Radion stabilization with(out) Gauss-Bonnet interactions and inflation

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motivation

★ basic idea: we live on a brane in a higher-dimensional space-time

★ *Horava & Witten*: 11d model with 10d branes (M-theory motivated)

★ many 5d models with 4d branes have been discussed since then

★ distance between the branes should be stabilized

★ radion - a scalar field related to that distance → to be stabilized

extension considered here

★ interactions of the higher order in the curvature tensor

⊛ α' expansion in string theories
$\longrightarrow$ Gauss-Bonnet (GB) term

$$\mathcal{R}^2_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$$

★ modeling the radion by introducing an additional bulk scalar field (*Goldberger & Wise*)
$\longrightarrow$ 5d model described by the action ($S^1/\mathbb{Z}_2$ orbifold)

$$\mathcal{S} = \int \mathrm{d}^5x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[\mathcal{R} + \alpha\mathcal{R}^2_{GB}\right] - \frac{1}{2}(\nabla\Phi)^2 - V(\Phi) - \sum_{i=1}^{2}\delta(y-y_i)U_i(\Phi)\right\}$$

★ ansatz for the metric and the scalar field

⊛ $\mathrm{d}s^2 = a(y)^2\left\{-\mathrm{d}t^2 + e^{2Ht}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j + \mathrm{d}y^2\right\}$

⊛ $\Phi = \phi(y)$

background equations of motion & boundary conditions

★ scalar eom. $\rightarrow \phi'' + 3\frac{a'}{a}\phi' - a^2 V' = 0$

★ tensor eom. $\rightarrow \left\{\frac{a''}{a} - 2\left(\frac{a'}{a}\right)^2 + H^2\right\}\frac{\xi}{a^2} + \frac{1}{3}\phi'^2 = 0$

$$\rightarrow 3\left\{\left(\frac{a'}{a}\right)^2 - H^2\right\}\left[1 + \frac{\xi}{a^2}\right] - \frac{1}{2}\phi'^2 + a^2 V = 0$$

⊛ where $\xi = a^2 - 4\alpha\left\{\left(\frac{a'}{a}\right)^2 - H^2\right\}$

★ scalar bc. $\rightarrow \lim_{y\rightarrow y_i^\pm} \frac{\phi'}{a} = \pm\frac{1}{2}U_i'$

★ tensor bc. $\rightarrow \lim_{y\rightarrow y_i^\pm} \left\{\frac{a'}{a^4}\left[a^2 - 4\alpha\left(\frac{1}{3}\left(\frac{a'}{a}\right)^2 - H^2\right)\right]\right\} = \mp\frac{1}{6}U_i$

scalar perturbations

★ generalized longitudinal gauge

⊛ $ds^2 = a^2 \left\{(1+2F_1)\left[-dt^2 + e^{2Ht}\delta_{ij}dx^i dx^j\right] + (1+2F_2)dy^2\right\}$

⊛ $\Phi = \phi + F_3$

★ linearized Einstein equations

⊛ $\frac{\xi'}{\xi}F_1 + \frac{a'}{a}F_2 = 0$

⊛ $(\xi F_1)' + \frac{1}{3}a^2\phi' F_3 = 0$

⊛ $$\frac{\xi}{a^2}\left\{(\Box + 4H^2)F_1 + 4\frac{a'}{a}F_1' - 4\left(\frac{a'}{a}\right)^2 F_2\right\} + \frac{1}{3}\phi'^2 F_2 + $$
$$+ \left\{\frac{1}{3}\phi'' + \frac{a'}{a}\phi'\right\} F_3 - \frac{1}{3}\phi' F_3' = 0$$

★ boundary conditions

⊛ $\lim_{y\to y_i^\pm}\left\{F_3' - F_2\phi'\right\} = \pm\frac{1}{2}aF_3U_i''$

variables elimination and separation

★ perturbations are not independent - F_2 and F_3 can be eliminated

★ defining $F_1(t,\vec{x},y) = \sum_{m^2} F_{m^2}(y) \left\{ \int \mathrm{d}^3k f_{(m^2,k)}(t) e^{i\vec{k}\vec{x}} \right\}$

★ dynamical equation of motion → $F''_{m^2} + 2\left\{ 2\frac{\xi'}{\xi} - \frac{a'}{a} - 2\frac{\phi''}{\phi'} \right\} F'_{m^2}$

$$+ \left\{ \frac{\xi''}{\xi} - \frac{\xi' a'}{\xi a} - 2\frac{\xi'\phi''}{\xi\phi'} - \frac{a^3\xi'}{3a'\xi^2}(\phi')^2 + m^2 + 4H^2 \right\} F_{m^2} = 0$$

★ separation constant m^2
→ scalars mass squared in the effective 4d description

★ eliminating F_2, F_3 (and F''_1) → boundary conditions

$$\pm b_{1(2)} \lim_{y\to y_1^+(y_2^-)} \left\{ F'_{m^2} + \frac{\xi'}{\xi} F_{m^2} \right\} + \left[m^2 + 4H^2 \right] \lim_{y\to y_1^+(y_2^-)} F_{m^2} = 0$$

⊛ where $b_{1/2} = \lim_{y\to y_1^+/y_2^-} \left\{ \frac{1}{2} a U''_{1/2} \pm \frac{a'}{a} \mp \frac{\phi''}{\phi'} \right\}$

summing-up the problem

★ defining $Q_{m^2} = \xi F_{m^2}$

★ dynamical equation becomes

$$-(pQ')' + qQ = \lambda pQ$$

i.e. Sturm-Liouville differential equation, where

⊛ $p = \dfrac{3}{2a\phi'^2}$

⊛ $q = \dfrac{a^2\xi'}{2a'\xi^2}$

⊛ $\lambda = m^2 + 4H^2$

★ with non-standard boundary conditions

$$\frac{\partial Q}{\partial n}(y_i) - \frac{\lambda}{b_i} Q(y_i) = 0$$

radion mass

★ lowest eigenvalue

$$\lambda_0 = \min_Q \left\{ \frac{\int_{y_1}^{y_2} \left[pQ'^2 + qQ^2\right]}{\int_{y_1}^{y_2} \left[pQ^2\right] + b_1^{-1}(pQ^2)|_{y_1} + b_2^{-1}(pQ^2)|_{y_2}} \right\}$$

★ $\rightarrow$ radion mass bound

$$m_0^2 \leq -4H^2 + \frac{\int \mathrm{d}y \frac{a^2\xi'}{a'\xi^2}}{3\left\{\int \mathrm{d}y \frac{1}{a\phi'^2} + \sum \frac{1}{b_i a(y_i)\phi'^2(y_i)}\right\}}$$

★ inflating branes $(H^2 > 0)$
$\rightarrow$ stability of the interbrane distance for $\lambda_0 > 4H^2$

★ numerical calculations

stability conditions

★ static branes ($H = 0$): $\lambda = m^2 \rightarrow$ stability for $\lambda_0 > 0$

★ brane system is stable if

- ⊛ $\phi'(y) \neq 0$
- ⊛ $\dfrac{\xi'(y)}{a'(y)} > 0$
- ⊛ $b_i > 0$

$\rightarrow$ sufficient & necessary conditions

role of Gauss-Bonnet interactions

★ stability conditions $\rightarrow$ addition of GB interactions unimportant?

★ numerics $\rightarrow$ solutions with small $\alpha \neq 0$ differ form those with $\alpha = 0$

★ qualitative analysis

- ⊛ GB with $\alpha < 0$: model dependent, in general worse stability
- ⊛ GB with $\alpha > 0$ (as predicted by the string theory):
  inter-brane distance decreases, radion mass squared increases
  $\rightarrow$ stability of the brane positions improves!

★ quantitative analysis: numerics