Energy loss distribution and maximum likelihood

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1 Transformations

The Laplace transform and its inverse, the Bromwich integral.

$$\mathcal{L}(s) = \mathcal{L}\left(f(t)\right) \equiv \int\limits_{-\infty}^{\infty} f(t) e^{-st} dt \qquad\qquad f(t) = \frac{1}{2\pi i} \int\limits_{\gamma - i\infty}^{\gamma + i\infty} e^{st} f(s) ds \tag{1}$$

The Landau distribution

$$\mathcal{L}\left(\phi(t)\right) = s^s \qquad\qquad \phi(t) = \frac{1}{2\pi i} \int\limits_{\gamma - i\infty}^{\gamma + i\infty} e^{st} e^{s \log s} ds \tag{2}$$

2 Energy loss distribution

If the average measured charge is unity, the distribution of the measured energy loss t can be well approximated by

$$f(t|1) \approx N(a,b,c) \cdot \phi(at+b) e^{-c(at+b)} \tag{3}$$

where a, b and c are parameters, a is different sector by sector (contains calibration constants), b and c are different chamber by chamber.

The function has to be normalized and has to have average value of unity.

$$\int f(t|1) dt = 1 \qquad\qquad \int t f(t|1) dt = 1 \tag{4}$$

Hence

$$N \int \phi(at+b) e^{-c(at+b)} dt = \frac{N}{a} \int \phi(x) e^{-cx} dx = \frac{N}{a} \mathcal{L}\left(\phi(t)\right) = N \frac{c^c}{a} = 1 \tag{5}$$

$$\begin{aligned} N \int t \phi(at+b) e^{-c(at+b)} dt = \frac{N}{a} \int \frac{x-b}{a} \phi(x) e^{-cx} dx = \\ \frac{1}{a} \left(\frac{1}{c^c} \int x \phi(x) e^{-cx} dx - b \right) = -\frac{1}{a} \left(\frac{d \log \mathcal{L}\left(\phi(t)\right)}{dc} + b \right) = \\ -\frac{\log c + 1 + b}{a} = 1 \end{aligned} \tag{6}$$

Thus

$$f(t|1) = \frac{a}{c^c} \cdot \phi(at+b) \cdot e^{-c(at+b)} \qquad c = e^{-(a+b+1)} \tag{7}$$

For charge values different from unity convolution can be used that is easily performed with help of Laplace transform.

$$\mathcal{L}\left(f(t|q)\right) = \mathcal{L}\left(f(t|1)\right)^q \tag{8}$$

$$2\pi i \cdot f(t|1) = \frac{a}{c^c} \int\limits_{\gamma-i\infty}^{\gamma+i\infty} e^{(s-c)(at+b)} e^{s \log s} ds = \frac{1}{c^c} \int e^{st} e^{\frac{s}{a}b + \left(\frac{s}{a}+c\right)\log\left(\frac{s}{a}+c\right)} ds \tag{9}$$

$$\mathcal{L}\left(f(t|1)\right) = \frac{1}{c^c} e^{\frac{s}{a}b + \left(\frac{s}{a}+c\right)\log\left(\frac{s}{a}+c\right)} \tag{10}$$

It is enough to construct the function

$$2\pi i \cdot f(t|q) = \frac{1}{c^{qc}} \int e^{st} e^{q\left(\frac{s}{a}b + \left(\frac{s}{a}+c\right)\log\left(\frac{s}{a}+c\right)\right)} ds \tag{11}$$

$$f(t|q) = \frac{a}{q \cdot c^{qc}} \cdot \phi\left(\frac{at}{q} + b - \log q\right) \cdot e^{-c(at+bq)} \tag{12}$$