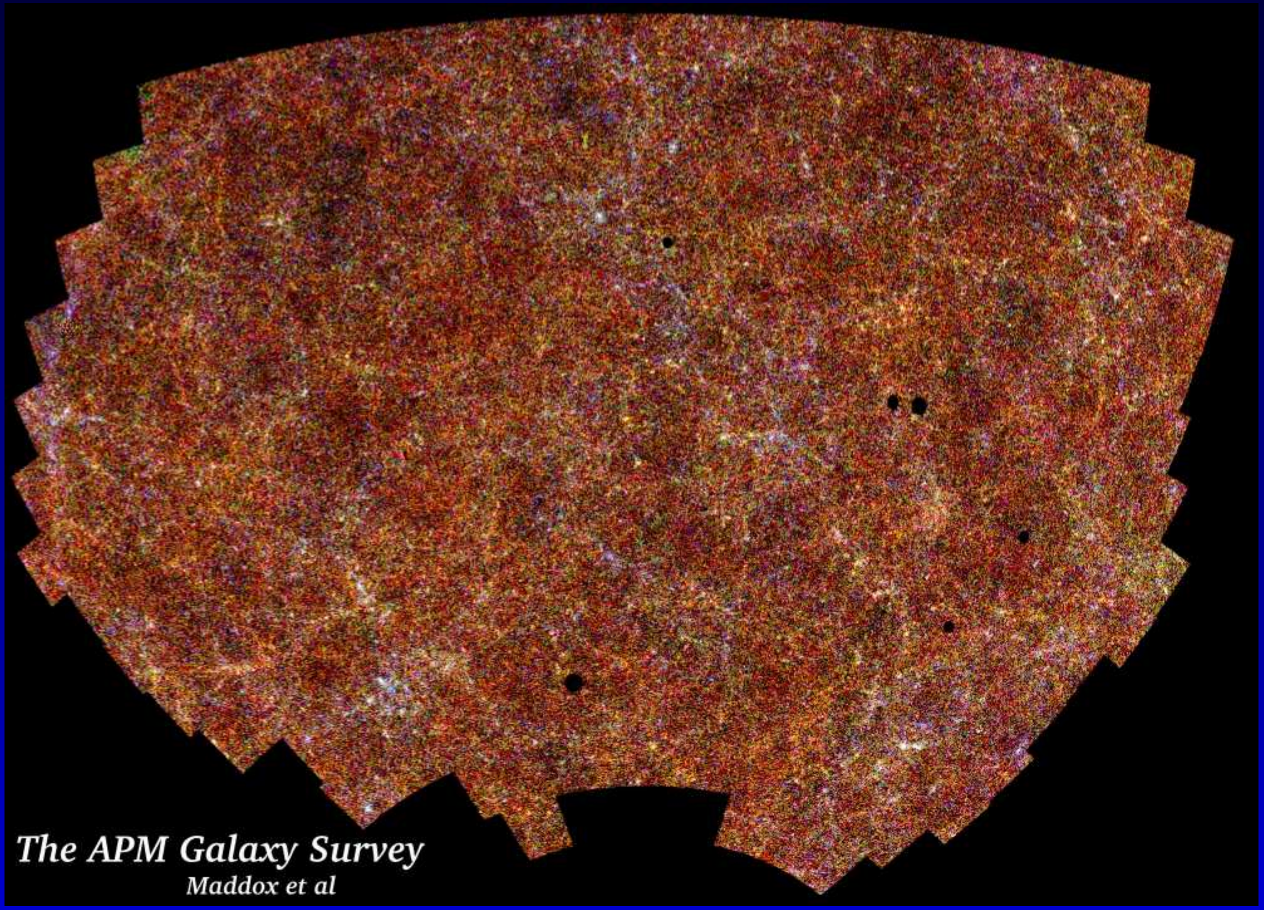
Dark Matter: Distribution

Direct observations tell us about galaxies.

Dark Matter: Distribution

Direct observations tell us about galaxies.



Correlation function

for uncorrelated distribution

$$dP(r) = ndV$$

Correlation function

$$dP(r) = ndV(1 + \xi(r))$$

for correlated distribution

Correlation function

$$dP(r) = ndV(1 + \xi(r))$$

for correlated distribution

Power spectrum:

$$P(k) = FT(\xi(r))$$

Bias

Simple bias :

$$\frac{\delta\rho}{\rho})_g = b \times \frac{\delta\rho}{\rho})_{DM}$$

so

$$\xi_g(r) = b^2 \xi_{DM}(r)$$

Bias

Simple bias :

$$\frac{\delta\rho}{\rho})_g = b \times \frac{\delta\rho}{\rho})_{DM}$$

so

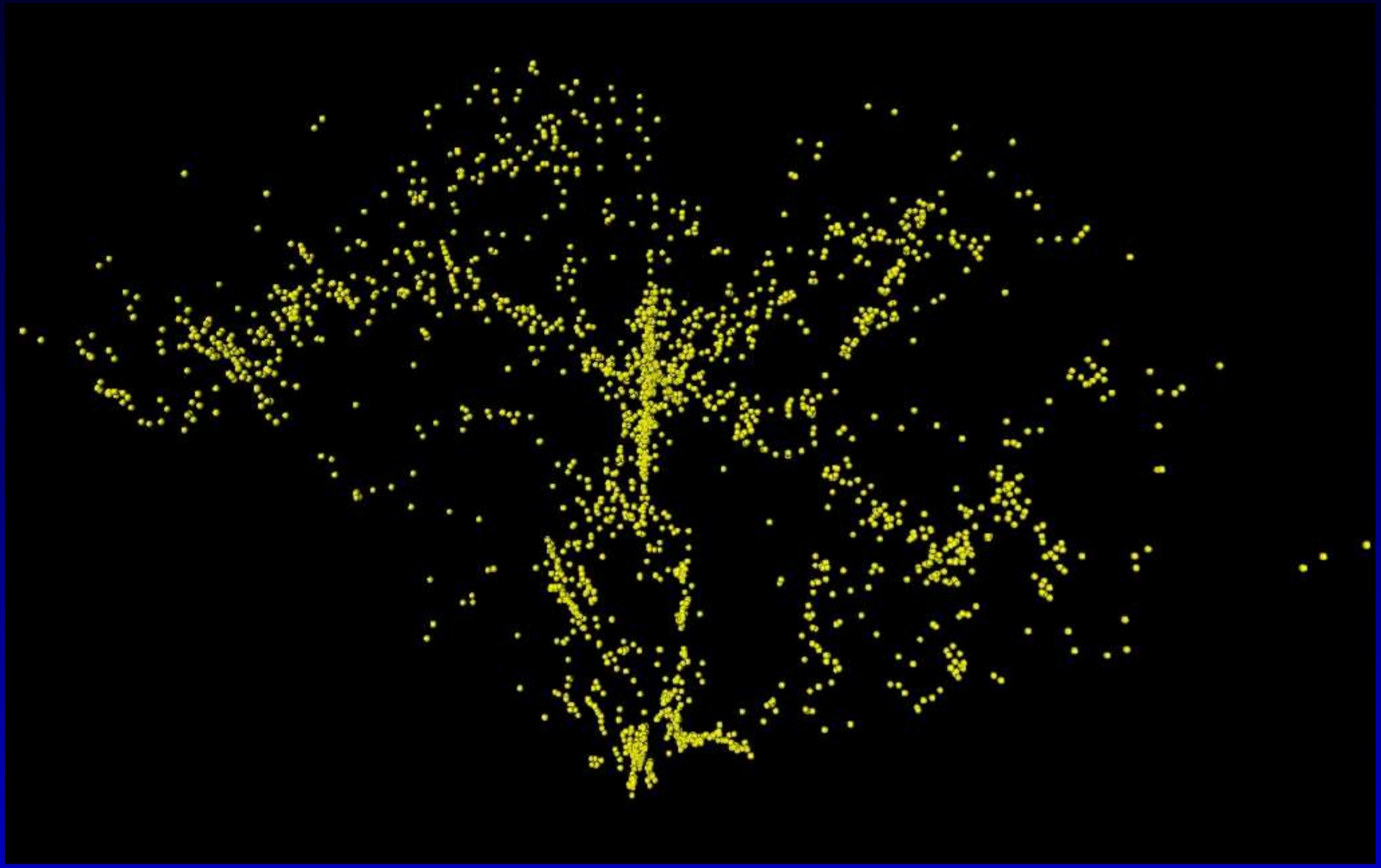
$$\xi_g(r) = b^2 \xi_{DM}(r)$$

one might have more complicated relation between galaxies and DM, and the bias can be a function of scale:

$$b(r)$$

...

Space distribution



Cfa Slice $\sim$ 1000 galaxies.

Galaxy correlation function

From the smallest scale up to $10h^{-1}$Mpc:

$$\xi(r) = (r/r_0)^\gamma$$

with:

$r_0 \approx 5.5h^{-1}$Mpc:

$\gamma \approx -1.77$:

So:

$$\sigma_8 = \sqrt{< \frac{\Delta N}{N} >_{R=8h^{-1}\mathrm{Mpc}}} \approx 1$$

Galaxy correlation function

From the smallest scale up to $10h^{-1}$Mpc:

$$\xi(r) = (r/r_0)^\gamma$$

with:

$r_0 \approx 5.5h^{-1}$Mpc:

$\gamma \approx -1.77$:

So:

$$\sigma_8 = \sqrt{< \frac{\Delta N}{N} >_{R=8h^{-1}\mathrm{Mpc}}} \approx 1$$

(there has been some debate on the possible fractal nature of the distribution)

Galaxy correlation function

Improving scales by a factor of ten:

Galaxy correlation function

Improving scales by a factor of ten:

need for $\sim 10^6$ galaxies...

Galaxy correlation function

Improving scales by a factor of ten:

need for $\sim 10^6$ galaxies...

this has motivated 2dF and SDSS surveys

Galaxy correlation function

Improving scales by a factor of ten:

need for $\sim 10^6$ galaxies...

this has motivated 2dF and SDSS surveys

The correlation is weaker on large scale...

Galaxy correlation function

Improving scales by a factor of ten:

need for $\sim 10^6$ galaxies...

this has motivated 2dF and SDSS surveys

The correlation is weaker on large scale...

How to improve LSS measurements ?

ξ on large scales

Select red galaxies:

ξ on large scales

Select red galaxies:

- They are bright: sample extends up to $z \sim 0.5$

ξ on large scales

Select red galaxies:

- They are bright: sample extends up to $z \sim 0.5$
- They are biased with $b = 2$ so ξ is boosted by a factor 4...

ξ on large scales

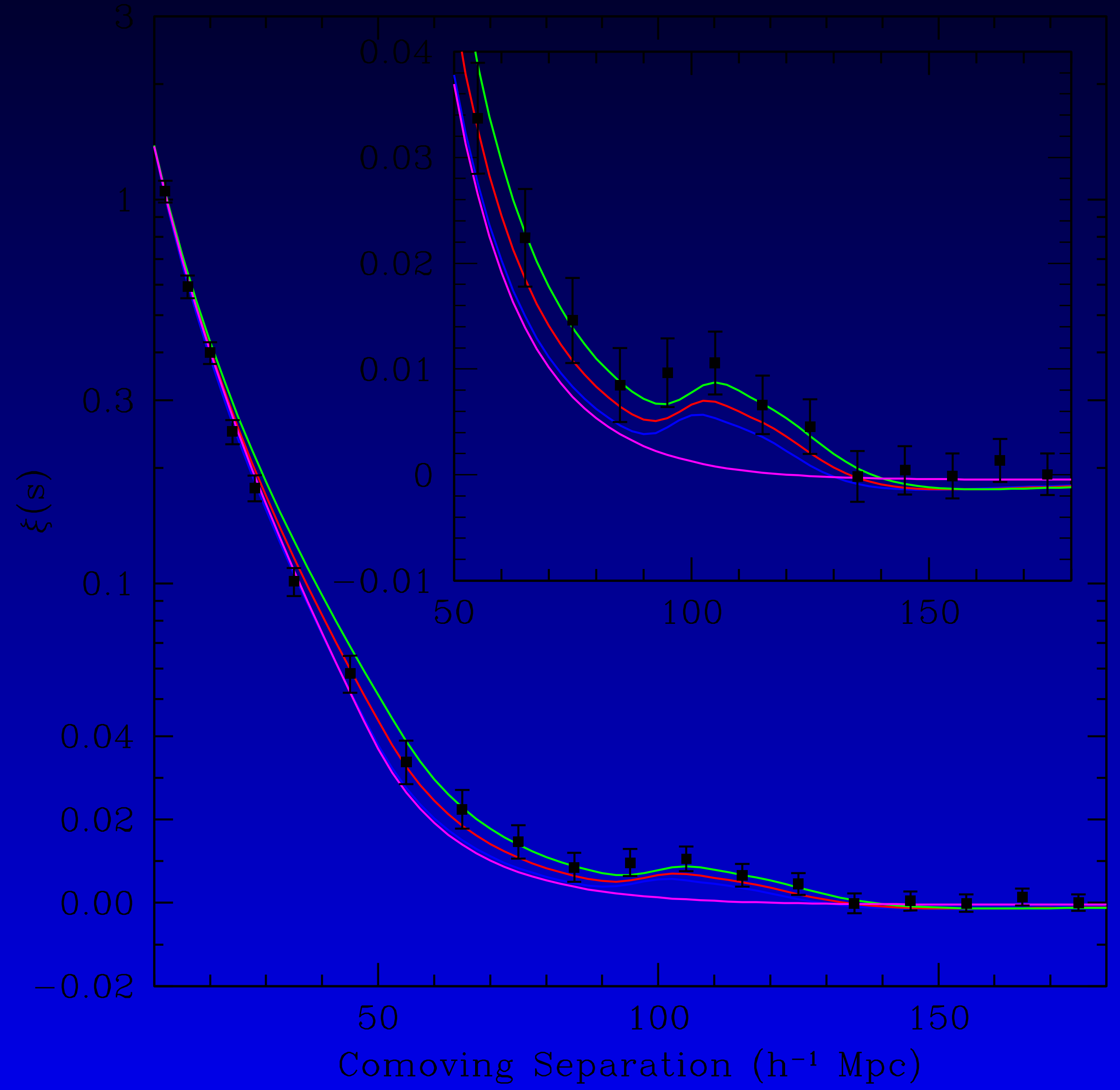
Select red galaxies:

- They are bright: sample extends up to $z \sim 0.5$
- They are biased with $b = 2$ so ξ is boosted by a factor 4...

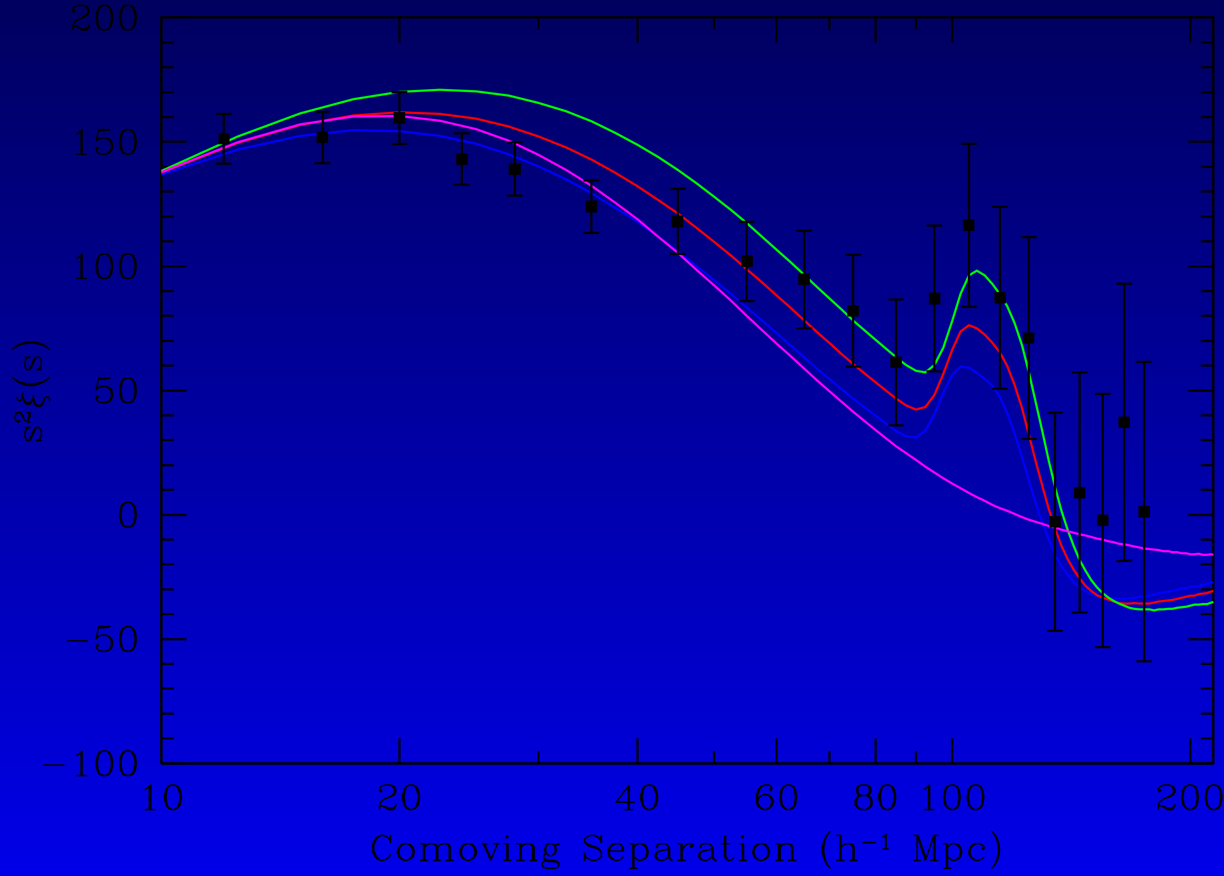
50 000 galaxies left...

Results : ξ on large scales

Results : ξ on large scales



Results : ξ on large scales



Origin of clustering

The question of the origin of structure in the Universe is an old question.

Origin of clustering

The question of the origin of structure in the Universe is an old question.

- Lemaître (1933) is the first to propose and study gravitational instability in an expanding universe.

Origin of clustering

The question of the origin of structure in the Universe is an old question.

- Lemaître (1933) is the first to propose and study gravitational instability in an expanding universe.
- Turbulence...

Origin of clustering

The question of the origin of structure in the Universe is an old question.

- Lemaître (1933) is the first to propose and study gravitational instability in an expanding universe.
- Turbulence...
- Cosmic explosions...

Origin of clustering

The question of the origin of structure in the Universe is an old question.

- Lemaître (1933) is the first to propose and study gravitational instability in an expanding universe.
- Turbulence...
- Cosmic explosions...
- Defects...

Gravitational instability

A key-point : seeds are needed.

Gravitational instability

A key-point : seeds are needed.
Linear theory:

$$\delta(z) = D(z, ...)\delta_0$$

Gravitational instability

A key-point : seeds are needed.
Linear theory:

$$\delta(z) = D(z, ...)\delta_0$$

D : growing mode.

Gravitational instability

A key-point : seeds are needed.
Linear theory:

$$\delta(z) = D(z, ...)\delta_0$$

D : growing mode.
Initial fluctuations are specified by (inflation ?):

Gravitational instability

A key-point : seeds are needed.
Linear theory:

$$\delta(z) = D(z, ...)\delta_0$$

D : growing mode.
Initial fluctuations are specified by (inflation ?):

- being adiabatic

Gravitational instability

A key-point : seeds are needed.
Linear theory:

$$\delta(z) = D(z, \ldots)\delta_0$$

D : growing mode.
Initial fluctuations are specified by (inflation ?):

- being adiabatic
- being Gaussian

Gravitational instability

A key-point : seeds are needed.
Linear theory:

$$\delta(z) = D(z, ...)\delta_0$$

D : growing mode.
Initial fluctuations are specified by (inflation ?):

- being adiabatic
- being Gaussian
- $P_i(k) = \hat{\delta}_i(k)\hat{\delta}_i^*(k)$

What you get

The “final” power spectrum depends on:

What you get

The “final” power spectrum depends on:

- The physics of fluctuations evolution

What you get

The “final” power spectrum depends on:

- The physics of fluctuations evolution
- The nature of dark matter (hot, cold, warm, ...)

What you get

The “final” power spectrum depends on:

- The physics of fluctuations evolution
- The nature of dark matter (hot, cold, warm, ...)

this is summarized through the tranfer function:

$$\hat{\delta}_f(k) = T(k)\hat{\delta}_i(k)$$

giving:

$$P_f(k) = P_i(k) * T^2(k)$$

From your preferred model...

Specified your scenario:

From your preferred model...

Specified your scenario:

- Early universe $\longrightarrow$ initial conditions and physics

From your preferred model...

Specified your scenario:

- Early universe $\longrightarrow$ initial conditions and physics
- Dark matter

From your preferred model...

Specified your scenario:

- Early universe $\rightarrow$ initial conditions and physics
- Dark matter
- Dark energy

From your preferred model...

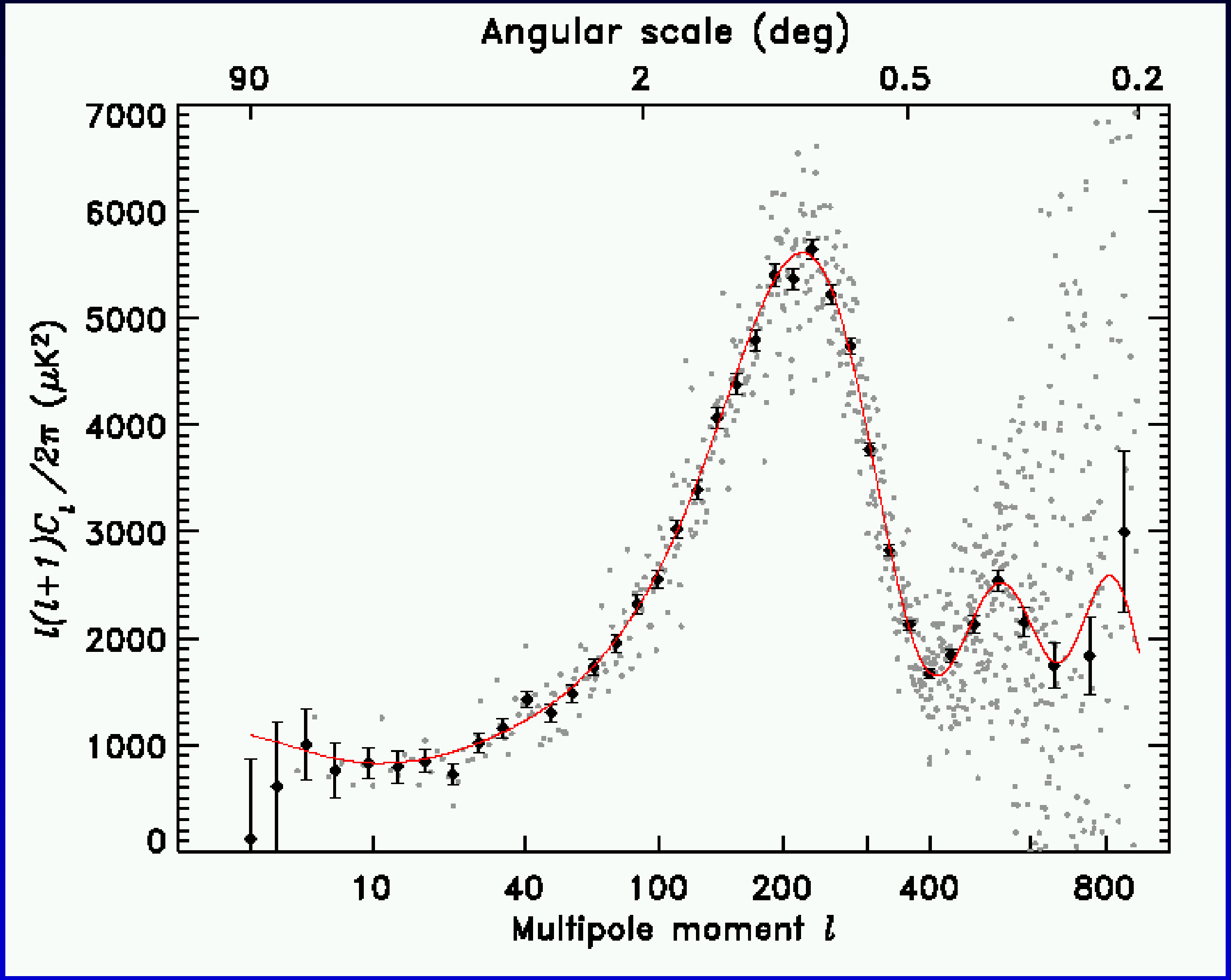
Specified your scenario:

- Early universe $\longrightarrow$ initial conditions and physics
- Dark matter
- Dark energy

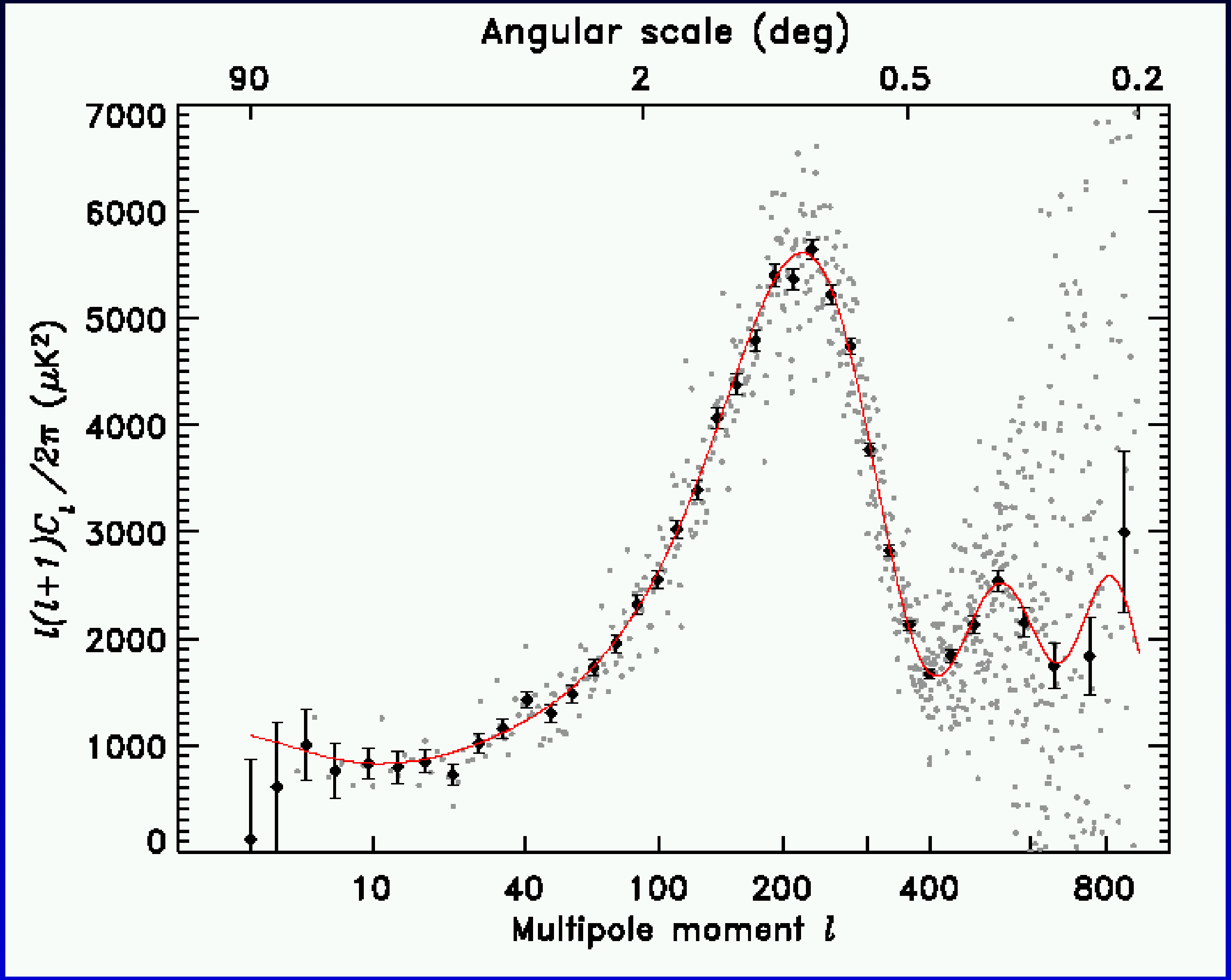
You get (through CMBfast or CAMB or ...) CMB C_l and $P(k)$ allow to test your model!

ΛCDM is successful...

ΛCDM is successful...



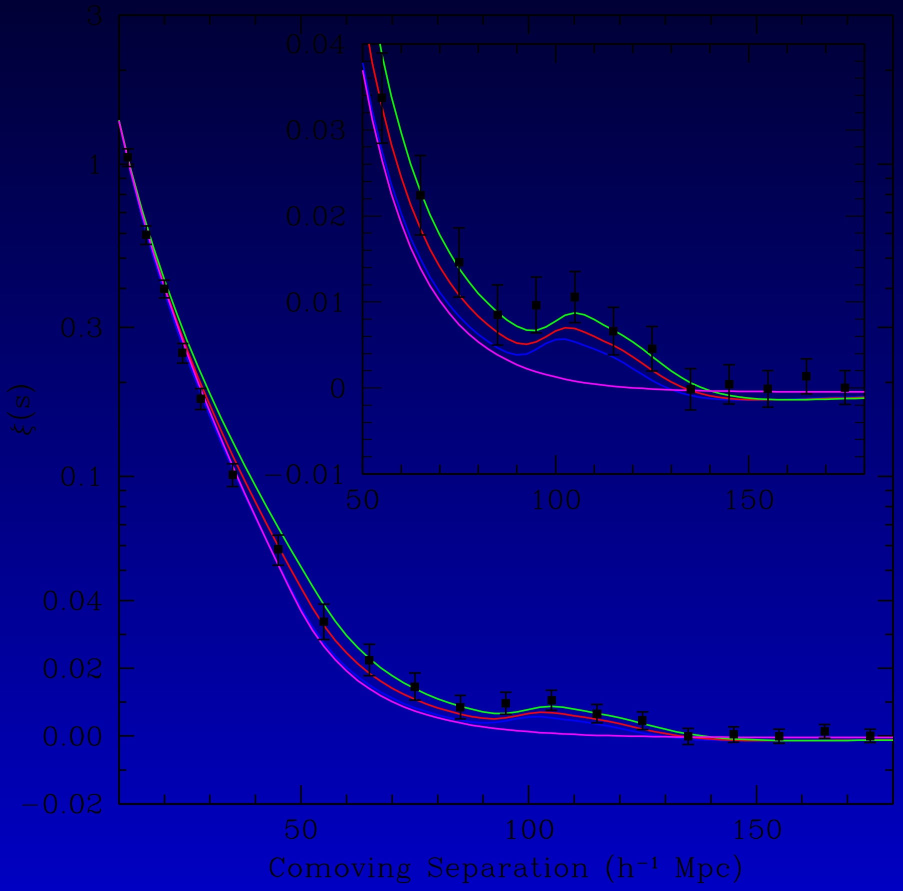
ΛCDM is successful...



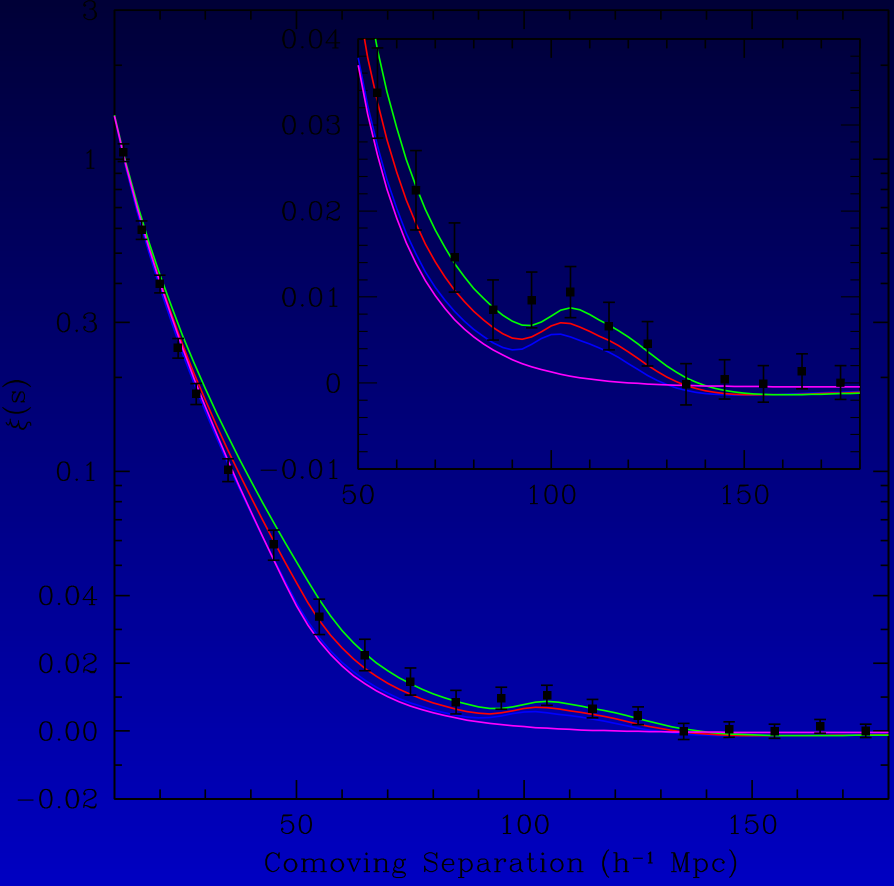
Predictive...

and successful!

and successful!



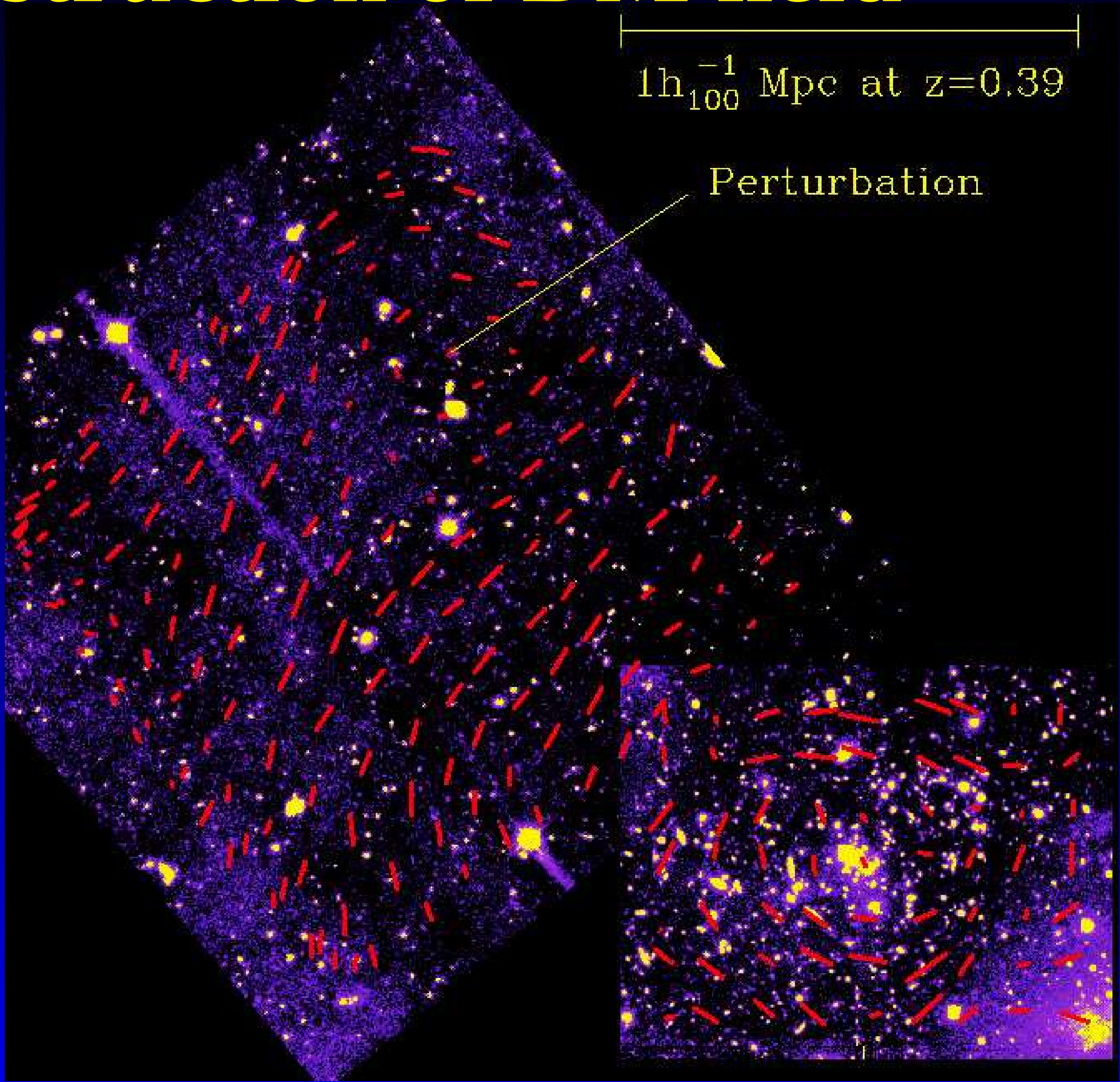
and successful!



It is difficult to compete with ΛCDM...

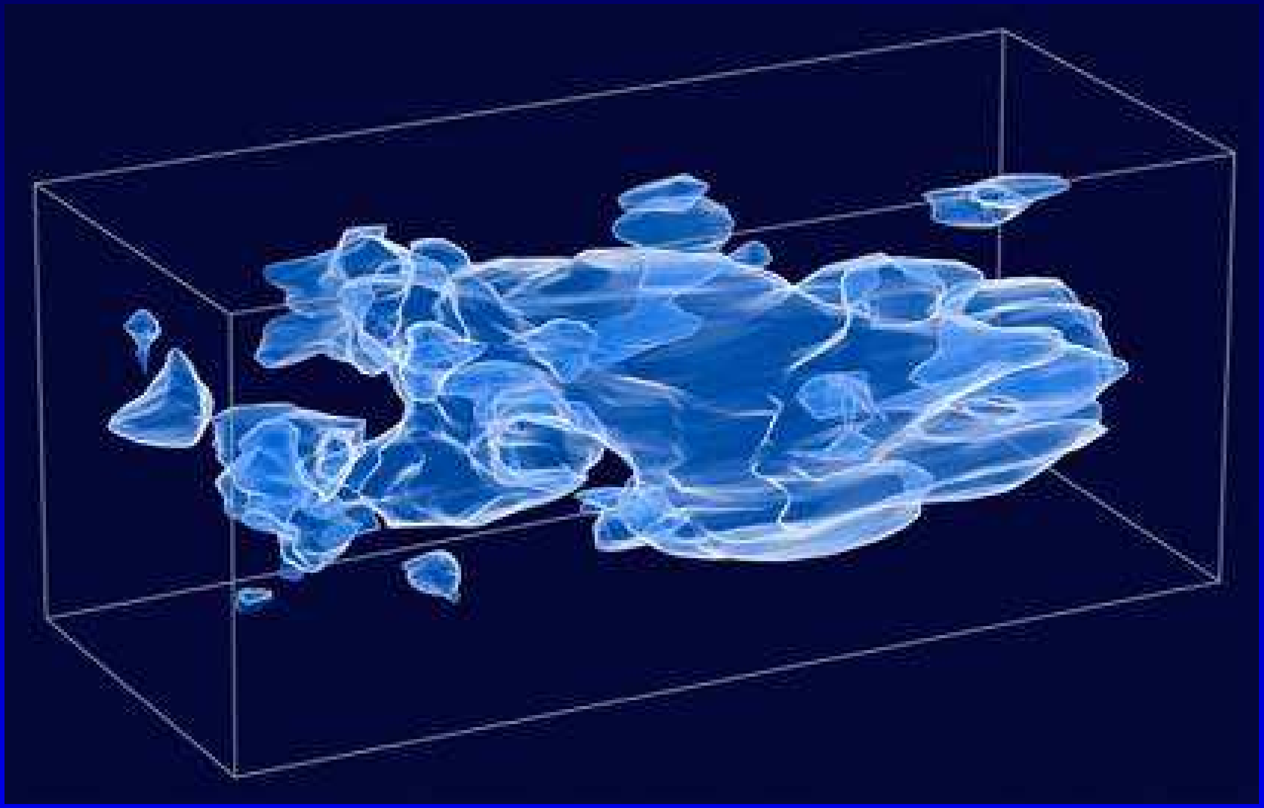
reconstruction of DM field

reconstruction of DM field



reconstruction of DM field

Weak shear surveys.



Beyond linear regime

Motivation: Understanding structure formation

Beyond linear regime

Motivation: Understanding structure formation

- LSS

Beyond linear regime

Motivation: Understanding structure formation

- LSS
- Clusters

Beyond linear regime

Motivation: Understanding structure formation

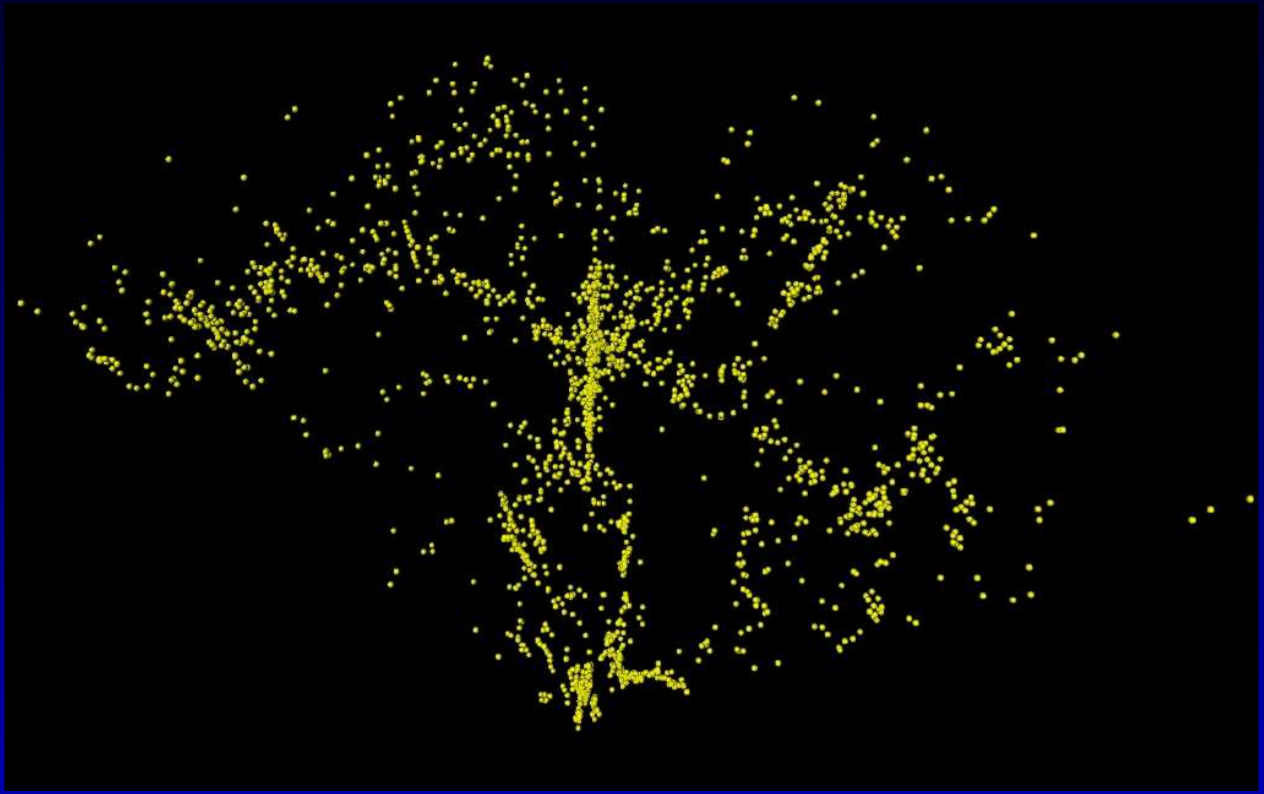
- LSS
- Clusters
- Galaxies

Beyond linear regime

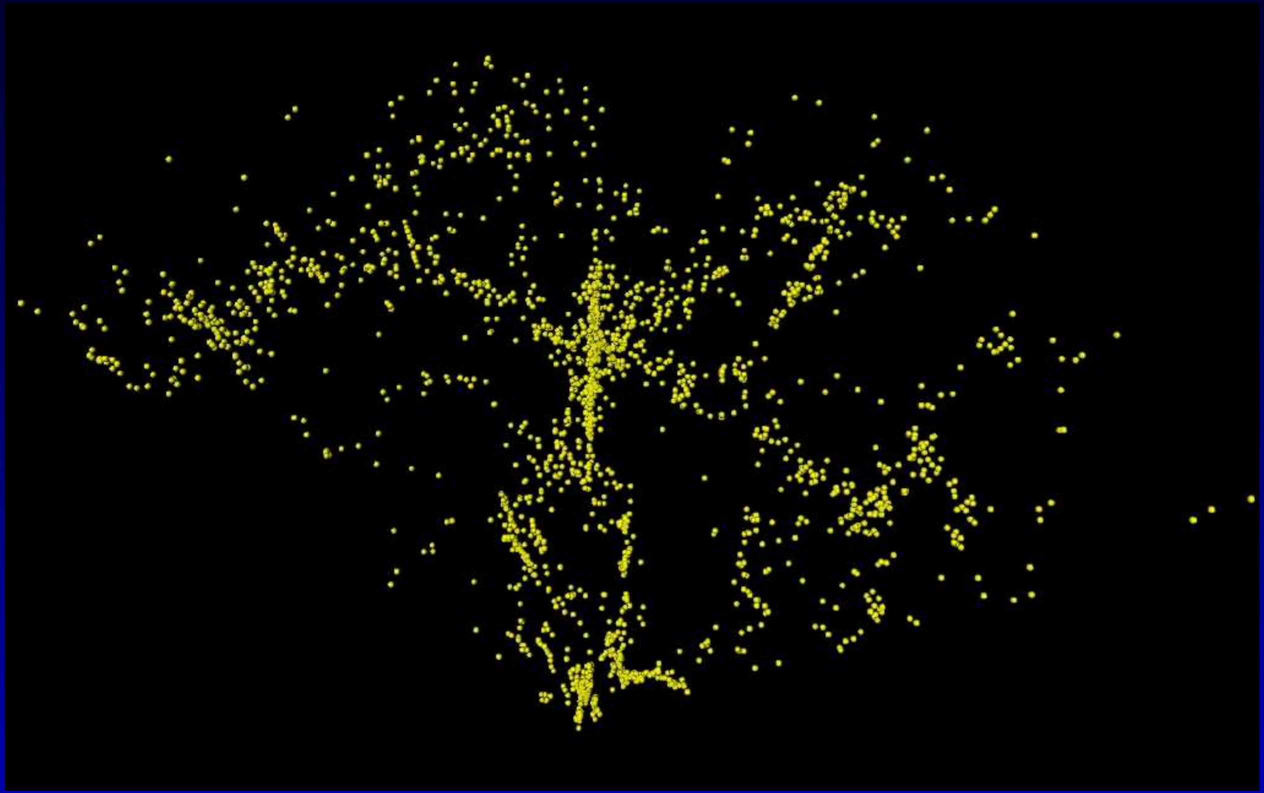
Motivation: Understanding structure formation

- LSS
- Clusters
- Galaxies
- Stars

Understanding structures formation?



Understanding structures formation?



connection between LSS and galaxy formation ?

Exact solutions

1D solutions:

Exact solutions

1D solutions:

- Planar solution (Zel'dovich, 1970)

Exact solutions

1D solutions:

- Planar solution (Zel'dovich, 1970)
- Spherical Collapse (Lemaître, 1933)

Exact solutions

1D solutions:

- Planar solution (Zel'dovich, 1970)
- Spherical Collapse (Lemaître, 1933)

Could this be transformed in an useful approximation ?

Perturbative approach

Perturbative approach

Troubles : non-linear features are extremely rapid and complex.

Perturbative approach

Troubles : non-linear features are extremely rapid and complex.

Illustration: the spherical model.

Perturbative approach

Troubles : non-linear features are extremely rapid and complex.

Illustration: the spherical model.

In GR Birkoff's theorem is an analog of Gauss theorem:
Under spherical symmetry the dynamics of a region($< R$) is independent of what is outside (and of the inner profile).

$\Omega_M > 1$ solution

No cosmological constant, no pressure.

$\Omega_M > 1$ solution

No cosmological constant, no pressure.

$R(t)$ can be developped as :

$$H_0\, t = \frac{\Omega_0}{2(\Omega_0 - 1)^{3/2}}(\phi - \sin(\phi))$$

$$\frac{1}{1+z} = \frac{R(t)}{R_0} = \frac{\Omega_0}{2(\Omega_0 - 1)}(1 - \cos(\phi))$$

$\Omega_M > 1$ solution

No cosmological constant, no pressure.

$R(t)$ can be developped as :

$$H_0\, t = \frac{\Omega_0}{2(\Omega_0 - 1)^{3/2}}(\phi - \sin(\phi))$$

$$\frac{1}{1+z} = \frac{R(t)}{R_0} = \frac{\Omega_0}{2(\Omega_0 - 1)}(1 - \cos(\phi))$$

Also describes the evolution of a spherical perturbation.

Evolution

From this one can compute the contrast density at maximum (within $\Omega_M = 1$ background):

$$1 + \Delta = \frac{9\pi^2}{16} \sim 5.55$$

while the linear amplitude is 1.01.. (linear regime : $\delta = \delta_0/(1+z)$) =

Evolution

From this one can compute the contrast density at maximum (within $\Omega_M = 1$ background):

$$1 + \Delta = \frac{9\pi^2}{16} \sim 5.55$$

while the linear amplitude is 1.01.. (linear regime : $\delta = \delta_0/(1+z)$) =

At time $2t_m$, ρ is diverging...while the linear amplitude is 1.68

Virialization

Virialization

At that $\sim$ time, the collapse reaches an equlibrium configuration, “virialized state”.

Virialization

At that $\sim$ time, the collapse reaches an equlibrium configuration, "virialized state".

Energy conservation:

$$E_c - \frac{GM}{R_f} = -\frac{GM}{R_i}$$

Virial theorem:

$$E_c = -\frac{E_p}{2}$$

Virialization

At that $\sim$ time, the collapse reaches an equlibrium configuration, “virialized state”.

Energy conservation:

$$E_c - \frac{GM}{R_f} = -\frac{GM}{R_i}$$

Virial theorem:

$$E_c = -\frac{E_p}{2}$$

so that:

$$R_f = \frac{R_i}{2}$$

and :

$$1 + \Delta = 18\pi^2 \sim 178$$

Simulations

Simulations

As soon as first gravitational N-body code was available, it was used for numerical simulations in an expanding universe...

Simulations

As soon as first gravitational N-body code was available, it was used for numerical simulations in an expanding universe...

Peebles (1970, with 50 particules...) validated the simple spherical collapse model.

Simulations

As soon as first gravitational N-body code was available, it was used for numerical simulations in an expanding universe...

Peebles (1970, with 50 particules...) validated the simple spherical collapse model.

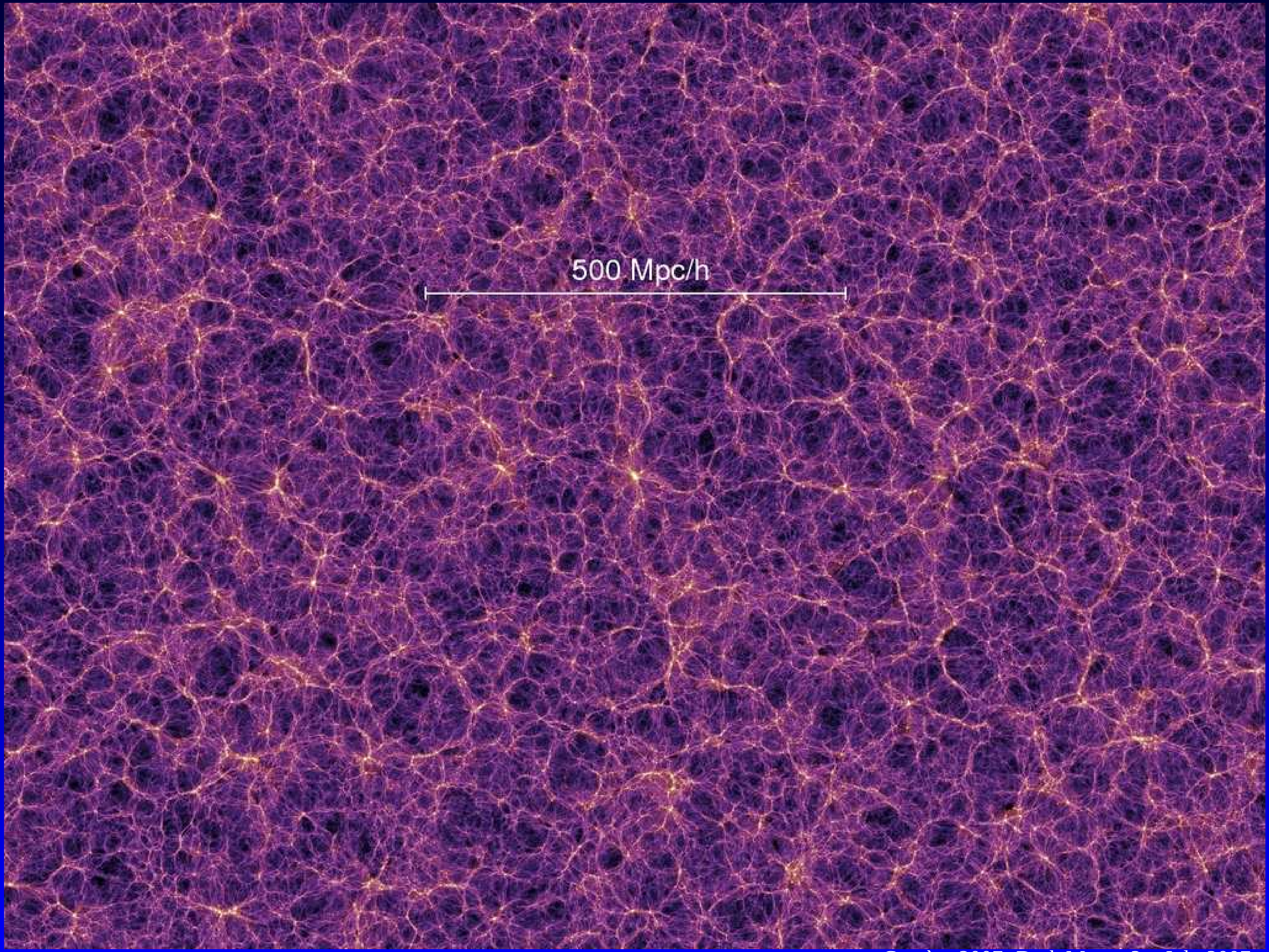
Now simulations can comprise $\sim 10^{10}$ particles, allowing movies and to include non-gravitational physics.

Results

Millenium simulation.

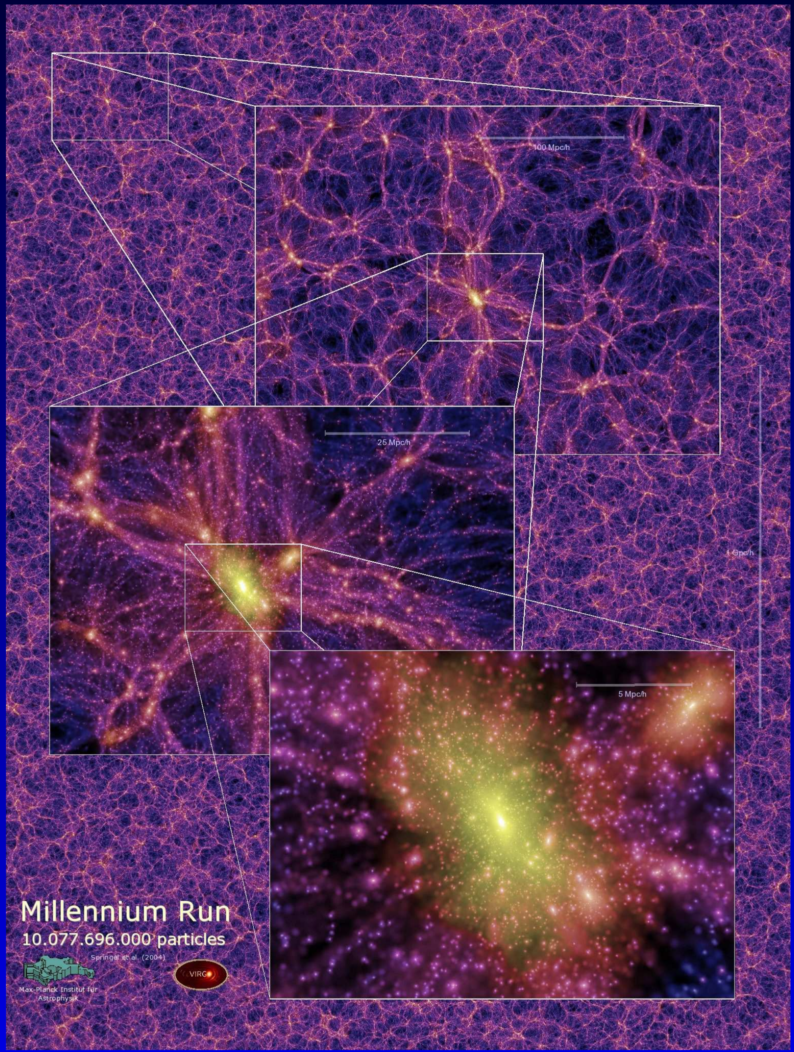
Results

Millenium simulation.



Results

Millenium simulation.

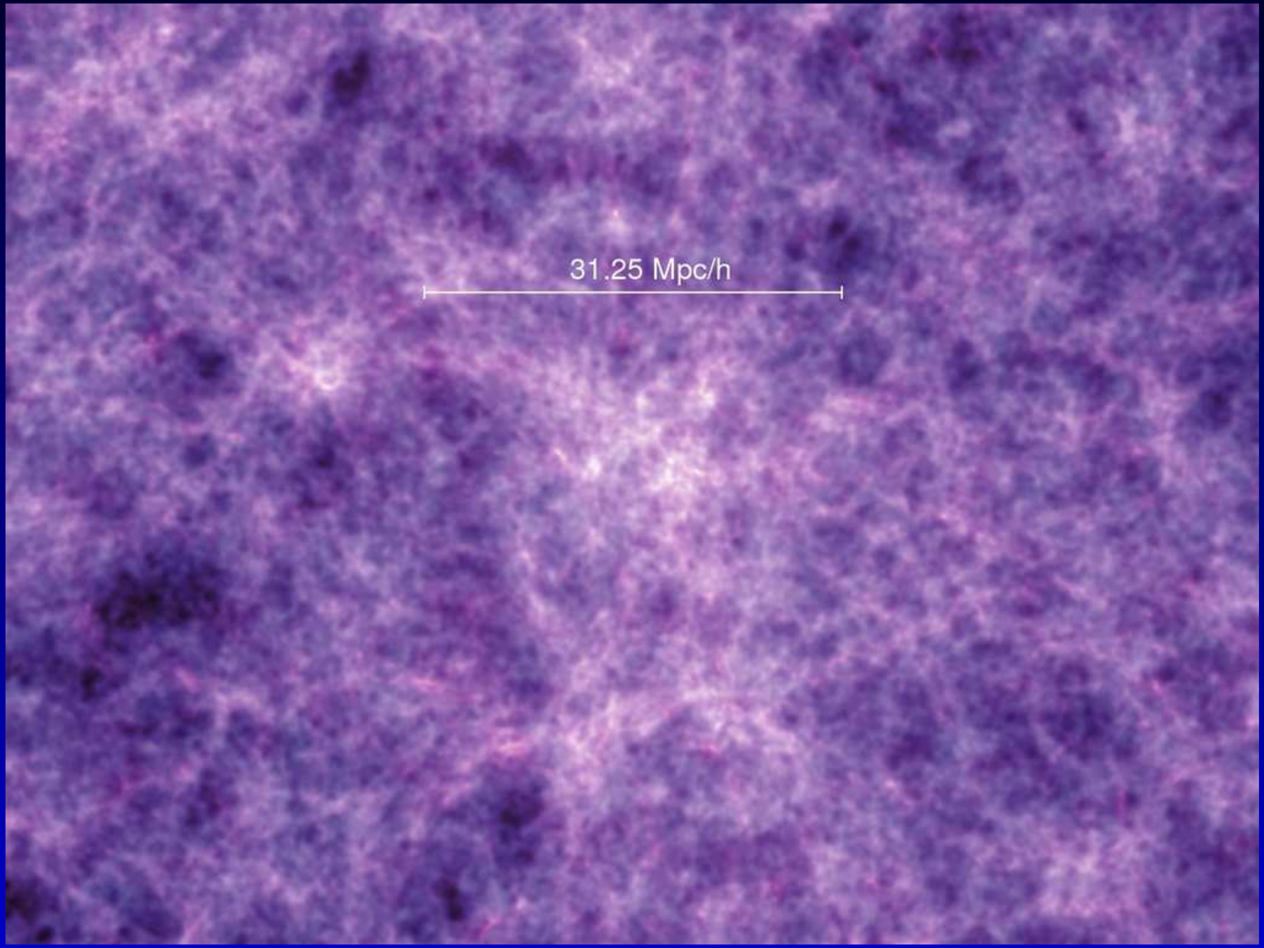


Zooming

Millenium simulation. Zooming

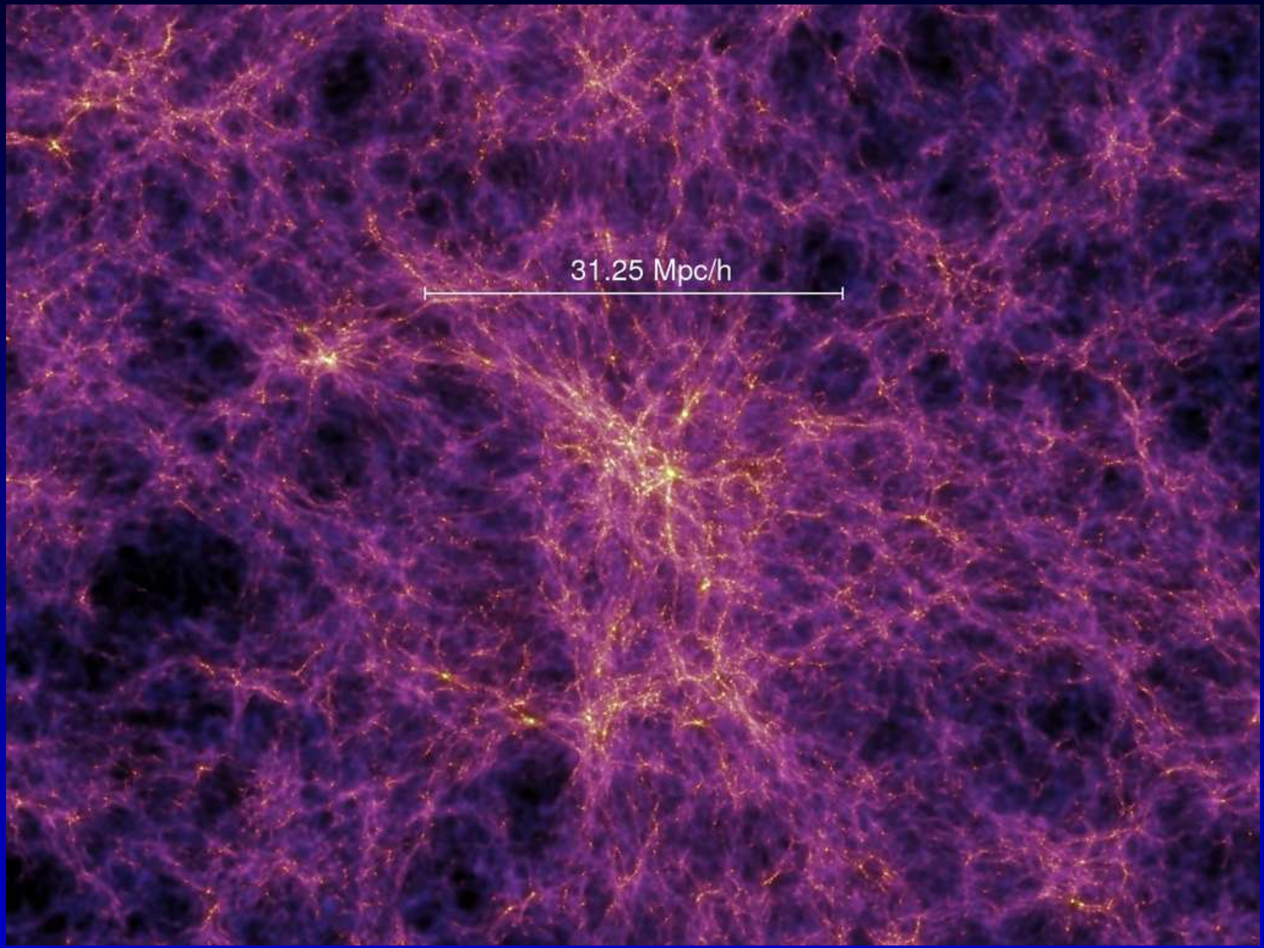
Following formation

Following formation



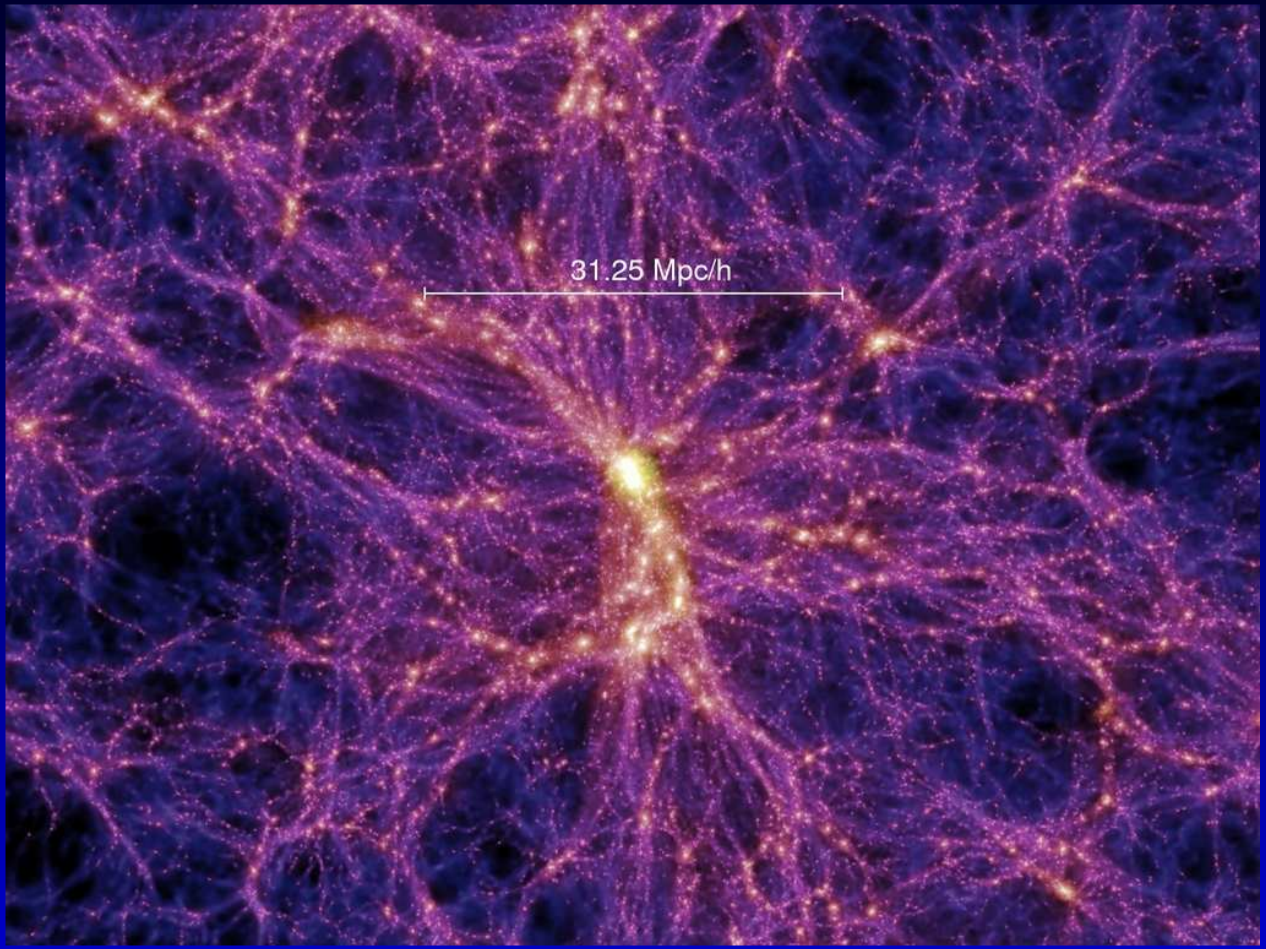
$z = 18$

Following formation



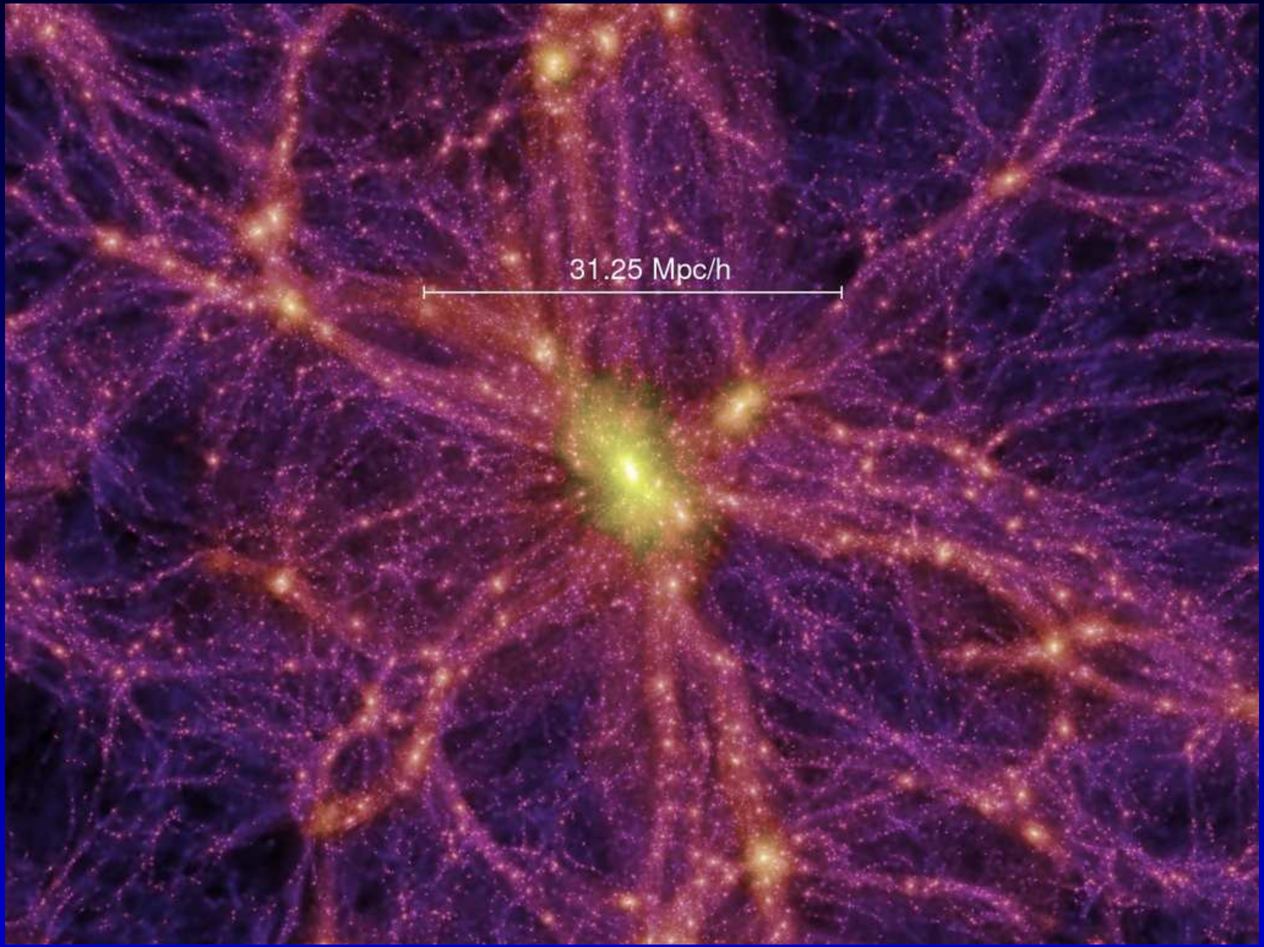
$z = 5.7$

Following formation



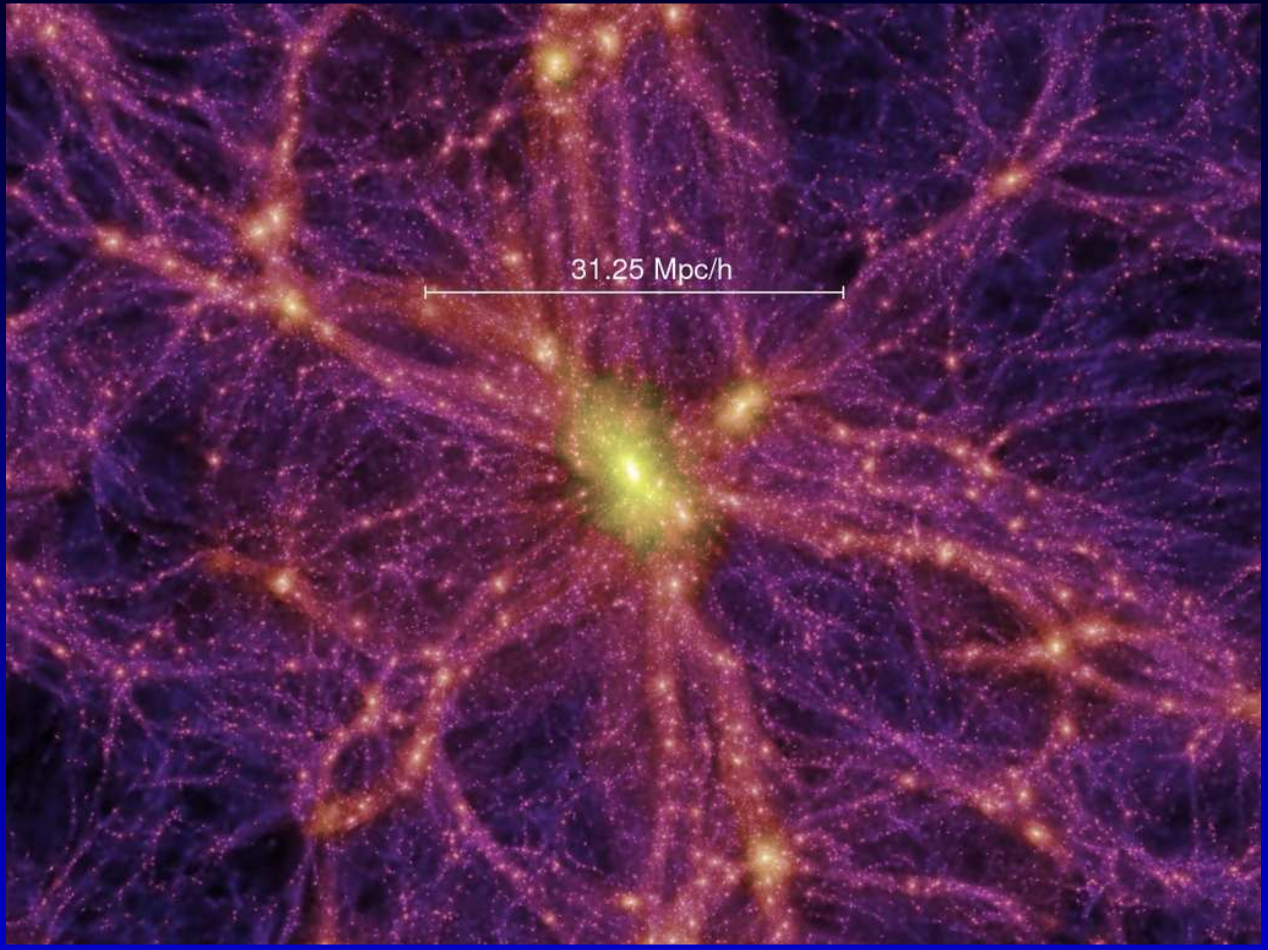
$z = 1.4$

Following formation



$z = 0$

Following formation



hierarchical structure formation

HKLM prescription

Hamilton, Kumar, Lu, Matthews (1991)

HKLM prescription

Hamilton, Kumar, Lu, Matthews (1991)

Spherical collapse in $\Omega_m = 1$:

$$\Delta = F(\delta)$$

(self similar in time)

but radius changes (and mass is conserved):

$$(1+\Delta)r^3 = (1+\delta)r_0^3 \approx r_0^3$$

(δ is the linear amplitude).

HKLM prescription

Number of neighbours in excess :

$$N_b = \int_0^r \xi(r) dV$$

so

$$\Delta \equiv \overline{\xi}$$

HKLM prescription

Number of neighbours in excess :

$$N_b = \int_0^r \xi(r) dV$$

so

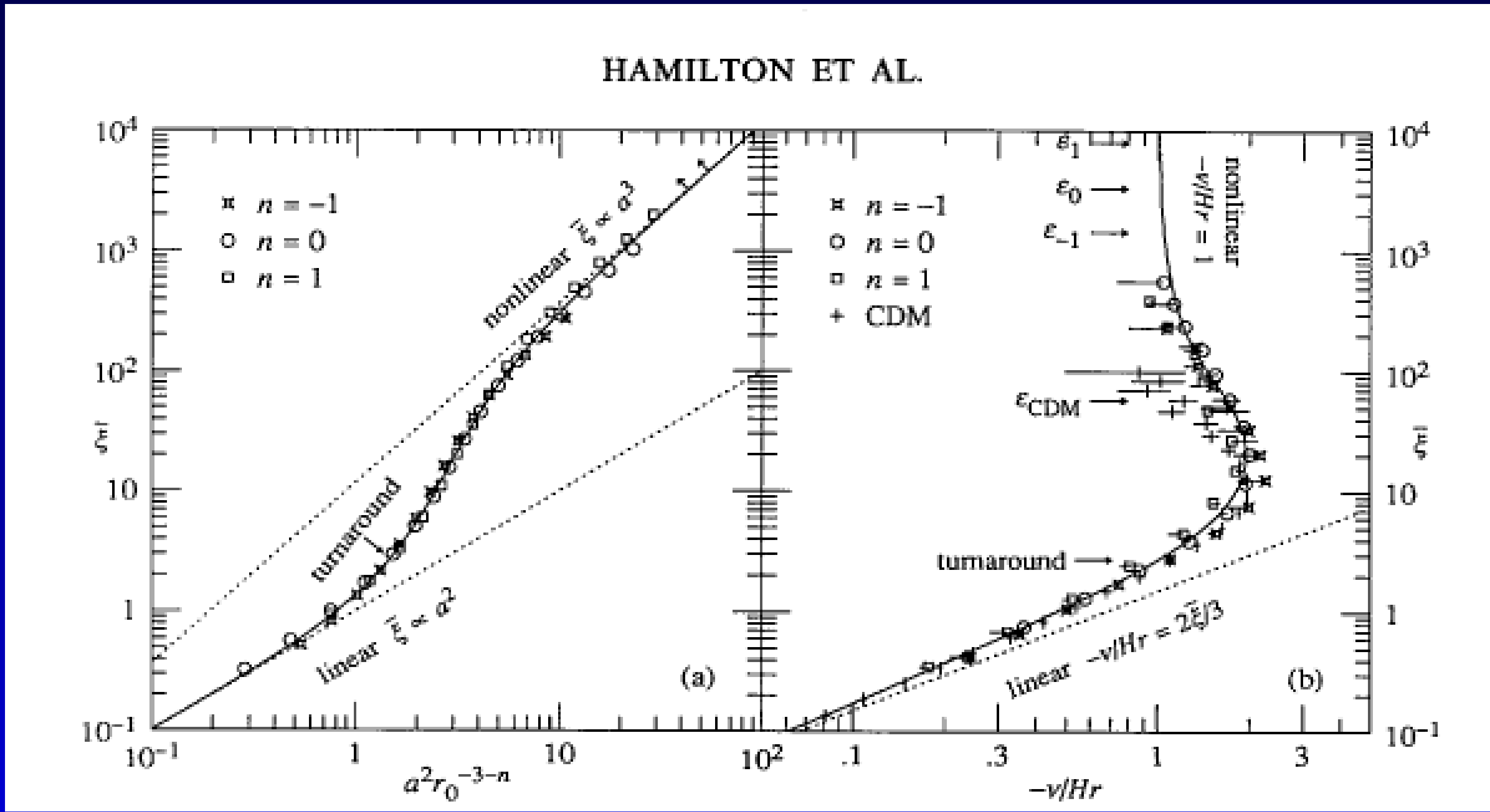
$$\Delta \equiv \bar{\xi}$$

Anzatz:

$$\bar{\xi} r^3 = F(\bar{\xi}_0 r_0^3)$$

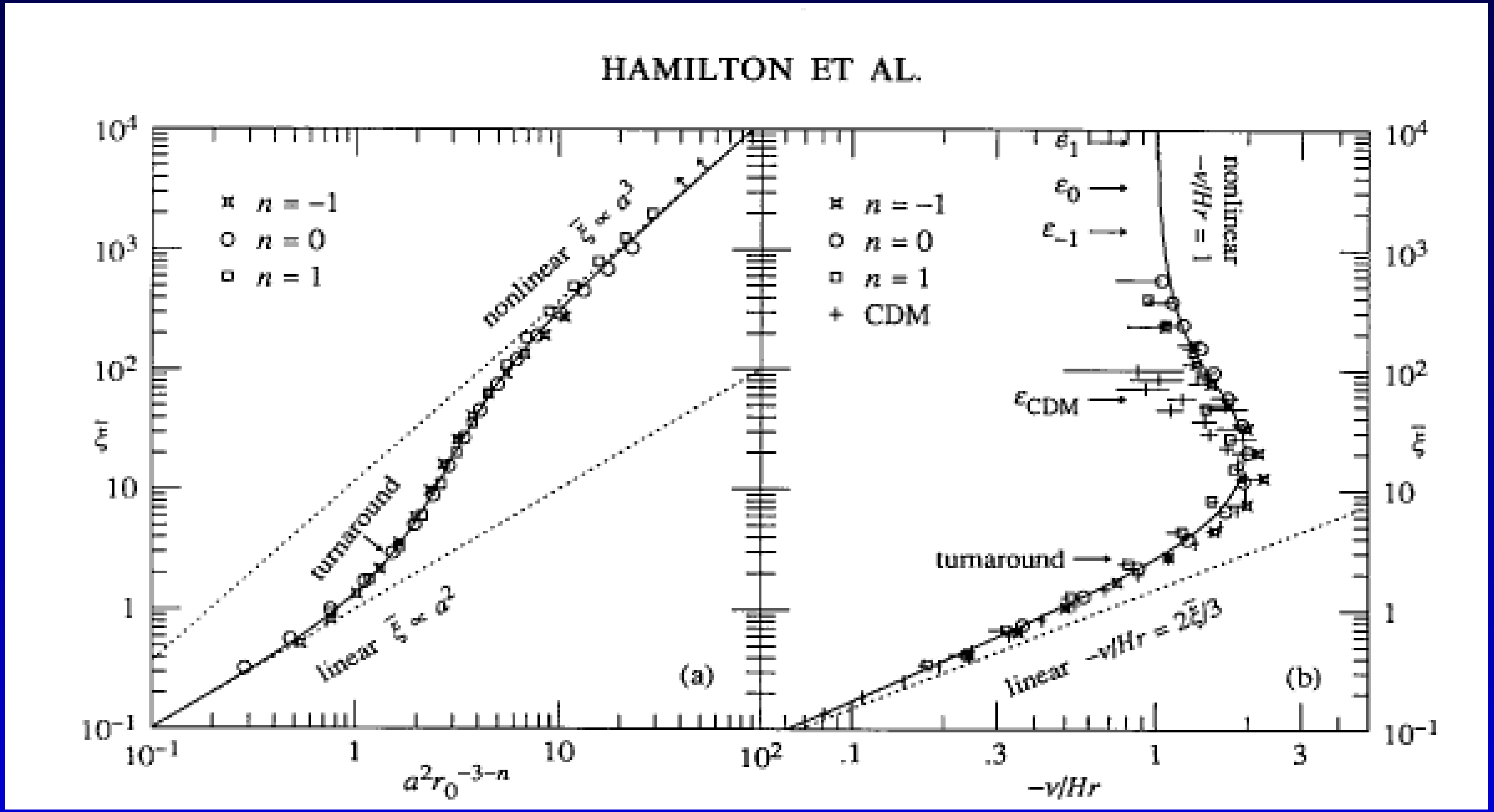
HKLM prescription

Comparison with numerical simulations:



HKLM prescription

Comparison with numerical simulations:



Peacock and Dodds have reformulated this in Fourier Space...and provide formula for arbitrary cosmology.

Back to the correlation function

Why is the correlation function is a power law?

Back to the correlation function

Why is the correlation function is a power law?

CDM provide the appropriate shape to explain both the (power law) non-linear shape of ξ and the linear shape on large scale.

NFW profile

From numerical simulations DM halo appear to be well fitted by the so-called NFW profile:

$$\frac{\rho(r)}{\rho_c} = \frac{\delta_c}{(r/r_c)(1. + r/r_c)^2}$$

Two parameters: mass in some radius (for instance $\Delta = 200$) and one parameter: the concentration c : $r_c = r_{200}/c$

NFW profile

From numerical simulations DM halo appear to be well fitted by the so-called NFW profile:

$$\frac{\rho(r)}{\rho_c} = \frac{\delta_c}{(r/r_c)(1. + r/r_c)^2}$$

Two parameters: mass in some radius (for instance $\Delta = 200$) and one parameter: the concentration c : $r_c = r_{200}/c$

Prediction: halos are more or less identical...

Scaling laws

From the simple spherical model and mass-radius relation:

$$M \propto \Delta \Omega_m \rho_c (1+z)^3 R_v^3$$

or

$$R_v \propto \left(\frac{M}{\Delta(\Omega_m, ...)\Omega_m} \right)^{1/3} \frac{1}{1+z}$$

Scaling laws

From the simple spherical model and mass-radius relation:

$$M \propto \Delta\Omega_m\rho_c(1+z)^3 R_v^3$$

or

$$R_v \propto \left(\frac{M}{\Delta(\Omega_m,\ldots)\Omega_m}\right)^{1/3}\frac{1}{1+z}$$

one can infer the scaling of velocity dispersion with mass and redshift:

$$V^2 \propto \frac{GM}{R} \propto \Omega_m\Delta M^{2/3}(1+z)$$

and it works...

The mass function

Inspired from Press and Schechter (1974)
The density field $\rho(x)$ has to be smoothed:

$$\tilde{\delta}(x) = \int \delta(x+u) W_R(u) du$$

and

$$\overline{\tilde{\delta}^2(x)} = \sigma^2(R)$$

The mass function

Inspired from Press and Schechter (1974)

The density field $\rho(x)$ has to be smoothed:

$$\tilde{\delta}(x) = \int \delta(x+u) W_R(u) du$$

and

$$\overline{\tilde{\delta}^2(x)} = \sigma^2(R)$$

For a top hat window (!):

$$M(R) = \frac{4\pi}{3} R^3 \overline{\rho}$$

The mass function

dV will be included in a NL object with mass greater than M if included in a fluctuation of radius $> R$ and witch is satisfying the non linear criteria.

$$\int_M^{+\infty} mn(m)dm = \overline{\rho} \int \mathcal{F}_\delta(\delta)s(\delta)d\delta \sim \overline{\rho} \int_{\delta_{NL}}^{+\infty} \mathcal{F}_\delta(\delta)d\delta$$

The mass function

dV will be included in a NL object with mass greater than M if included in a fluctuation of radius $> R$ and witch is satisfying the non linear criteria.

$$\int_M^{+\infty} mn(m)dm = \overline{\rho} \int \mathcal{F}_\delta(\delta)s(\delta)d\delta \sim \overline{\rho} \int_{\delta_{NL}}^{+\infty} \mathcal{F}_\delta(\delta)d\delta$$

for a sharp threshold:

$$\int_M^{+\infty} mn(m)dm = \overline{\rho} \int_{\nu_{NL}}^{+\infty} \mathcal{F}(\nu)d\nu$$

The mass function

Following the spherical model:

$$\nu_{NL} = \frac{\delta_{NL}}{\sigma(M)}$$

Just derive against M:

$$N(M) = -\frac{\rho}{M^2\sigma(M)}\delta_{NL}\frac{\ln\sigma}{\ln M}\mathcal{F}(\nu_{NL})$$

The mass function

Following the spherical model:

$$\nu_{NL} = \frac{\delta_{NL}}{\sigma(M)}$$

Just derive against M:

$$N(M) = -\frac{\rho}{M^2\sigma(M)}\delta_{NL}\frac{\ln\sigma}{\ln M}\mathcal{F}(\nu_{NL})$$

Press and Schechter use a Gaussian:

$$\mathcal{F}(\nu) = \sqrt{\frac{2}{\pi}}\exp(-\frac{\nu^2}{2})$$

The mass function

Following the spherical model:

$$\nu_{NL} = \frac{\delta_{NL}}{\sigma(M)}$$

Just derive against M:

$$N(M) = -\frac{\rho}{M^2\sigma(M)}\delta_{NL}\frac{\ln\sigma}{\ln M}\mathcal{F}(\nu_{NL})$$

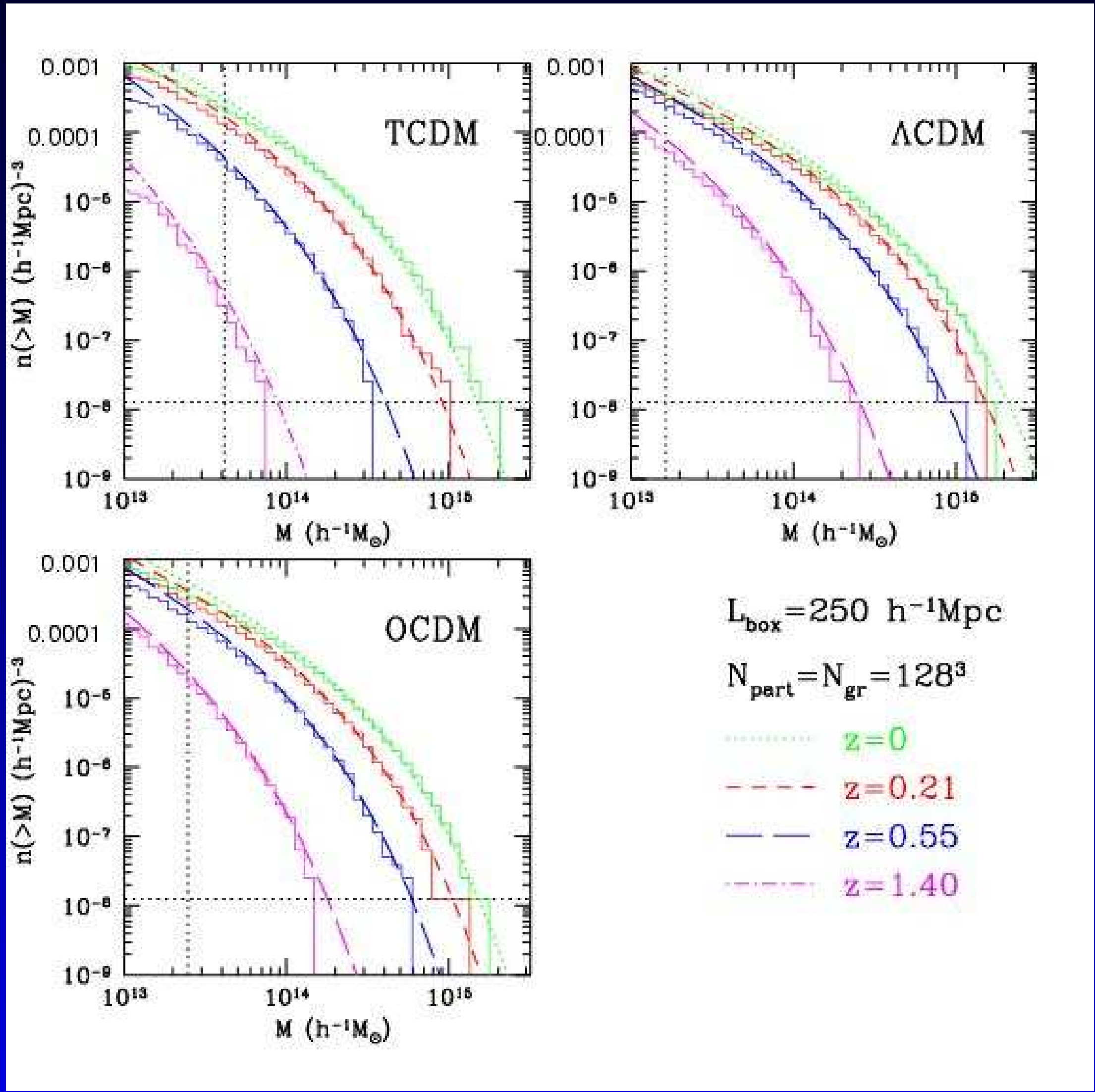
Press and Schechter use a Gaussian:

$$\mathcal{F}(\nu) = \sqrt{\frac{2}{\pi}}\exp(-\frac{\nu^2}{2})$$

and test it against numerical simulations...

But...

But...



It actually works!

Jenkins formula

More recent expression for $\mathcal{F}$ from Jenkins et al. (2001):

$$\mathcal{F}(\nu) = \sqrt{\frac{2A}{\pi}} C \exp(-0.5 A \nu^2)(1. + (1./(A\nu)^2)^Q)$$

with $A = 0.707$ $C = 0.3222$ $Q = 0.3$.

Jenkins formula

More recent expression for $\mathcal{F}$ from Jenkins et al. (2001):

$$\mathcal{F}(\nu) = \sqrt{\frac{2A}{\pi}} C \exp(-0.5A\nu^2)(1. + (1./(A\nu)^2)^Q)$$

with $A = 0.707$ $C = 0.3222$ $Q = 0.3$.
Allows to investigate structure formation:

History of individual structure is missing: merging tree $\longrightarrow$ semi-analytical method "SAM" in order to model galaxy formation:assembly/evolution.

Conclusions

- There is a convincing modeling of dark matter distribution and evolution in both linear and non-linear regimes to constrain cosmological scenario.

Conclusions

- There is a convincing modeling of dark matter distribution and evolution in both linear and non-linear regimes to constrain cosmological scenario.
- Warning: data come through “light” which is coming from baryons and this was almost not discussed in these lectures...