EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
ORGANISATION EUROPEENNE POUR LA RECHERCHE NUCLEAIRE

CERN - PS DIVISION

PS/DI Note 99-22

ACCUMULATION WITH A HOLLOW ELECTRON BEAM

N. Madsen

Geneva, Switzerland
November 16, 1999

1 Introduction

The Low Energy Ion Ring (LEIR) is to be used for lead ion accumulation for injection into LHC [1]. In order to obtain as high an accumulation rate and intensity in each delivery as possible it is planned to build a new electron cooler for LEIR. The purpose of the electron cooling is to compress the lead ions in phase space in order to enhance the planned multi-turn, 3D stacking scheme.

As preparation for the design of a new electron cooler and as test of the feasibility of the proposed stacking scheme, a number of experiments were carried out on the LEAR machine[1] with the old electron cooler [2]. In these experiments it was found that the lifetime of the stored stack of highly charged lead ions was strongly influenced by the capture of cooling electrons, and also that this depends highly on the exact charge state of the ion [3]. Thus the embedding of the ion beam into an electron beam cause limitations to the lifetime. Another problem is the intensity limitation due to loss of Landau damping and due to large tune shifts, which can be caused by so-called 'over-cooling'. This arises due to the reduced frequency spreads with the small momentum spread (due to longitudinal cooling) and the small beam size (transverse cooling), and furthermore due to the increased space charge impedance (tune shifts) with the increased density in the cold beam.

In this note we investigate a possible mechanism for solving these problems. The idea, due to Parkhomchuk, is to use a hollow electron beam [5]. Detailed investigations of how to create such a beam have been carried out by Sharapa and Shemyakin [6]. A hollow electron beam would mean that the stacked beam, once it has become small enough, stops interacting with the electron beam. Thus it will not be cooled further than desired, and furthermore no longer be exposed to electron capture processes.

Table 7 in Ref. [1] gives an overview of some relevant beam parameters and is reproduced in Table 1.

Beam at T/A = 4.2 MeV/u		After	After stacking	
		injection	coasting	bunched
Intensity	N	$1.2{\cdot}10^8$	$1.2{\cdot}10^9$	$1.2{\cdot}10^9$
Emittances	ϵ_h [π mm mrad]	2.1	10	10
	ϵ_v [π mm mrad]	2.1	5	5
Momentum spread	σ_p/p [10^{-3}]	1	0.5	1
	σ_p [m/s]	$3{\cdot}10^4$	$1.5{\cdot}10^4$	$3{\cdot}10^4$
Bunching factor	B_f	1	1	0.4
Max. Tune shift	$\Delta\hat{Q}_v$	0.009	0.033	0.084
IBS Growth time	τ_{IBS} [s]	11	2	19

Table 1: Beam conditions, resulting maximum tune shifts and intra beam scattering times for three different situations on the 4.2 MeV/u injection flat top. The emittances are $1\times\sigma$ emittances. The phase space area over which the injected beam is slowly painted is of order 50 π mm mrad [2].

The electron cooling times estimated in Ref. [1] are approximately 2 orders of magnitude

[1] The LEIR machine is the same as the LEAR, the name change was proposed to take account of LEAR's out phasing as an anti-proton ring, and the modifications made to work as an ion accumulator.

lower than the intra beam scattering (IBS) growth times calculated[2], thus this was not a point of concern. The estimate was confirmed by (some of) the measurements recently published in Ref. [2]. However, with a hollow electron beam the cooling time tends to be reduced, and it is desirable to know how much it will be reduced in order to establish whether IBS growth becomes important, and also whether the speed of merging the injected pulse into the stack will be affected.

2 A hollow electron beam

Figure 1 shows the geometry of the electron beam used for these calculations. Depicted is the three degrees of freedom, the longitudinal s and the two transverse x and y, horizontal and vertical respectively.

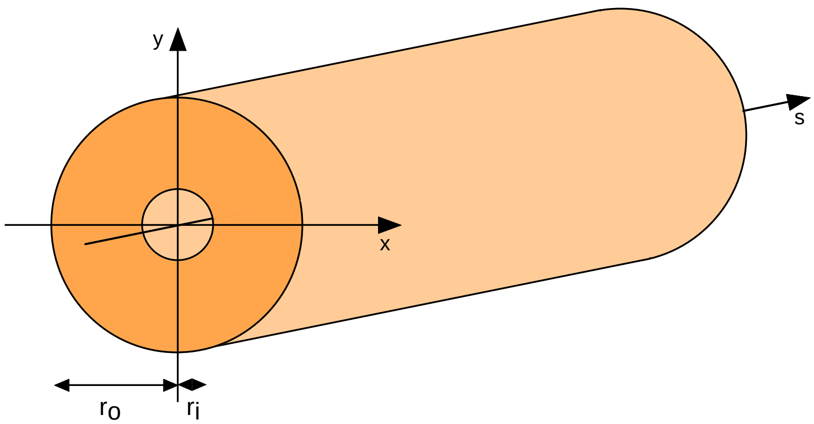

Figure 1: A hollow electron beam. The radius of the beam is typically r_o = 25 mm, whereas the hole is typically r_i = 9 mm.

As the beam particles undergo betatron oscillations it is not the direct physical overlap which is relevant for future calculations, but rather the phase space overlap, which is equivalent to calculating the average time a beam particle spends embedded in the electron beam. The overlap is thus a unit less quantity which can attain a maximum value of 1 - equal to the situation where the particle in question is always embedded in the electron beam. This does not mean that the particle is necessarily cooled, as the particle may be 'out of reach' in terms of its velocity, i.e. the force of interaction may be zero because the particle is moving to fast to interact with the electrons - as this depends on the exact nature of the force employed in the calculation we let this question rest for a moment.

This overlap can be used to estimate the influence from a hollow beam in terms of the reduction in the cooling force (completely ignoring the velocity and/or position dependence of the friction force), and more important the reduction in the electron capture rate as a function of the emittance of the ion beam. Assuming no coupling between horizontal and vertical motion this

[2]Cooling and growth times in this note refer, unless otherwise stated, to the e-folding time of the change of the amplitude of the betatron motion (or the magnitude of the momentum spread in the longitudinal case) - i.e. assuming a change in amplitude given by $\sim \exp(-t/\tau)$ where τ is the e-folding time.

overlap is given by

$$\eta_\beta(\epsilon_x, \epsilon_y) = \frac{1}{(2\pi)^2} \int\int_{r_i^2<r^2<r_o^2} d\phi_x\, d\phi_y\ ;\ r^2 = \epsilon_x \beta_x \cos^2\phi_x + \epsilon_y \beta_y \cos^2\phi_y \tag{1}$$

where $\beta_{x,y}$ are the horizontal and vertical beta function in the cooler, $\epsilon_{x,y}$ are the horizontal and vertical emittances, and $\phi_{x,y}$ are the horizontal and vertical betatron phases.

For the calculations throughout this note the parameter values listed in Table 2 have been used unless otherwise stated. The choice of beta functions at the electron cooler of 8 m in both transverse planes may not be realistic, however, for the calculations the important fact is the relation between the hole size and the beta function, as this gives the maximum beam emittance inside the hole. This emittance has in these calculations been chosen to be 10 π mm mrad in each dimension dimension (Table 2). As will be shown in Figure 2 this means that a beam of equal horizontal and vertical emittance of 5 π mm mrad will be without contact with the electron beam. Thus the calculations are valid assuming that this is the desired stacking emittance. If another emittance is desired it suffices to change either the beta functions or the hole radius accordingly - the choice of beta function is therefore not a limitation in the study.

Parameter	Symbol	Value
Horizontal Beta function	β_x	8.0 m
Vertical Beta function	β_y	8.0 m
Relativistic Factor	β	0.095
Electron Transverse Temperature	$\Delta_\perp$	0.1 eV
Emittance equivalent to $\Delta_\perp$	ϵ_e	400 π mm mrad
Radius of electron beam	r_o	25 mm
Equivalent emittance to radius	ϵ_o	78 π mm mrad
Radius of hole (typical)	r_i	9 mm
Equivalent emittance at hole-edge	ϵ_i	10 π mm mrad

Table 2: Values used for the calculations. The hole size is chosen to accommodate the stacked beam of 10 π mm mrad. The values of the beta functions have been chosen to be equal for simplicity.

Equation (1) depends on both the horizontal and the vertical emittance, and in Figure 2.a the dependence of the overlap on the two emittances are shown. As the two emittances are often almost equal, and a more quantitative visualization is desirable Figure 2.b shows η_β for $\epsilon_x = \epsilon_y$ as a function of the magnitude of ϵ.

Of interest is the necessary radius of the hole in order to contain some fraction of the stacked beam completely out of the electron beam to avoid electron capture. As a function of the emittance to be contained the necessary radius of the hole is given by

$$r_i(\epsilon_x, \epsilon_y) = \sqrt{\epsilon_x \beta_x + \epsilon_y \beta_y} \tag{2}$$

which with the parameters from Table 2 gave an emittance of 5 π mm mrad, as shown in Figure 2. From this figure it can also be extracted that particles with an order of magnitude larger transverse emittance are essentially completely embedded in the electron beam. With these considerations in mind the beta functions can be tailored to match the necessary emittance and the experimentally feasible hole sizes.

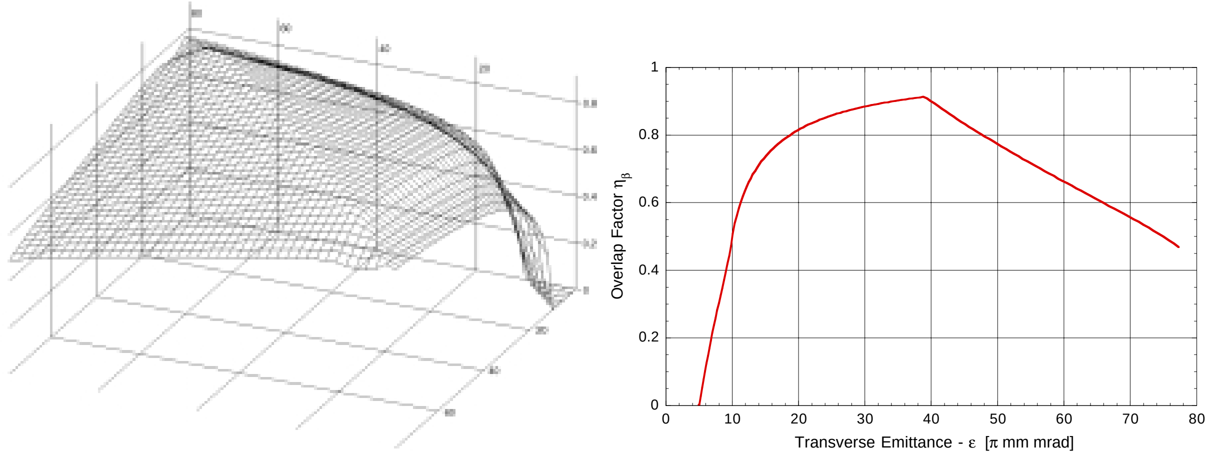

Figure 2: Left: A calculation of η_β (vertical axis) as a function of ϵ_x and ϵ_y (in units π mm mrad) - the axis are not marked, as the beta functions are equal in the two dimensions, thus the plot is symmetric around $\epsilon_x = \epsilon_y$. Right: The same calculation for $\epsilon = \epsilon_x = \epsilon_y$ - the diagonal on the left plot.

Finally we expect the longitudinal cooling force to be practically decoupled from the transverse relative velocity, and thus the longitudinal cooling force will be reduced with a fraction equal to the reduction in overlap as given by equation (1).

The over-cooling mentioned in the introduction will be reduced with reduced overlap between electron beam and ion beam. For the cold ions in the hole of the electron beam the equilibrium emittances and momentum spread will be determined by the equilibration due to IBS with the limiting condition that particles which increase their temperature to become embedded in the electron beam will be cooled into the cold core again. Thus whether the beam will be overcooled transversely depends the choice of maximum emittance not to be cooled relative to the number of particles in the ion beam, which in turn gives the maximum density obtained in the stacked beam, and thereby also the tune-shift induced. As the IBS grows with increased density, and the transverse dimensions are much warmer than the longitudinal, the equilibrium longitudinal temperature will be considerably larger than with electron cooling of the core - thus longitudinal over-cooling will not be a problem when using a hollow beam.

3 Detailed study

The transverse cooling force depends on the transverse relative velocity and the smaller the relative velocity between the electrons and the ions the larger the friction (until we reach the linear regime). Thus it is usually assumed that most of the friction damping takes place at large betatron amplitudes, where the transverse velocity is small. Therefore, and also because the amplitude of cold ions is small, the introduction of a hole in the electron beam will mainly influence the already cold ions, which is exactly the effect desired with the hole.

In the calculations on the hollow beam carried out here it will be assumed that the hollow beam is obtained by simply "cutting out" the central part of a solid beam. Thus the current of the hollow beam will be smaller by the ratio of the area of the hollow beam and the solid beam. As will be pointed out it suffices to multiply by the reciprocal of this ratio to obtain a calculation where a constant total electron current is assumed.

Table 3 lists emittances and corresponding velocities of a few selected parameters for the calculations.

Parameter	Usual units	Velocity
Transverse e-beam temperature	0.1 eV	$1.9{\cdot}10^5$ m/s
Longitudinal e-beam temperature	$1{\cdot}10^{-4}$ eV	$5.9{\cdot}10^3$ m/s
Max. $v_\perp$ with emittance at e-beam edge	78 π mm mrad	$8.8{\cdot}10^4$ m/s
Transverse stacking space LEIR	50 π mm mrad	$7{\cdot}10^4$ m/s
Max. $v_\perp$ with emittance at hole edge	10 π mm mrad	$3.2{\cdot}10^4$ m/s
Longitudinal stacking space LEIR (σ_p/p)	$3{\cdot}10^{-3}$	$8.4{\cdot}10^4$ m/s
Final momentum spread in stack	$5{\cdot}10^{-4}$	$1.4{\cdot}10^4$ m/s

Table 3: Various parameters in the unit they are usually stated in and in unit of m/s for comparisons.

The values in Table 3 shows that in most of the parameter range of the transverse emittances for the stacking in LEIR (up to 50 π mm mrad), the transverse emittance is such that the corresponding transverse velocities are smaller than the transverse electron velocity, and will thus be in the linear range of the electron cooling force [7]. In this case the cooling force actually increases with amplitude, and it is therefore not immediately obvious how strong the influence of the hole will be. However, in the case of a magnetized electron beam, the transverse temperature becomes irrelevant and the flip-over from linear friction to an inverse square in relative velocity is at the longitudinal velocity [7], which according to Table 3 means that most of our range of emittances will be in the inverse square law regime.

In order to estimate the effect more quantitatively the following model friction force is used

$$F_x(v_x, v_y) = -v_x \frac{1}{1 + (v_x^2 + v_y^2 + v_z^2)^{3/2}/\Delta_e^3} \tag{3}$$

where we will keep the longitudinal velocity component v_z at a constant value for the calculations in order to simplify them (and reduce calculation time). For the Δ_e either the transverse $\Delta_\perp$ or the longitudinal $\Delta_\parallel$ electron velocity will be used depending on whether magnetization is assumed or not.

Including friction the transverse equation of motion in the smooth approximation is given by

$$\frac{d^2x}{dt^2} + \omega_\perp^2 x = \frac{1}{m} F_\perp(\frac{dx}{dt}) \tag{4}$$

where $\omega_\perp = Q_\perp \omega_0$ where ω_0 is the revolution frequency and $Q_\perp$ is the transverse tune, and where m is the mass of the particle. The solution in the case of a friction force linear in velocity, i.e. $F_\perp(v) = -kv$, where k is a constant describing the strength of the friction force, with $k/m < \omega_\perp$, is given by

$$x(t) = A \exp(-\frac{kt}{2m}) \cos(\omega_\perp t + \phi_0) \tag{5}$$

where ϕ_0 is an arbitrary initial phase and A the initial amplitude of the oscillation. Thus the cooling time (e-folding) is given by

$$\tau_{cool} = \frac{2m}{k} \tag{6}$$

For a non-linear friction force the method of variation of parameters can be used to find an approximate solution for slow damping [8]. The solution to the homogeneous version of equation (4) is used as generating solution

$$x(t) \;=\; A\cos(\omega_\perp t + \phi_0) \tag{7}$$

$$v(t) \;=\; \frac{dx}{dt} = -A\omega_\perp \sin(\omega_\perp t + \phi_0) \tag{8}$$

One finds that

$$\frac{dA}{dt} = \frac{1}{\omega_\perp}\langle \sin(\omega_\perp t + \phi_0)\frac{1}{m}F_\perp(v(t))\rangle_c \tag{9}$$

where the average is over one betatron cycle.

In order to calculate the reduction in the damping caused by a hole in the electron beam the difference in equation (9) between integrating over all of the cycle and only over the part of the cycle where there is interaction with the hollow beam should be calculated.

In terms of the parameters in Table 2 the transverse relative velocities are given as

$$v_{x,y}^2 = (\epsilon_{x,y}/\beta_{x,y}) \cdot v_0^2 \sin^2 \phi_{x,y} \tag{10}$$

where $v_0 = \beta c$ is the longitudinal velocity of the ion beam. This leads to the following expression for the relative reduction in the damping of the horizontal amplitude of a particle of emittance ϵ_x

$$\eta_x(\epsilon_x, \epsilon_y) = \frac{\int\int_{r_i^2<r^2<r_o^2} \sin\phi_x F_x(v_x, v_y) d\phi_x\, d\phi_y}{\int\int \sin\phi_x F_x(v_x, v_y) d\phi_x\, d\phi_y} \tag{11}$$

Figure 3.left shows a η_x as a function of the transverse emittance ($\epsilon_x = \epsilon_y$) for a hole of r_i = 9 mm, for two different longitudinal velocities, and for two different electron beam temperatures (Δ_e). The two different electron beam models, the non-magnetized and magnetized electron beams, as discussed above are used. We observe again that the damping goes to zero at the emittance at which no overlap is present.

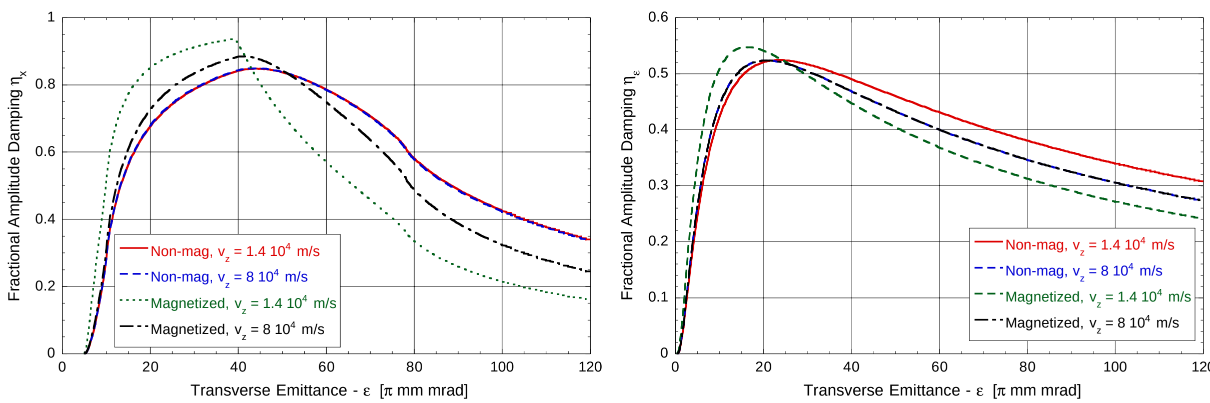

Figure 3: Left: Calculation of η_x as a function of the transverse (single particle) emittance ($\epsilon_x = \epsilon_y$). Right: Average damping reduction in a Gaussian beam ($\eta\epsilon_x$ from equation (12)). The magnetized curves have been calculated with $\Delta_e = \Delta_\parallel$. The calculations assume a constant electron density, thus the current is lower in the hollow beam than in the solid beam to which it is compared. As discussed in the text the increase in the fractional damping induced by introducing a constant current instead is 15%.

Figure 3.left shows that the reduction in damping rate resembles the reduction in overlap very much. However, of interest is rather the average damping time reduction of the RMS emittance of a full beam, than the reduction in damping of a single particle. A reasonable estimate of this can be found by averaging the reduction over a Gaussian beam profile. The average can be found as

$$\eta_{\epsilon_x}(\epsilon_x, \epsilon_y) = \int_0^\infty \frac{\eta_x(\epsilon_x, \epsilon_y)}{2\epsilon_{beam}} \exp\left(-\frac{\epsilon}{2\epsilon_{beam}}\right) d\epsilon \tag{12}$$

Figure 3.right shows the beam profile averaged reduction of the horizontal damping rate as a function of the transverse emittance for several different situations. The result of this averaging is naturally that the maximum cooling efficiency is reduced somewhat, and that some parts of the ion beam are always interacting with the electron beam. The influence of magnetization is mainly that the reduction in damping becomes influenced by the longitudinal velocity spread. However with the parameter values foreseen for LEIR the variation is rather small, and it will therefore not be necessary to go into more detailed calculations to predict the influence of the hollow beam on the cooling times.

We have in the calculations above assumed a constant electron density, which implies that the total current in the hollow beam is lower, thus it is only natural that the cooling time is reduced. However, we can instead try to keep the current constant. As the cooling rate usually scales linearly with density [9], we only need to multiply with the relative area of the solid beam to the hollow beam, i.e. by a factor $r_o^2/(r_o^2 - r_i^2) = 1.15$. Thus we can increase the density by 15% if we wish to keep the current constant. This has the consequence that for a few single particle emittances in the calculation in Figure 3.left the hollow beam is more efficient than the solid beam. In general however it means 15% better efficiency, thus a cooling time reduction of the order 2 in the emittance range expected to be filled by multiple stacking.

4 Space-charge in the electron beam

Another necessary consideration in order to be able to evaluate the feasibility of a hollow beam is space charge effects, which cause dispersion in the electron beam. The calculation will be for a constant current, as this is the 'worst case', because the electric fields, and therefore the potentials will be 15% larger than if a constant density is assumed.

The electric field from a hollow electron beam of uniform density and constant total current (charge) can be found by Gauss' law to be [10]

$$E(r) = \frac{\lambda}{2\pi\epsilon_0} \times \begin{cases} 0 & ;\ r < r_i \\ \frac{r^2 - r_i^2}{r(r_o^2 - r_i^2)} & ;\ r_i < r < r_o \\ 1/r & ;\ r_o < r < b \end{cases} \tag{13}$$

where r_i is the radius of the hole, r_o of the electron beam, b of the vacuum chamber and $\lambda = I_e/v_e$ is the longitudinal charge density in the lab-frame (I_e is the electron current and v_e is the electron velocity). This electric field gives a potential distribution over the electron beam equivalent to

$$\Phi_{sc}(r) = \frac{\lambda}{4\pi\epsilon_0} \times \left[2 \ln\frac{b}{r_o} - \frac{r^2 - r_o^2}{r_o^2 - r_i^2} + 2\frac{r_i^2}{r_o^2 - r_i^2} \ln\frac{r}{r_o}\right]\ ;\ r_i \leq r \leq r_o \tag{14}$$

which in an accelerated but non-relativistic electron beam can be approximated to the following relative velocity variation across the electron beam (assuming that the acceleration potential

$\Phi_0 \ll \Phi_s c$)

$$\frac{\Delta v}{v}(r) = \frac{\lambda}{8\pi\epsilon_0 \Phi_0 (r_o^2 - r_i^2)} \left[r^2 - r_i^2 \left(1 + 2 \ln \frac{r}{r_i} \right) \right] \; ; \; r_i \leq r \leq r_o \tag{15}$$

which is zero at the boundary of the hole (per definition).

If $r_i \ll r_o$ the velocity variation over the electron beam will be the same as for a non-hollow beam. Figure 4 shows a few examples of how the velocity offset will vary across the electron beam for various hole radii.

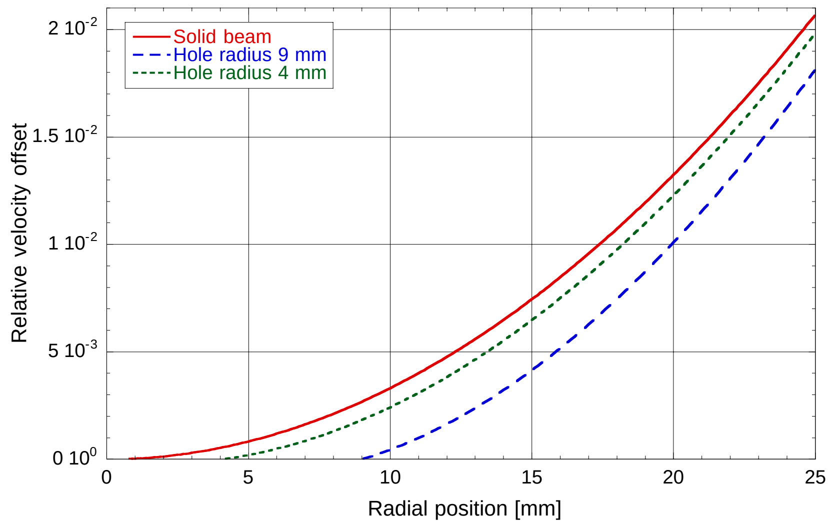

Figure 4: Velocity offset as a function of radial position in an electron beam. For the calculation a constant electron current of 0.3 A has been assumed, i.e. the density of electrons increase correspondingly when the hole is increased.

Figure 4 shows that even for a rather large hole (9 mm) of about 36% of the radius, the velocity offset is only altered about 12% at the edge of the electron beam. The dependency on the radial position is also quite close to the solid beam, as the logarithmic correction is very small. Thus the influence of the hole on the space-charge effects is rather weak even for a large hole, and therefore not likely to be a concern.

5 Dispersion

Experiments suggest that it is beneficial to have ring dispersion in the electron cooler, both due to the dispersion in the electron cooler [2] but also due to single particle effects [11]. As a final point it is therefore necessary to investigate how a hollow beam may change these effects.

Due to dispersion (first order) the horizontal closed orbit position of an off momentum particle will be shifted as follows

$$\Delta x = D_x \frac{\Delta p}{p_0} = D_x \gamma^2 \frac{\Delta v}{v_0} \tag{16}$$

where D_x is the local value of the dispersion function, γ is the relativistic factor, and p_0 is the mean beam momentum.

This means that particles with enough momentum offset may be shifted into the electron beam if the dispersion in the electron cooler is large enough. However, as the relative velocities between electrons and ions in this case would be rather large the cooling could become slow, and the main effect of this overlap would be a possible slight increase electron capture processes. With our example hole size, and a dispersion of 5 m, the velocity offset needed is $5{\cdot}10^4$ m/s, small compared to the transverse electron velocity of $2{\cdot}10^5$ m/s, which therefore means that cooling will be effective, and hence cool the particle to central orbit. Even in a magnetized beam, this offset is only a factor 2 above the longitudinal thermal velocity of the electrons, and the cooling will therefore not be reduced much.

Electron capture processes are most efficient if the relative velocity between the ion and the electron to be captured is of the order of the orbital velocity of the electron around the ion. The orbital velocity of the outer electron is typical of the order of 1% of the speed of light, and thus the electron capture is not significantly enhanced in the above example [12].

Another possibility would be that the cooling enhancement due to the combined dispersion of the ion and the electron beam as argued in Ref. [2] may be altered by the hollowness. However, in the previous section it was found that the dispersion of the electron beam is only altered weakly with the introduction of a hole, and this effect is therefore negligible.

6 Conclusion

The feasibility of using a hollow electron beam for cooling and stacking ions in a low energy accumulation ring was studied. The emphasis was put on the balance between the benefits of reducing the losses due to electron capture processes and the possibility to reduce the risk of over-cooling on the one hand and the change in the cooling efficiency due to the hole on the other hand.

The reduction in cooling efficiency for the emittance range foreseen was of order 2 both in the case of a magnetized electron beam and in the case of non-magnetized electron cooling. The cooling time with the hollow beam used in this study is improved about 15% if the electron current is assumed constant instead of the electron density. The reduction in cooling rate found is negligible compared to the IBS rate which is 2 orders of magnitude lower than the predicted (and experimental) rate for the solid electron beam use for reference. The decrease of electron capture may however be large with a suitable choice of hole size, and the reduction in longitudinal cooling of cold particle equally large. The risk of transverse over-cooling depends on the exact choice of stacking emittance in relation to the number of stacked particles, and more details on the exact setup would be needed to determine to what extend a hollow beam helps. The longitudinal over-cooling is avoided with a hollow beam, as only IBS will cool the stacked particles, and the transverse temperatures are far larger than the usual instability limits in the longitudinal dimension.

However, as noted in Ref. [2] the cooling time in the stacking experiments was about a factor two smaller than the design goal. Thus even though a hollow beam seems like a feasible idea for LEIR it would have to be combined with an additional electron beam which could supply some cooling of the central orbit particles. As the large emittance particles are cooled into the hole of the hollow electron beam this beam could be quite intense, and therefore compensate for the lack of cooling in the center, whereas a weaker one could be used for keeping the stack cold.

References

[1] P. Lefèvre and D. Möhl, *A Low Energy Accumulation Ring of Ions for LHC*, Internal Report, CERN/PS 93-62.

[2] J. Bosser, C. Carli, M. Chanel, C. Hill, A. Lombardi, R. Maccaferri, S. Maury, D. Möhl, G. Molinari, S. Rossi, E. Tanke, G. Tranquille, M. Vretenar, *Experimental Investigation of Electron Cooling and Stacking of Lead Ions in a Low Energy Accumulation Ring*, Internal Note, CERN/PS 99-033, Submitted to Part. Accel. (1999).

[3] S. Baird, J. Bosser, C. Carli, M. Chanel, P. Lefèvre, R. Ley, R. Maccaferri, S. Maury, I. Meshkov, D. Möhl, G. Molinari, F. Motsch, H. Mulder, G. Tranquille, F. Varenne, *Measurement of the lifetime of Pb*$^{52+}$*, Pb*$^{53+}$ *and Pb*$^{54+}$ *beams at 4.2 MeV per nucleon subject to electron cooling.*, Phys. Lett. B **361**, 184 (1995).

[4] J. Bosser, C. Carli, M. Chanel, N. Madsen, S. Maury, D. Möhl, and G. Tranquille, *Stability of Cooled Beams*, Internal Note, CERN/PS 99-044, Submitted to Nucl. Inst. & Meth. A (1999)

[5] V. Parkhomchuk, Private Communication.

[6] A.N. Sharapa and A.V. Shemyakin, *Electron cooling device without bending magnets*, Nucl. Inst, & Meth. A **336**, 6 (1993)

[7] I.N. Meshkov, *Electron cooling: Status and perspectives*, Phys. Part. Nucl. **25**, 631 (1994)

[8] W. J. Cunningham, *Introduction to Nonlinear Analysis*, (McGraw-Hill Book Company, New York, 1958), p. 257 - 259.

[9] N. Madsen, *Simulations of Electron Cooling using 'BETACOOL'*, Internal Note, CERN PS/DI 99-20.

[10] J.D. Jackson, *Classical Electrodynamics*, Second Edition, (John Wiley & sons, New York, 1975), p. 30-33

[11] I. Lauer, U. Eisenbarth, M. Grieser, R. Grimm, P. Lenisa, V. Luger, T. Schätz, U. Schramm, D. Schwalm, and M. Weidemüller, *Transverse laser-cooling of a fast stored ion beam through dispersive coupling*, Phys. Rev. Lett. **81**, 2052 (1998)

[12] B.H. Bransden and C. J. Joachain, *Physics of Atoms and Molecules*, (Longman Scientific & Technical, Essex, 1983), p. 27 - 36