Measuring deviations from a cosmological constant: a field–space parametrisation

Elisabetta Majerotto

Institute of Cosmology & Gravitation

In collaboration with Robert Crittenden and Federico Piazza

Based on

Crittenden, Majerotto, Piazza, Phys.Rev.Lett.98:251301,2007

Outline

1. Motivation

Outline

Outline

Outline

1. Motivation
2. Quintessence
3. Field space parametrisation of Dark Energy
4. Thawing vs freezing models

Outline

Outline

Motivation

Studies of Dark Energy are hindered by the lack of specific alternatives to Λ.

Motivation

Studies of Dark Energy are hindered by the lack of specific alternatives to Λ.

- Once we allow that the DE density varies with time, how it evolves appears often as a free function

Motivation

Studies of Dark Energy are hindered by the lack of specific alternatives to Λ.

- Once we allow that the DE density varies with time, how it evolves appears often as a free function
- $\Rightarrow$ The dynamical evolution of DE is quantified by phenomenological fitting models

Motivation

Studies of Dark Energy are hindered by the lack of specific alternatives to Λ.

- Once we allow that the DE density varies with time, how it evolves appears often as a free function
- $\Rightarrow$ The dynamical evolution of DE is quantified by phenomenological fitting models
- But these models have often little theoretical motivation and may be misleading or difficult to explain (e.g. cross the phantom divide)

Motivation

Studies of Dark Energy are hindered by the lack of specific alternatives to Λ.

- Once we allow that the DE density varies with time, how it evolves appears often as a free function
- $\Rightarrow$ The dynamical evolution of DE is quantified by phenomenological fitting models
- But these models have often little theoretical motivation and may be misleading or difficult to explain (e.g. cross the phantom divide)

We develop a description of DE based on the dynamics of the scalar field exact in the limit $w \to -1$

Quintessence

Quintessence: a scalar field ϕ rolling down its potential $V(\phi)$ that comes to dominate energy density of the Universe *(C. Wetterich 1988, B. Ratra & P.J.E. Peebles, 1988)*

Quintessence

Quintessence: a scalar field ϕ rolling down its potential $V(\phi)$ that comes to dominate energy density of the Universe *(C. Wetterich 1988, B. Ratra & P.J.E. Peebles, 1988)*

- The evolution of DE is determined by $V(\phi)$: for every $w(z)$ there is a $V(\phi)$ able to produce it **but** not any $V(\phi)$ is equally likely

Quintessence

Quintessence: a scalar field ϕ rolling down its potential $V(\phi)$ that comes to dominate energy density of the Universe *(C. Wetterich 1988, B. Ratra & P.J.E. Peebles, 1988)*

- The evolution of DE is determined by $V(\phi)$: for every $w(z)$ there is a $V(\phi)$ able to produce it **but** not any $V(\phi)$ is equally likely
- $\Rightarrow$ in Bayesian statistics contest what priors to put on the functional space of $w(z)$?

Quintessence

Quintessence: a scalar field ϕ rolling down its potential $V(\phi)$ that comes to dominate energy density of the Universe *(C. Wetterich 1988, B. Ratra & P.J.E. Peebles, 1988)*

- The evolution of DE is determined by $V(\phi)$: for every $w(z)$ there is a $V(\phi)$ able to produce it **but** not any $V(\phi)$ is equally likely
- $\Rightarrow$ in Bayesian statistics contest what priors to put on the functional space of $w(z)$?
- We might expect that relevant $\Delta\mathcal{L} \sim \Delta\phi \sim$ some"smoothness scale" M

Quintessence

Quintessence: a scalar field ϕ rolling down its potential $V(\phi)$ that comes to dominate energy density of the Universe *(C. Wetterich 1988, B. Ratra & P.J.E. Peebles, 1988)*

- The evolution of DE is determined by $V(\phi)$: for every $w(z)$ there is a $V(\phi)$ able to produce it **but** not any $V(\phi)$ is equally likely
- $\Rightarrow$ in Bayesian statistics contest what priors to put on the functional space of $w(z)$?
- We might expect that relevant $\Delta\mathcal{L} \sim \Delta\phi \sim$ some"smoothness scale" M
- $V(\phi_0) \sim M_P^2 H_0^2$ (present energy density) and $V''(\phi_0) \lesssim H_0^2$

Quintessence

Quintessence: a scalar field ϕ rolling down its potential $V(\phi)$ that comes to dominate energy density of the Universe *(C. Wetterich 1988, B. Ratra & P.J.E. Peebles, 1988)*

- The evolution of DE is determined by $V(\phi)$: for every $w(z)$ there is a $V(\phi)$ able to produce it **but** not any $V(\phi)$ is equally likely
- $\Rightarrow$ in Bayesian statistics contest what priors to put on the functional space of $w(z)$?
- We might expect that relevant $\Delta\mathcal{L} \sim \Delta\phi \sim$ some"smoothness scale" M
- $V(\phi_0) \sim M_P^2 H_0^2$ (present energy density) and $V''(\phi_0) \lesssim H_0^2$

$\Rightarrow$ We can introduce the "**smoothness scale**" M by defining:

$$V(\phi) = M_P^2 H_0^2 f(\phi/M)$$

assuming that f and its derivatives are of order ≤ 1

Field space parametrisation of Dark Energy

Inflation

The dynamics is independent of the initial condition $\Rightarrow$ define **slow roll parameters** that substantially describe the evolution:

$$\begin{aligned}\epsilon &\propto (V'/V)^2 \\ \eta &\propto V''/V\end{aligned}$$

Field space parametrisation of Dark Energy

Inflation

The dynamics is independent of the initial condition $\Rightarrow$ define **slow roll parameters** that substantially describe the evolution:

$$\begin{aligned} \epsilon &\propto (V'/V)^2 \\ \eta &\propto V''/V \end{aligned}$$

Late Universe

Quintessence is effectively **late time inflation**, but Dark Matter makes things more complicated

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \\ 6H^2 = \rho_m + \rho_\phi, \end{aligned}$$

The acceleration term $\beta \equiv \frac{\ddot{\phi}}{3H\dot{\phi}}$ is not negligible any more.

Field space parametrisation of Dark Energy

$$1 + w \equiv 1 + \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{2}{3} \frac{V'^2}{6H^2(1+\beta)^2\rho_\phi}$$

Field space parametrisation of Dark Energy

$$1 + w \equiv 1 + \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{2}{3} \, \frac{V'^2}{6H^2(1+\beta)^2 \rho_\phi}$$

$$\kappa(\phi) \equiv \frac{V'}{V(1+\beta)}$$

Field space parametrisation of Dark Energy

$$1 + w \equiv 1 + \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{2}{3}\,\frac{V'^2}{6H^2(1+\beta)^2\rho_\phi}$$

$$\kappa(\phi) \equiv \frac{V'}{V(1+\beta)}$$

Since typically in thawing models (see later in the talk) β moves smoothly from $1/2$ during matter domination to 0 during DE domination, we can assume $\kappa(\phi)$ small

Field space parametrisation of Dark Energy

$$1 + w \equiv 1 + \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{2}{3}\,\frac{V'^2}{6H^2(1+\beta)^2\rho_\phi}$$

$$\kappa(\phi) \equiv \frac{V'}{V(1+\beta)}$$

Since typically in thawing models (see later in the talk) β moves smoothly from $1/2$ during matter domination to 0 during DE domination, we can assume $\kappa(\phi)$ small

$$1 + w = \frac{1}{6}\kappa^2(\phi)(1-w)^2\Omega_\phi$$

$$\frac{d\phi}{d\ln a} = -\kappa(\phi)\Omega_\phi(1-w)$$

Field space parametrisation of Dark Energy

$$1 + w \equiv 1 + \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{2}{3}\,\frac{V'^2}{6H^2(1+\beta)^2\rho_\phi}$$

$$\kappa(\phi) \;\equiv\; \frac{V'}{V(1+\beta)}$$

Since typically in thawing models (see later in the talk) β moves smoothly from $1/2$ during matter domination to 0 during DE domination, we can assume $\kappa(\phi)$ small

If $1 + w \ll 1$ then drop terms $\mathcal{O}(1+w)$ and $\Omega_\phi \to \Omega_\Lambda(a) = \frac{\Omega_{\phi 0}}{\Omega_{\phi 0}+(1-\Omega_{\phi 0})a^{-3}}$

$$\begin{aligned} 1 + w &= \frac{1}{6}\kappa^2(\phi)(1-w)^2\Omega_\phi \\ \frac{d\phi}{d\ln a} &= -\kappa(\phi)\Omega_\phi(1-w) \end{aligned}$$

Field space parametrisation of Dark Energy

$$1 + w \equiv 1 + \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{2}{3}\,\frac{V'^2}{6H^2(1+\beta)^2\rho_\phi}$$

$$\kappa(\phi) \equiv \frac{V'}{V(1+\beta)}$$

Since typically in thawing models (see later in the talk) β moves smoothly from $1/2$ during matter domination to 0 during DE domination, we can assume $\kappa(\phi)$ small

If $1 + w \ll 1$ then drop terms $\mathcal{O}(1 + w)$ and $\Omega_\phi \to \Omega_\Lambda(a) = \frac{\Omega_{\phi 0}}{\Omega_{\phi 0} + (1 - \Omega_{\phi 0})a^{-3}}$

$$\begin{aligned} 1 + w &= \frac{2}{3}\kappa^2(\phi)\Omega_\Lambda(a) \\ \frac{d\phi}{d\ln a} &= -2\kappa(\phi)\Omega_\Lambda(a). \end{aligned}$$

Field space parametrisation of Dark Energy

- Typically in thawing models β moves smoothly from $1/2$ during matter domination to 0 during DE domination $\Rightarrow$ we can assume $\kappa(\phi)$ small

Field space parametrisation of Dark Energy

- Typically in thawing models β moves smoothly from $1/2$ during matter domination to 0 during DE domination $\Rightarrow$ we can assume $\kappa(\phi)$ small
- Only a small region $\Delta\phi$ of phase space is probed by observation:

$$1 \gg 1 + w(\phi) = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{(\Delta\phi)^2}{6\Omega_\phi}$$

$\Rightarrow$ recently the field has moved little in Planck units

Field space parametrisation of Dark Energy

- Typically in thawing models β moves smoothly from $1/2$ during matter domination to 0 during DE domination $\Rightarrow$ we can assume $\kappa(\phi)$ small
- Only a small region $\Delta\phi$ of phase space is probed by observation:

$$1 \gg 1 + w(\phi) = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{(\Delta\phi)^2}{6\Omega_\phi}$$

$\Rightarrow$ recently the field has moved little in Planck units

Linear approximation

$$\kappa(\phi) = \kappa_{\mathbf{0}} + \kappa_{\mathbf{1}}(\phi - \phi_{\mathbf{0}})$$

$$\kappa(a) = \kappa_0 \left[\frac{\Omega_\Lambda(a)}{a^3\Omega_{\phi 0}}\right]^{2\kappa_1/3}$$

$$\rho_\phi \propto \exp[\mathcal{I}(a)]$$

Where $\mathcal{I} \simeq \frac{1}{2\kappa_1}(\kappa^2(a) - \kappa_0^2)$

Thawing vs freezing models

Quintessence models have been separated into two classes *(R.R. Caldwell & E.V. Linder, 2005)*:

Thawing models

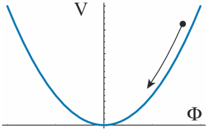
The field is fixed at early times at $w = -1$ and only begins to ”thaw” recently towards $w > -1$.
Typically $V(\phi) \propto \phi^n$.

Thawing vs freezing models

Quintessence models have been separated into two classes *(R.R. Caldwell & E.V. Linder, 2005)*:

Thawing models

The field is fixed at early times at $w = -1$ and only begins to ”thaw” recently towards $w > -1$.
Typically $V(\phi) \propto \phi^n$.



Thawing vs freezing models

Quintessence models have been separated into two classes *(R.R. Caldwell & E.V. Linder, 2005)*:

Freezing models

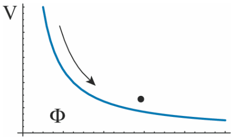
The field begins at $w > -1$ and rolls quickly down a potential at early times, and now starts to slow down as the potential flattens towards $w = -1$. Typically $V(\phi) \propto 1/\phi^n$.

Thawing vs freezing models

Quintessence models have been separated into two classes *(R.R. Caldwell & E.V. Linder, 2005)*:

Freezing models

The field begins at $w > -1$ and rolls quickly down a potential at early times, and now starts to slow down as the potential flattens towards $w = -1$. Typically $V(\phi) \propto 1/\phi^n$.



Thawing vs freezing models

Quintessence models have been separated into two classes *(R.R. Caldwell & E.V. Linder, 2005)*:

Thawing models

The field is fixed at early times at $w = -1$ and only begins to "thaw" recently towards $w > -1$.
Typically $V(\phi) \propto \phi^n$.

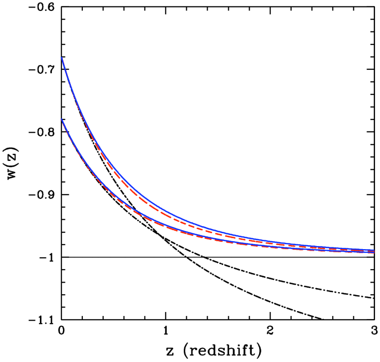
Freezing models

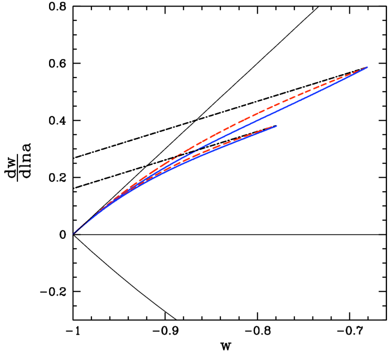
The field begins at $w > -1$ and rolls quickly down a potential at early times, and now starts to slow down as the potential flattens towards $w = -1$.
Typically $V(\phi) \propto 1/\phi^n$.

Our parametrisation works well for the thawing models. In fact...

Thawing vs freezing models

- solid blue → exact integration for a quadratic potential
- dashed red → slow roll parametrisation
- dot–dashed → linear ($w(a) = w_0 + w_a(1 - a)$) parametrisation





Comparison with a linear parametrisation

We can convert from our field space variables to redshift space variables:

Comparison with a linear parametrisation

We can convert from our field space variables to redshift space variables:

$$
\begin{aligned}
w(z=0) &= -1 + \frac{2\,\Omega_{\phi 0}\kappa_0^2}{3} \\
\left.\frac{dw}{d\ln a}\right|_{z=0} &= \frac{2}{3}\kappa_0^2\Omega_{\phi 0}[3 - \Omega_{\phi 0}(3 + 4\kappa_1)]\,.
\end{aligned}
$$

Comparison with a linear parametrisation

We can convert from our field space variables to redshift space variables:

$$\begin{aligned} w(z=0) &= -1 + \frac{2\,\Omega_{\phi 0}\kappa_0^2}{3} \\ \left.\frac{dw}{d\ln a}\right|_{z=0} &= \frac{2}{3}\kappa_0^2\Omega_{\phi 0}[3 - \Omega_{\phi 0}(3 + 4\kappa_1)]\,. \end{aligned}$$

With this conversion we can compare our parametrisation with a very used linear one *(M. Chevallier & D. Polarski, 2001, E.V. Linder. 2003)*:

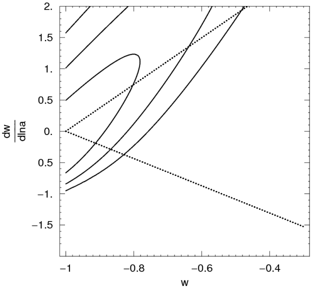
$$w(a) = w_0 + w_a(1 - a)$$

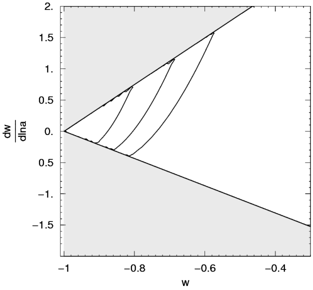
where $\frac{dw}{d\ln a}\big|_{z=0} = -w_a$.

Comparison with a linear parametrisation

Constant likelihood contours
resulting from SN (*Astier et al., 2005*), CMB (*Spergel et al., 2006*) and BAO (*Eisenstein et al., 2005*) data, fixing for simplicity $\Omega_{\phi 0} = 0.74$ and imposing κ_0 and κ_1 to be between 0 and 1.

- **left** **linear ($w(a) = w_0 + w_a(1-a)$) model** for the equation of state of DE
- **right** our **scalar field motivated** parametrisation

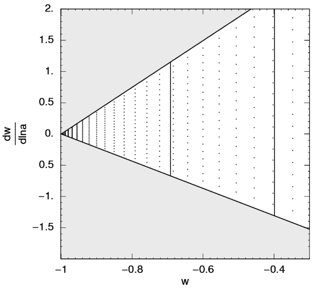


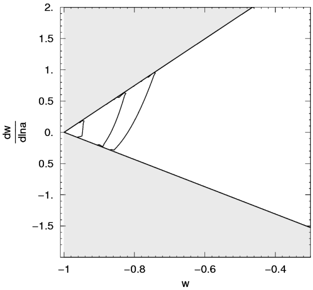


Comparison with a linear parametrisation

left prior from a uniform prior in $\kappa_0 - \kappa_1$ space = Jacobian of the transformation from our parametrisation to the linear, $|J| \propto [\Omega_{\phi 0}(1+w)]^{-3/2}$,

right posterior = prior $\times$ likelihood





Conclusions

- Approaching cosmological constant behaviour the evolution of quintessence DE is constrained by the requirement of a **smooth potential**

Conclusions

- Approaching cosmological constant behaviour the evolution of quintessence DE is constrained by the requirement of a **smooth potential**
- We developed a physical parametrisation apt to search for **small deviations** from Λ

Conclusions

- Approaching cosmological constant behaviour the evolution of quintessence DE is constrained by the requirement of a **smooth potential**
- We developed a physical parametrisation apt to search for **small deviations** from Λ

Future work

- The assumptions about $w(z)$ can affect dramatically the conclusions about DE (see *B.A. Bassett, P.S. Corasaniti & M. Kunz, 2004*) $\Rightarrow$ use this parametrisation for **projections of future experiments**
- **Extend** the parametrisation to models with coupling DE–DM (e.g. *L. Amendola (2000), M. Gasperini, F. Piazza & G. Veneziano (2002)*) or exotic kinetic term.