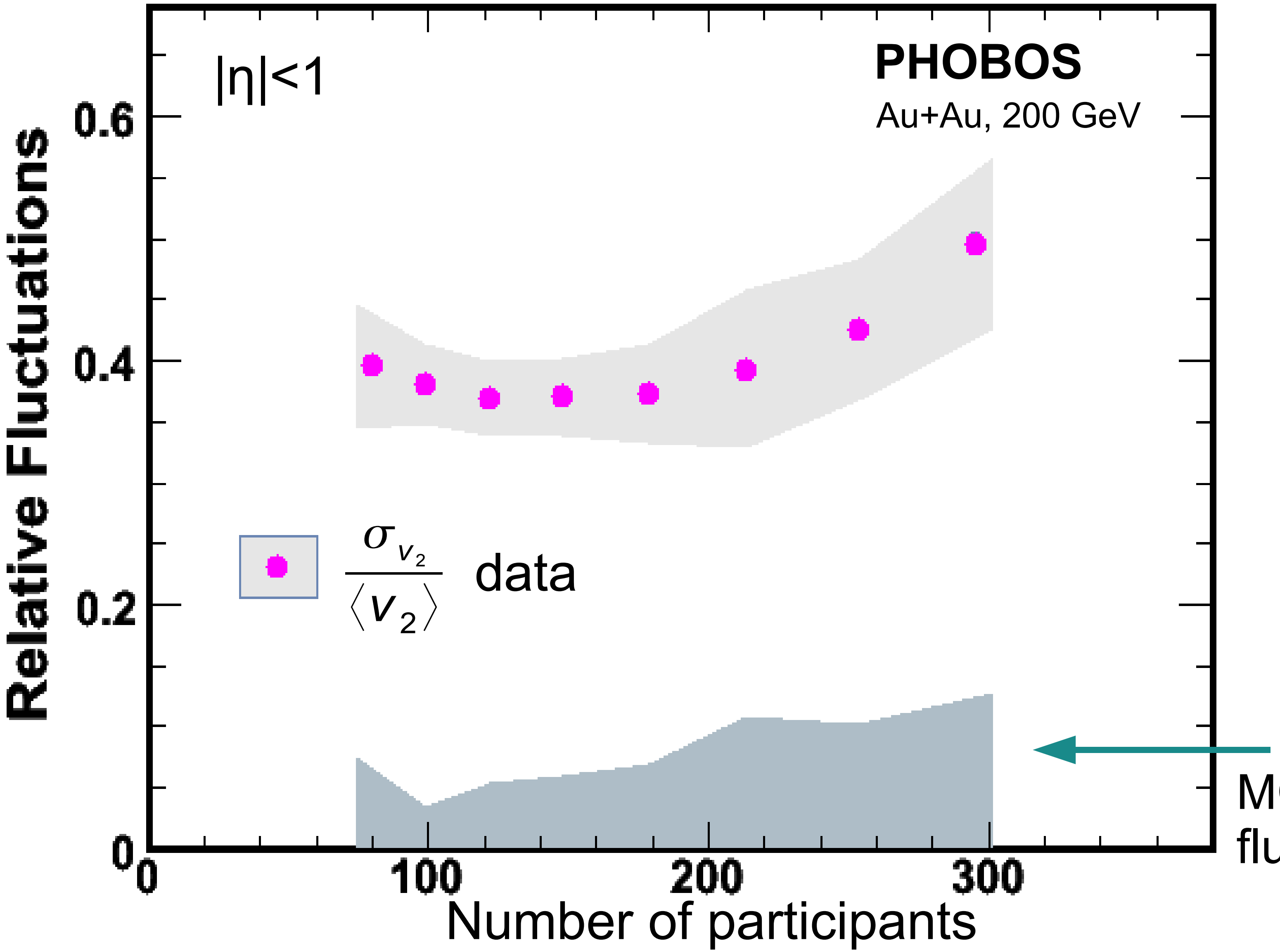
Kernel





Detector response

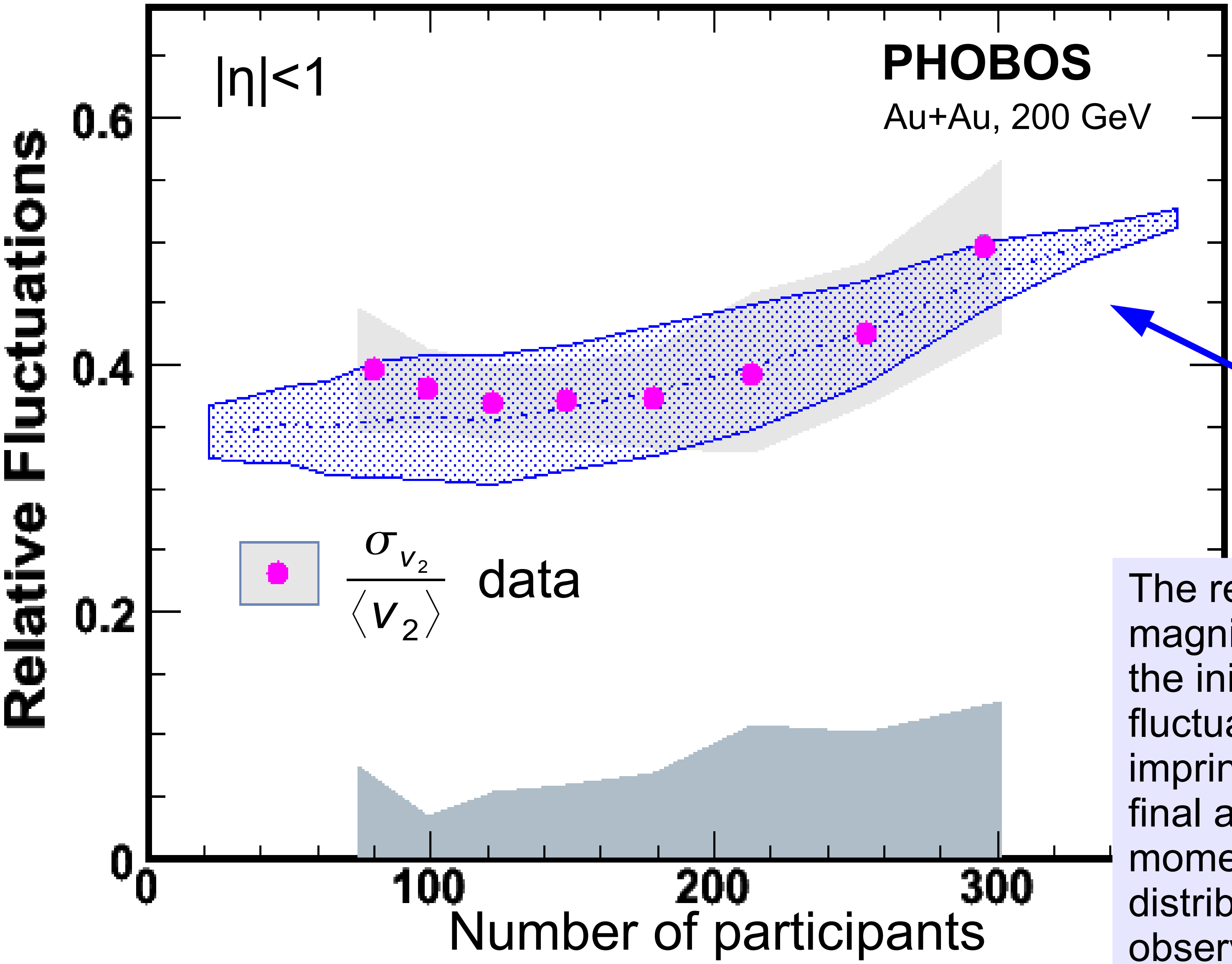
Relative elliptic flow fluctuations



MC with no fluctuations

nucl.-ex/0702036 (sub.to PRL)

Elliptic flow and collision geometry (4)



Participant eccentricity model prediction $\frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$

The relative magnitude of the initial state fluctuations are imprinted in the final azimuthal momentum distribution of the observed particles

nucl.-ex/0702036 (sub.to PRL)

Connection to Knudson and Reynolds number?

Define rel. flow fluctuations:

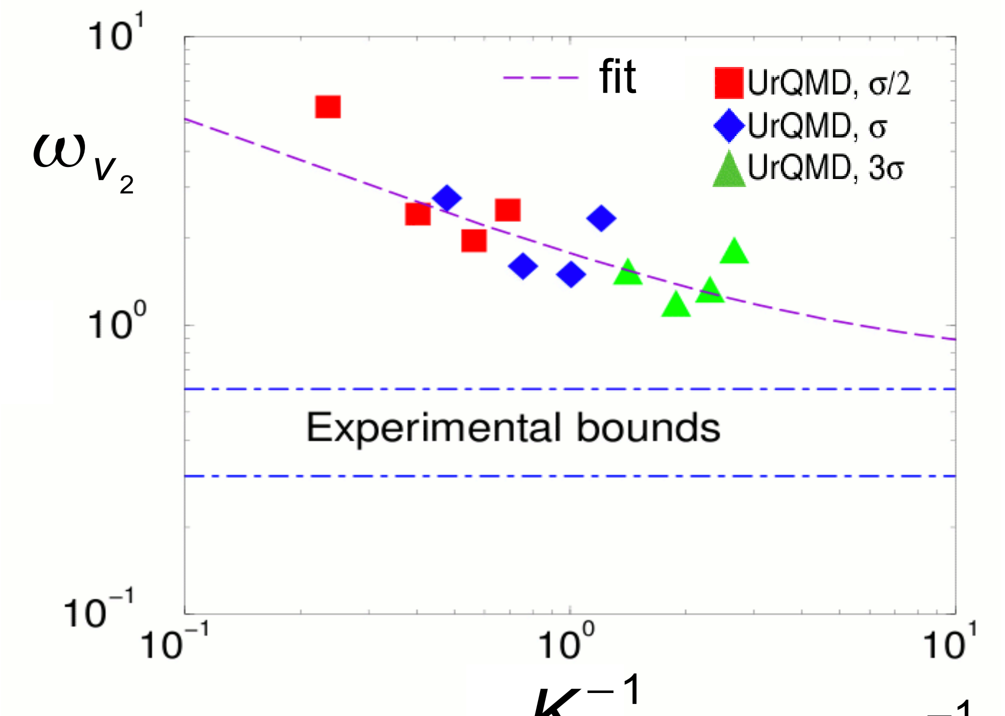
$$\omega_{v_2}^2 \equiv \frac{\sigma_{v_2}^2}{\langle v_2 \rangle^2} = \frac{\sigma_{\epsilon_{part}}^2}{\langle \epsilon_{part} \rangle^2} + \Delta_{dyn}^2$$

Define the inverse of the Knudson, the average number of collisions suffered by a dof in the system:

$$K_n^{-1} = L / \lambda$$

Assume Poissonian:

$$\Delta_{dyn} \sim \alpha \sqrt{K_n}$$



$$K_n^{-1} \sim R_e \sim \frac{\eta}{s}^{-1}$$

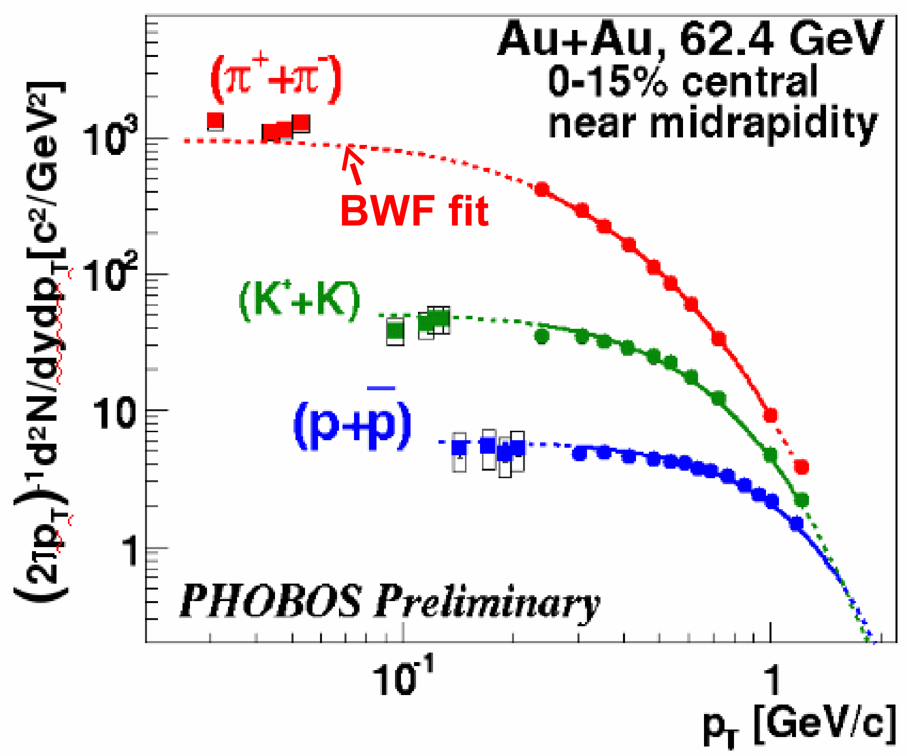
Conclusion/speculation(?) that viscosity must be large enough to avoid strong turbulences (that are not seen in the data)

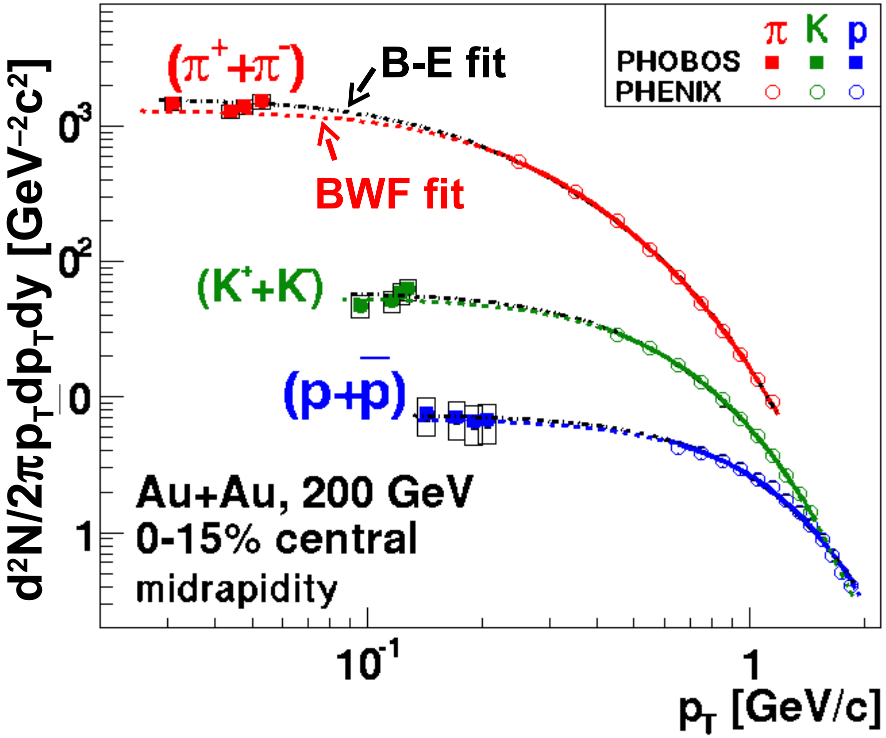
S.Vogel, G.Torrieri, M.Bleicher, nucl-th/0703031

Summary

- Heavy ion elliptic flow data at RHIC energies can be described by ideal hydrodynamics.
- Eccentricity fluctuations describe the eccentricity-scaled elliptic flow data across the Cu+Cu and Au+Au systems.
- We have measured the predicted elliptic flow fluctuations and found a relative magnitude of 40%.
  - The participant eccentricity predictions from a simple Glauber MC simulation, where the nucleon interaction points are interpreted event-by-event, are in striking agreement with the data.
- The postulated fluctuations in eccentricity are real and provide new insight in the initial conditions. Together with the measured flow fluctuation we may gain new access to properties of the liquid.

Properties of the medium





In a large volume + weakly interacting system, one expects the development of particles with long wavelengths.

PHOBOS WhitePaper

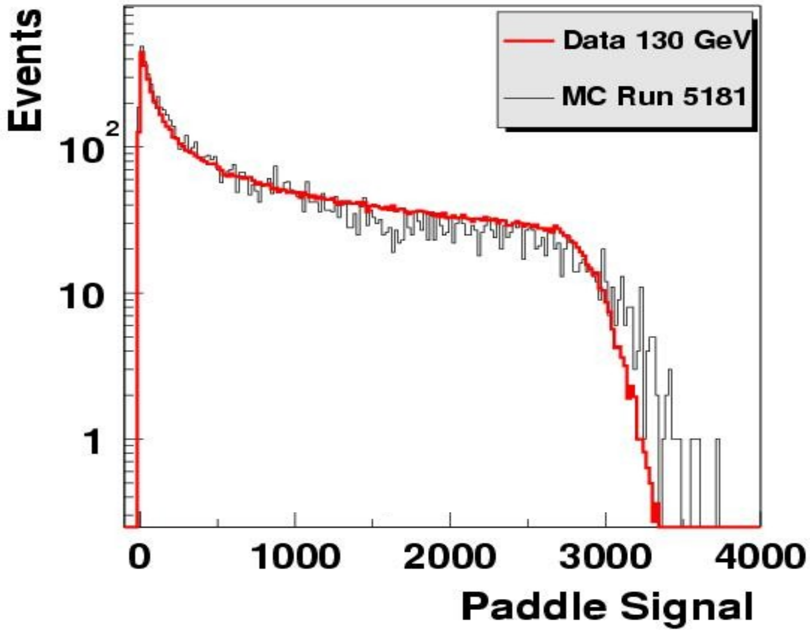
No evidence of enhanced particle production at very low p_T

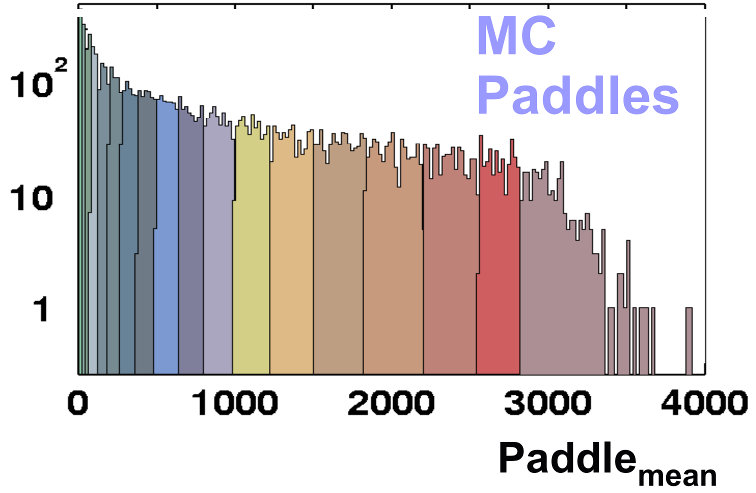
constraints $\longrightarrow$ <E>

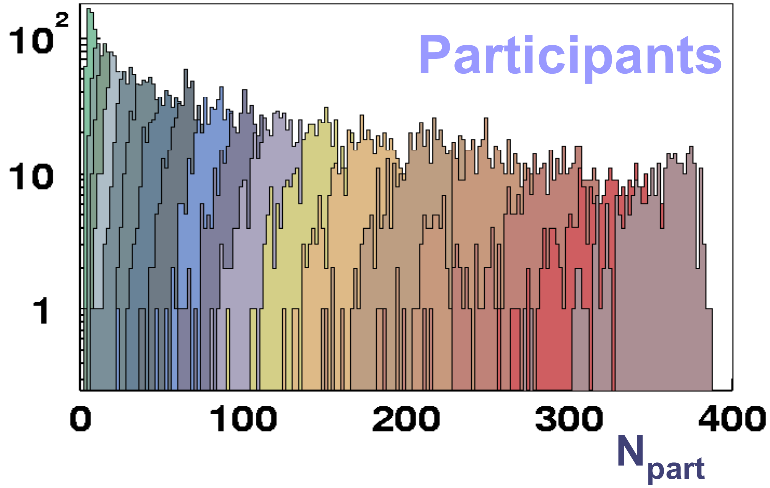
200 GeV PHOBOS: PRC 70, 051901 (R) (2004)
200 GeV PHENIX: PRC 69, 034909 (2004)
62.4 GeV PHOBOS prel. : QM05, nucl-ex/0510039

For details and fit params, see G.Veres, SQM'06

Relate centrality to data







Flow measurement in PHOBOS

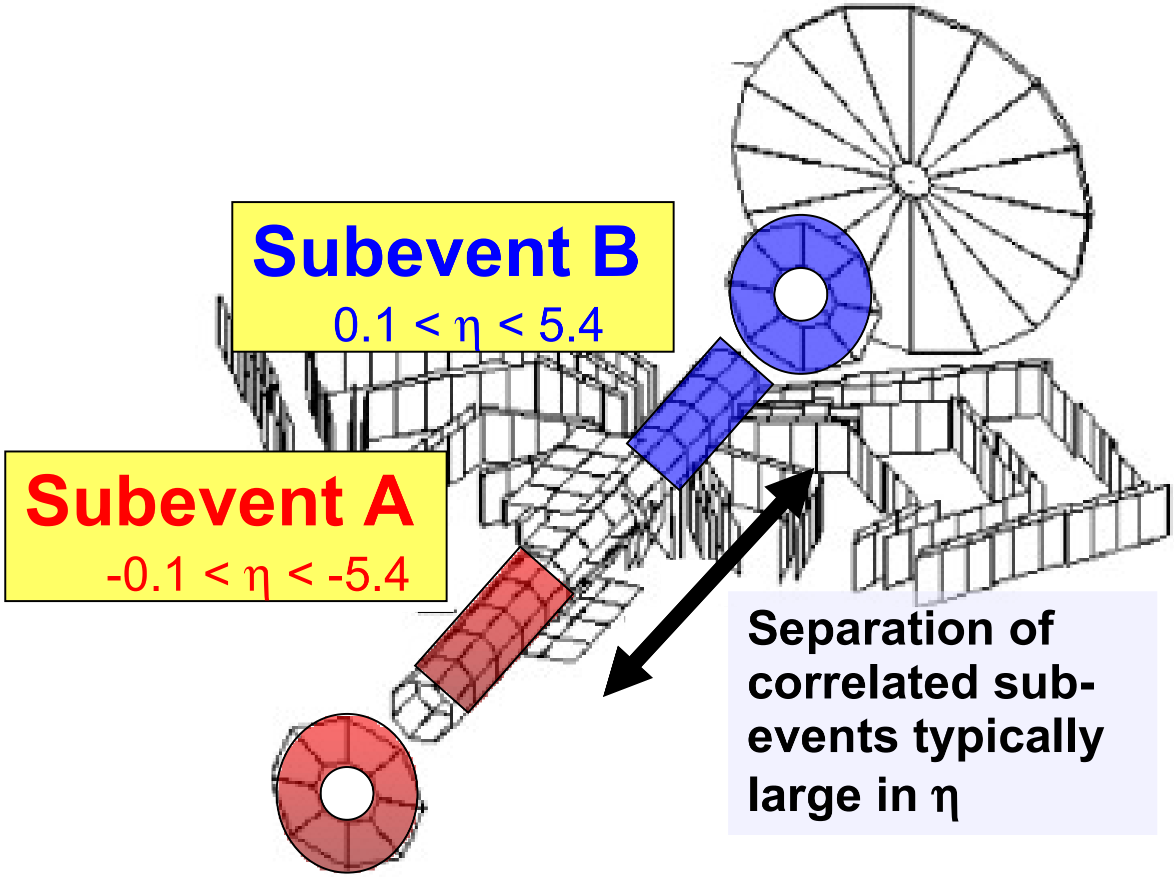
- Reaction-plane / Subevent technique
  - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts of the detector

$$\tan(2\psi_A) = \frac{\langle \sin(2\phi) \rangle_A}{\langle \cos(2\phi) \rangle_A}$$

$$v_2^{obs} = \langle \cos(2\phi - 2\psi_A) \rangle_B$$

$$v_2 = \frac{\langle v_2^{obs} \rangle_{events}}{\sqrt{\langle \cos(2\psi_A - 2\psi_B) \rangle_{events}}}$$

A.Poskanzer, S.Voloshin, nucl-ex/9805001


Subevent B

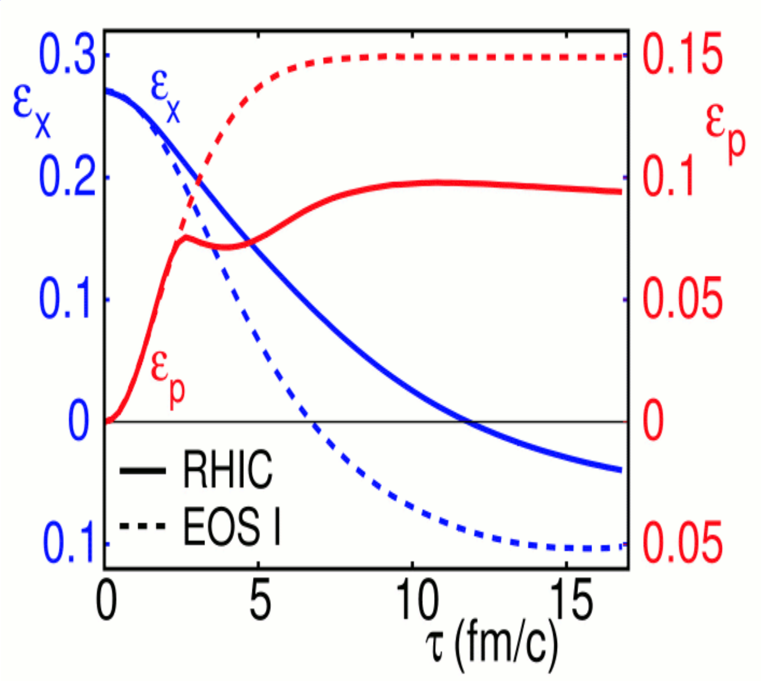
$0.1 < \eta < 5.4$

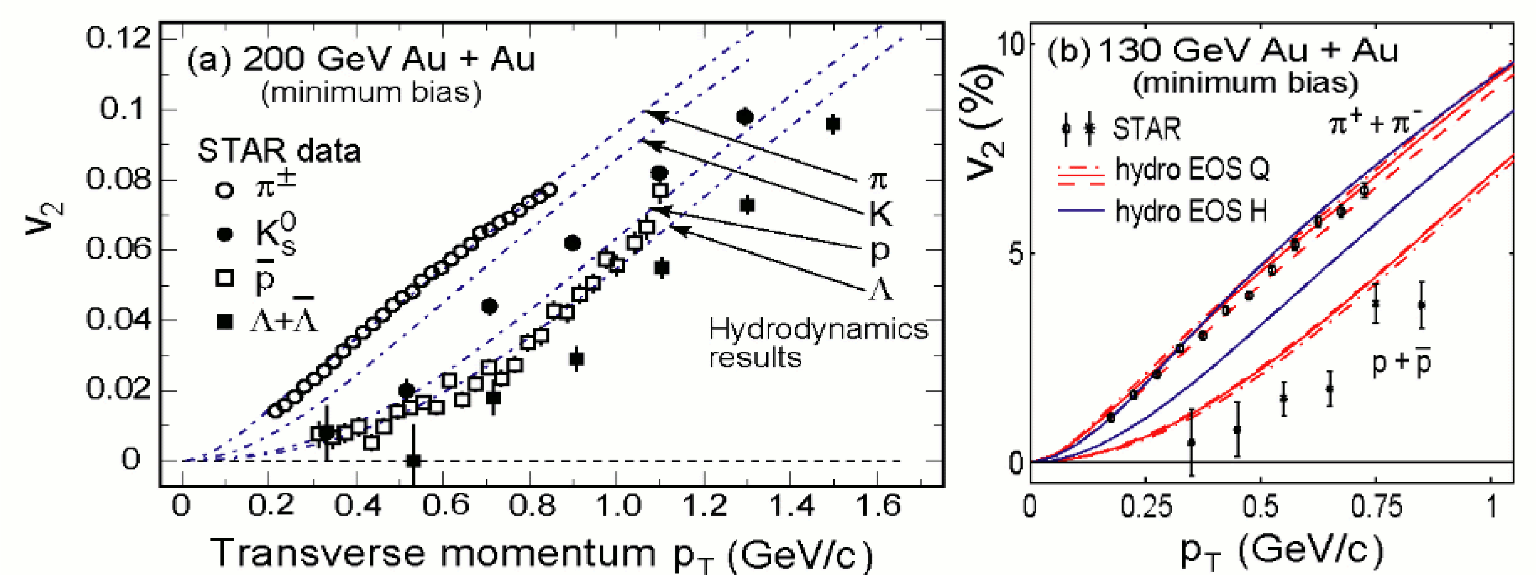
Subevent A

$-0.1 < \eta < -5.4$

Separation of correlated sub-events typically large in η

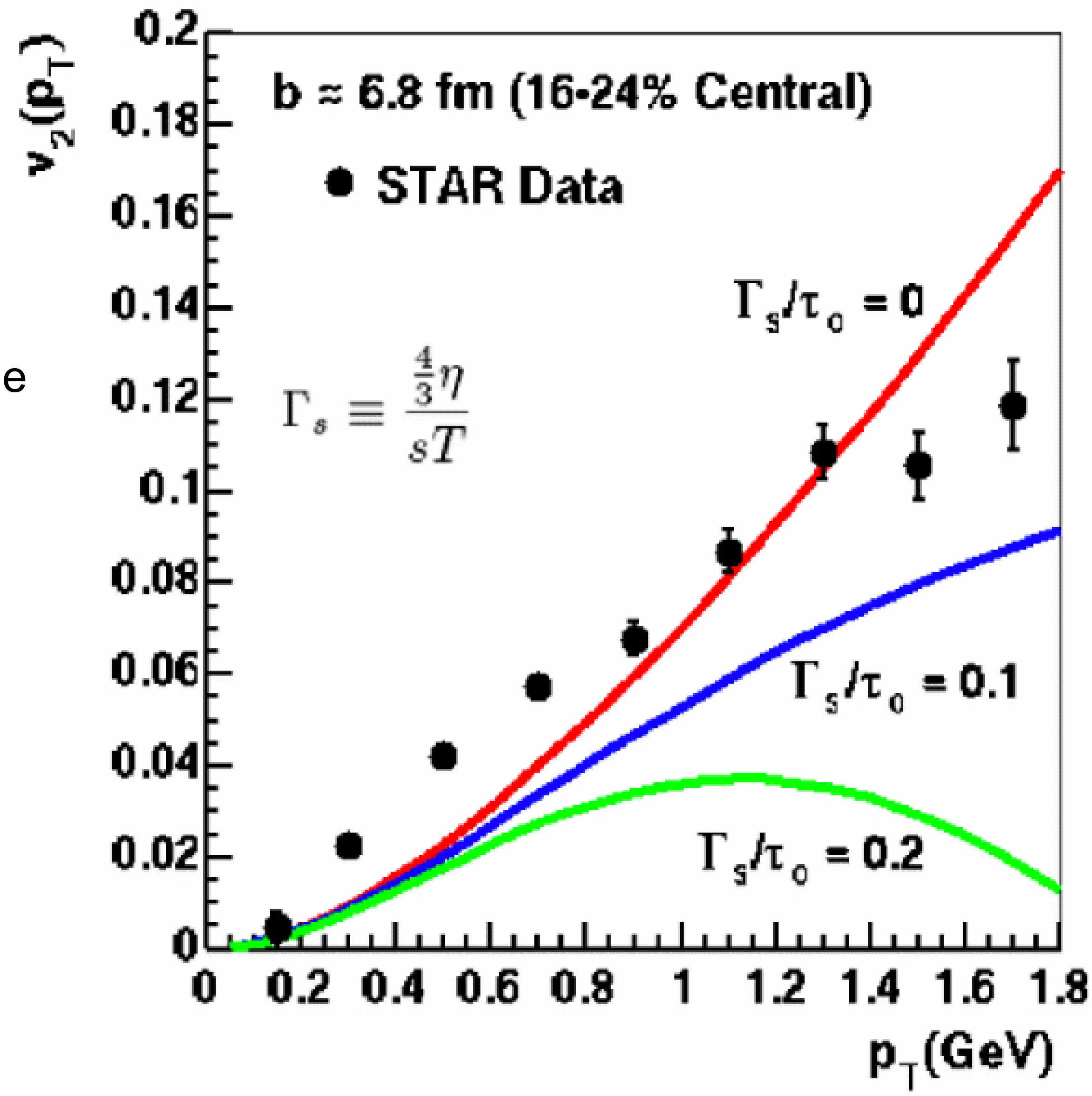
Self quenching and hydro success





Viscous corrections

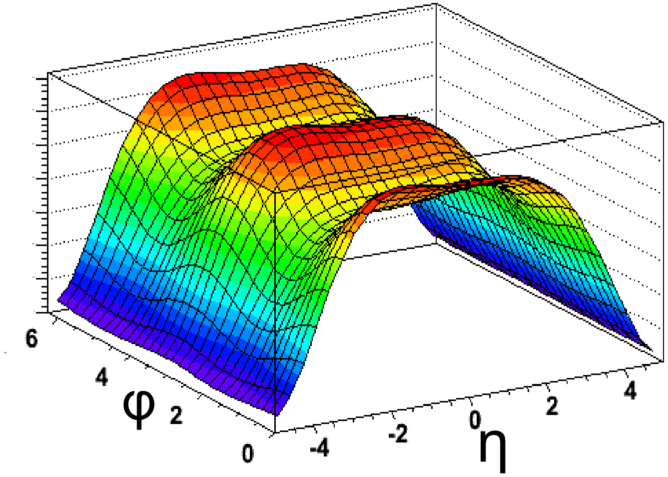
Sound attenuation length is approx the same as the mean free path



Event-by-event measurement of v_2^{obs}

- Event-by-event measurement of v_2^{obs}
  - Deal with acceptance effects
  - Use all available hit information

Probability distribution function



- Probability distribution function for hit positions:

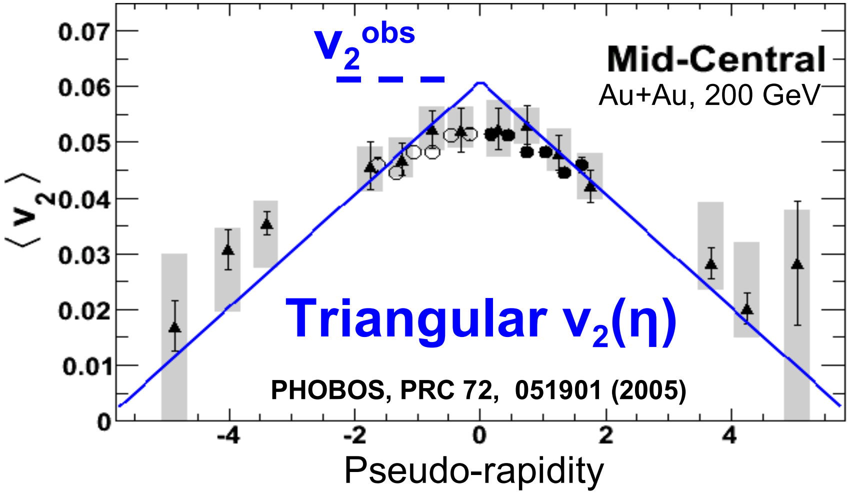
$$P(\eta,\phi;\ v_2^{obs},\phi_0)=\underbrace{p(\eta)}_{\text{Normalization incl. acceptance}}\underbrace{[1+2v_2(\eta)\cos(2\phi-2\phi_0)]}_{\text{Probability of hit in }(\phi,\eta)}$$

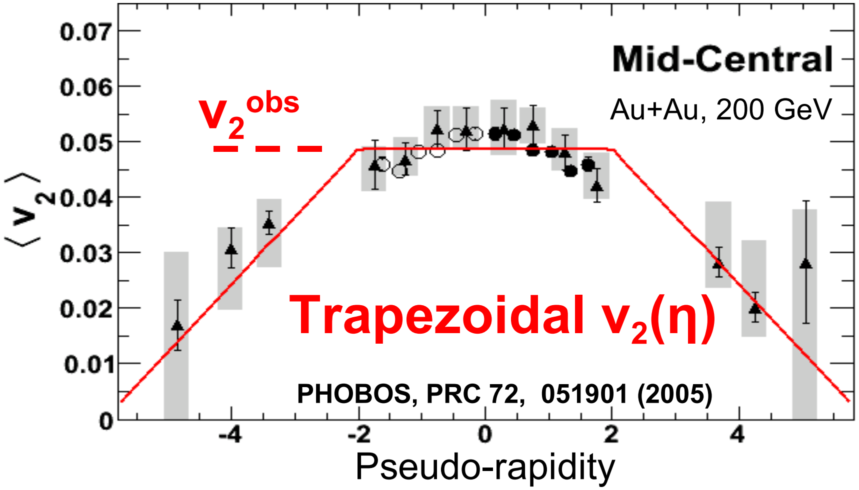
  - Maximize the likelihood function to obtain v_2^{obs} and φ^0 (event plane angle)

$$L(v_2^{obs},\ \phi_0)=\prod_{i=1}^{n}P(\eta_i,\phi_i;\ v_2^{obs},\phi_0)$$

nucl.-ex/0702036 (sub.to PRL), nucl-ex/0608025 (Proceedings of Science)

Event-by-event measurement of v_2^{obs}





$$P(\eta,\phi;\, v_2^{obs},\phi_0)=p(\eta)[1+2\underline{v_2(\eta)}\cos(2\phi-2\phi_0)]$$

$\uparrow$

Use known, measured shape

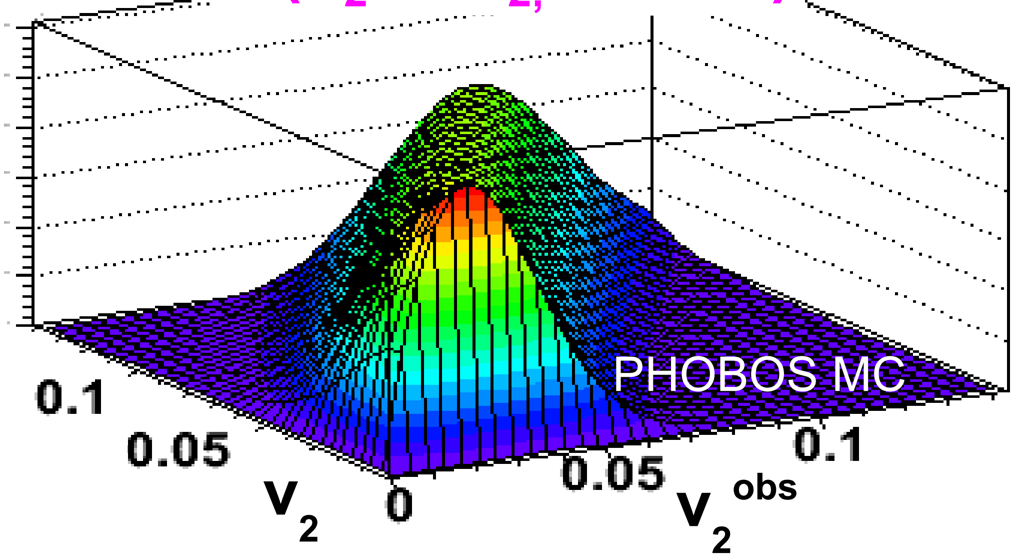
Analysis is run completely independent on triangular and trapezoidal shape. Results are averaged at the end.

nucl.-ex/0702036 (sub.to PRL),
nucl-ex/0608025 (Proceedings of Science)

Determining the kernel

- “Measure” and record the v_2^{obs} distribution in bins of v_2 and multiplicity (n) from large MC samples
    - $1.5 \cdot 10^6$ HIJING events
    - Modified φ to include triangular or trapezoidal flow

$K(v_2^{obs}, v_2,$ fixed n)



- Fit response function (ideal case)

$$K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma^2} e^{-\left(\frac{v_2^{obs}+v_2^2}{2\sigma^2}\right)} I_0\left(\frac{-v_2^{obs} v_2}{\sigma^2}\right)$$

(J.-Y.Ollitrault, PRD (1992) 46, 226)

- Changed to account for detector effects

$$v_2 \rightarrow (An+B)v_2 \qquad \sigma = \frac{C}{\sqrt{n}} + D$$

(suppression) (finite resolution)

nucl.-ex/0702036 (sub.to PRL),
nucl-ex/0608025 (Proceedings of Science)

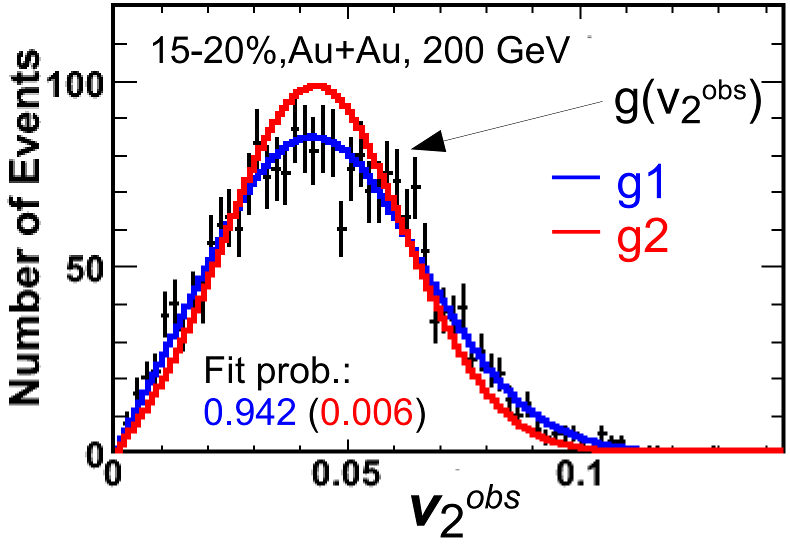
Extracting dynamical fluctuations

$$\underbrace{g(v_2^{obs})}_{\text{Measured}} = \int_0^1 \underbrace{K(v_2^{obs}, v_2)}_{\text{Constructed from MC}} \underline{f(v_2)}\, dv_2$$

Gaussian Ansatz:

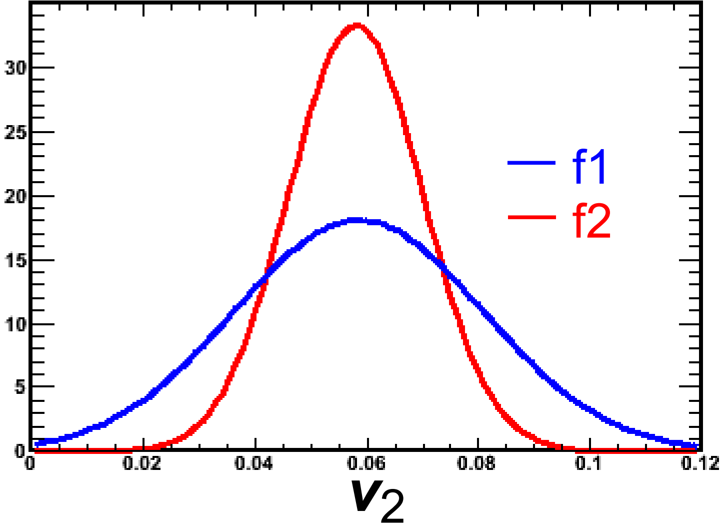
$$f(v_2) = \exp\left[\frac{-(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v_2}^2}\right]$$

Comparison with data $g(v_2^{obs})$



Use kernel + integrate

Different trials for Ansatz $f(v_2)$



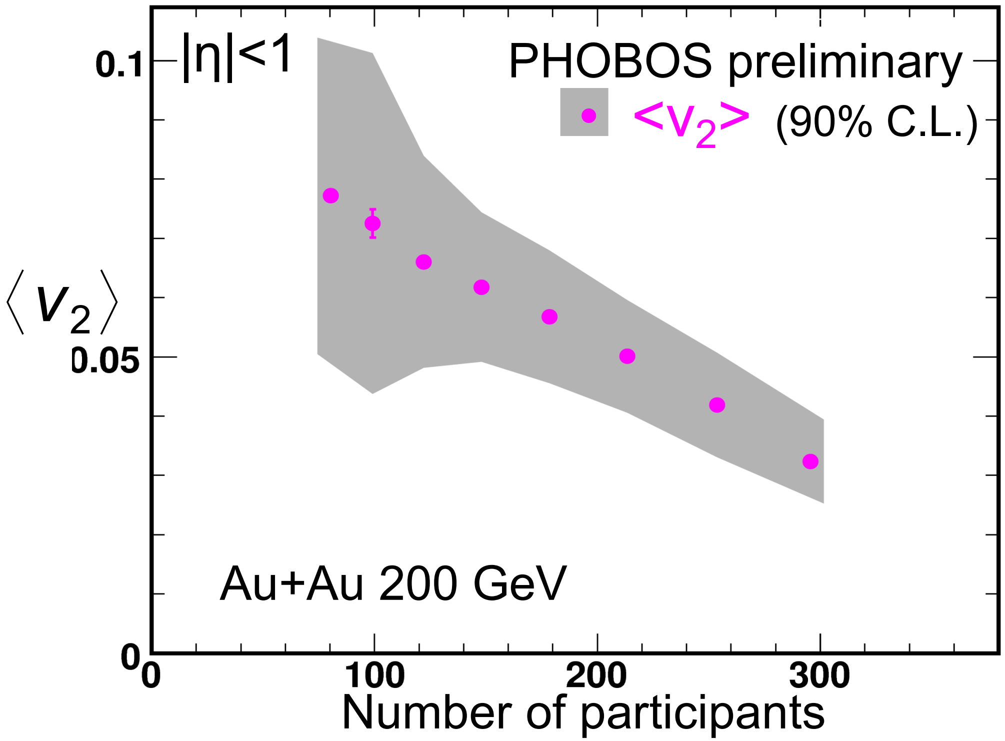
Compare expected $g(v_2^{obs})$ for trials with data:

Maximum-Likelihood fit → $<v_2>$ and σ_{v_2}

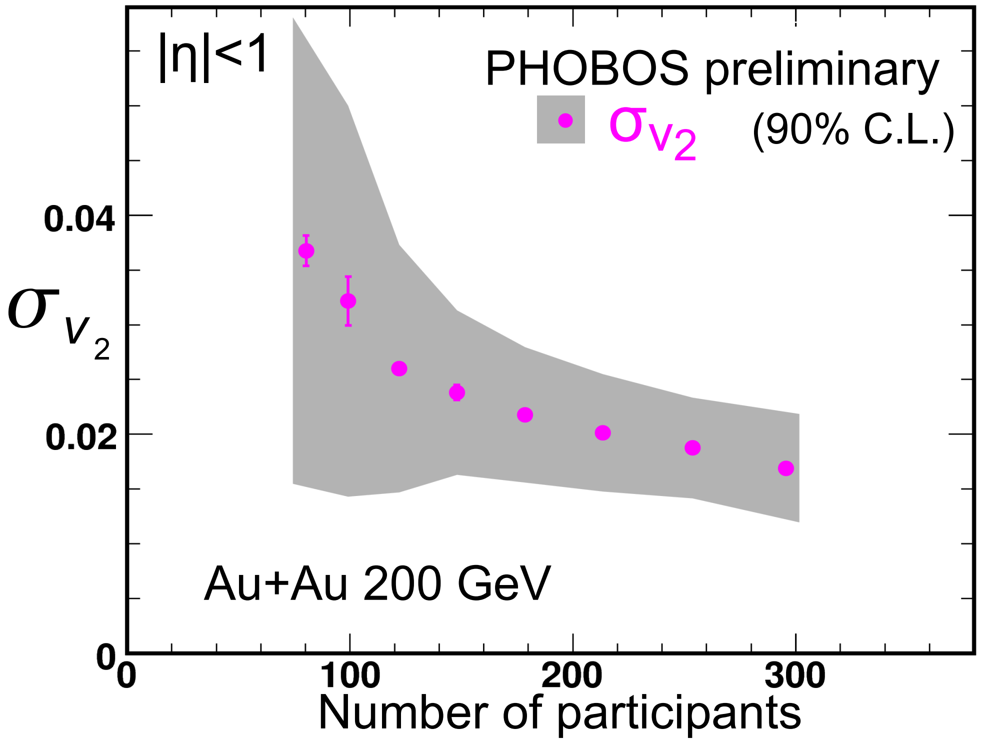
nucl.-ex/0702036 (sub.to PRL),
nucl-ex/0608025 (Proceedings of Science)

Elliptic flow fluctuations: $\langle v_2 \rangle$ and σ_{v_2}

Mean elliptic flow



Dynamical flow fluctuations



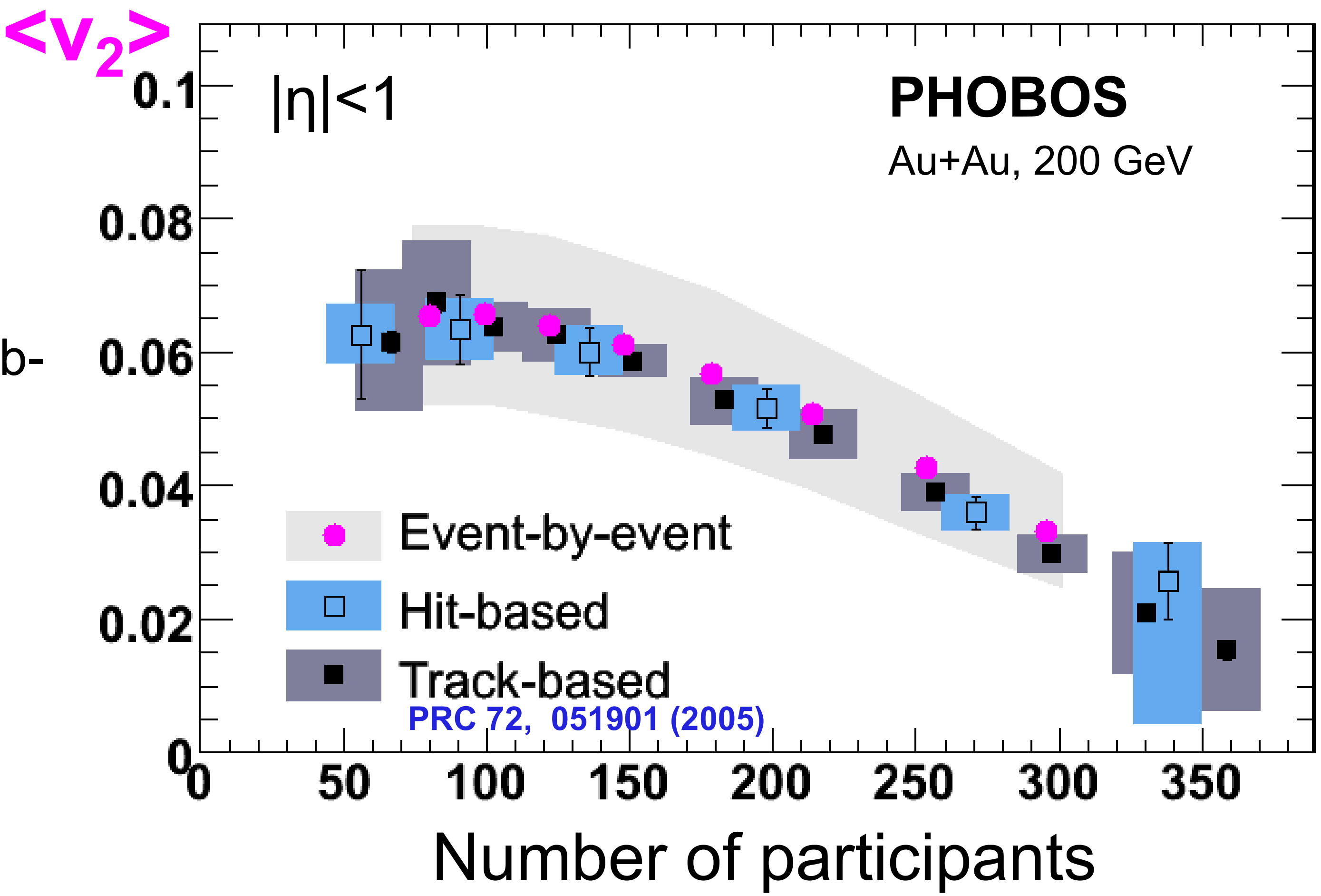
Systematic errors:

- Variation in η-shape
- Variation of $f(v_2)$
- MC response
- Vertex binning
- Φ_0 binning

"Scaling" errors cancel in the ratio: relative fluctuations, $\sigma_{v_2}/\langle v_2 \rangle$

Event-by-event mean v_2 vs published results

- Standard methods
  - Averaged over events to measure the mean
  - Hit- and track-based
  - Use reaction plane sub-event technique



Very good agreement of the event-by-event measured mean v_2 with the hit- and tracked-based, event averaged, published results