# Spin amplitudes and gauge-invariance: from PHOTOS Monte Carlo to QCD

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Web pages: http://wasm.home.cern.ch/wasm/goodies.html

http://piters.home.cern.ch/piters/MC/PHOTOS-MCTESTER/

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## $\mathcal{P}$urpose of the talk

**Analysis of spin amplitudes was essential to assure high precision of PHOTOS and KKMC Monte Carlo programs**

**As those projects became popular over the years it is tempting to verify if elements, techniques used there can be extended to QCD**

**It is not easy to separate aspects,** I must start from ...

-1- **phase space and crude distribution**: PHOTOS: Based on exact paramnetrization (presamplers for collinear and infrared singularities) for arbitrary number of charges and photons in final state.

-2- **Iteration properties** Phase space forces ME.

-3- **single emission** matrix elements for QED

-4- **Double emission** matrix elements for $e^+e^- \to \nu_e \bar{\nu}_e \gamma\gamma$; (KKMC QED +SM),

-5- **NEW**: matrix elements for $q\bar{q} \to Jgg$

*Main References*

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*Presentation*

- PHOTOS ( by E.Barberio, B. van Eijk, Z. W., P.Golonka) is used to simulate the effect of radiatiative corrections in decays, since 1989.

- Full events combining complicated tree structure of production and subsequent decays are fed from other generators.

- This is often source of technical difficulties (problems of event record grammar, see my other talks) or precision loss, but solution works and is used in simulation chains of most of today high energy physics experiments.

- Important for today: At every event decay branching, PHOTOS intervene. With certain probability extra photon(s) may be added and kinematics of other particles adjusted.

- PHOTOS works on four-momenta; to think of any extensions of algorithm phase space treatment had to be re-visited first.

## $\mathcal{P}$hase $\mathcal{S}$pace: (trivialities)

Let us recall the element of Lorentz-invariant phase space (*Lips*):

$$dLips_{n+1}(P) =$$

$$\frac{d^3 k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3 k_n}{2k_n^0(2\pi)^3} \frac{d^3 q}{2q^0(2\pi)^3} (2\pi)^4 \delta^4 \left( P - \sum_1^n k_i - q \right)$$

$$= d^4 p \delta^4(P - p - q) \frac{d^3 q}{2q^0(2\pi)^3} \frac{d^3 k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3 k_n}{2k_n^0(2\pi)^3} (2\pi)^4 \delta^4 \left( p - \sum_1^n k_i \right)$$

$$= d^4 p \delta^4(P - p - q) \frac{d^3 q}{2q^0(2\pi)^3} dLips_n(p \to k_1 ... k_n).$$

Integration variables, the four-vector $p$, compensated with $\delta^4 \left( p - \sum_1^n k_i \right)$, and another integration variable $M_1$ compensated with $\delta \left( p^2 - M_1^2 \right)$ are introduced.

*Pase Space Formula of the talk*

$$dLips_{n+1}(P \to k_1...k_n, k_{n+1}) = dLips_n^{+1 \ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1 \ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$

$$\{k_1, \ldots, k_{n+1}\} = \mathbf{T}(k_\gamma, \theta, \phi, \{\bar{k}_1, \ldots, \bar{k}_n\}). \tag{1}$$

1.  One can verify that if $dLips_n(P)$ is exact, this formula lead to exact parametrization of $dLips_{n+1}(P)$ as well

2.  Practical use: Take the confi gurations from n-body phase space.

3.  Turn it back into some coordinate variables.

4.  construct new kinematical confi guration from all variables.

5.  **Forget about temporary** $k_\gamma \theta \phi$. **From now on, only weight and four vectors count.**

6.  A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent at singularities. Simultaneous use of several $\mathbf{T}$ is possible and necessary/convenient if more than one charge is present in fi nal state.

**Phase Space: (main formula)**

If we choose

$$G_n \;:\; M^2_{2...n}, \theta_1, \phi_1, M^2_{3...n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\rightarrow\; \bar{k}_1 \ldots \bar{k}_n \qquad (2)$$

and

$$G_{n+1} \;:\; k_\gamma, \theta, \phi, M^2_{2...n}, \theta_1, \phi_1, M^2_{3...n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\rightarrow\; k_1 \ldots k_n, k_{n+1}$$

$$(3)$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \ldots, \bar{k}_n)). \qquad (4)$$

The ratio of the Jacobians (factors $\lambda^{1/2}$ etc.) form the factor $W_n^{n+1}$, which in our case is rather simple,

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1...n}, M^2_{2...n}/M^2_{1...n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2...n}/M^2)}, \qquad (5)$$

● All details depend on definition of $G_n$.

Phase Space: (multiply iterated)

By iteration, we can generalize formula (1) to the case of $l$ particles added and obtain:

$$dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) = \frac{1}{l!} \prod_{i=1}^{l} \left[ dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W_{n+i-1}^{n+i} \right]$$

$$\times dLips_n(P \to \bar{k}_1...\bar{k}_n), \tag{6}$$

$$\{k_1, \ldots, k_{n+l}\} = \mathbf{T}(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}(\ldots, \mathbf{T}(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \ldots, \bar{k}_n\})\ldots).$$

Note that variables $k_{\gamma_m}, \theta_{\gamma_m}, \phi_{\gamma_m}$ are used at a time of the $m-$th step of iteration only, and are not needed elsewhere in construction of the physical phase space; the same is true for invariants and angles $M_{2...n}^2, \theta_1, \phi_1, \ldots, \theta_{n-1}, \phi_{n-1} \to \bar{k}_1 \ldots \bar{k}_n$ of (2,3), which are also redefined at each step of the iteration. Also intermediate steps require explicit construction of temporary $\bar{k}'_1 \ldots \bar{k}'_n \ldots \bar{k}'_{n+m}$

We have got **exact distribution** of **weighted** events over $n + l$ body phase space.

### *Crude Ddistribution*

If we add arbitrary factors $f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i})$ and sum over $l$ we obtain:

$$\sum_{l=0}^{} \exp(-F)\frac{1}{l!} \prod_{i=1}^{l} f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) =$$

$$\sum_{l=0}^{} \exp(-F)\frac{1}{l!} \prod_{i=1}^{l} \left[ f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W_{n+i-1}^{n+i} \right] \times$$

$$dLips_n(P \to \bar{k}_1...\bar{k}_n), \tag{7}$$

$$\{k_1, \ldots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}(\ldots, \mathbf{T}(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \ldots, \bar{k}_n\})\ldots)\big),$$

$$F = \int_{k_{min}}^{k_{max}} dk_\gamma d\cos\theta_\gamma d\phi_\gamma f(k_\gamma, \theta_\gamma, \phi_\gamma).$$

● The Green  parts of rhs. alone, give crude distribution over tangent space (orthogonal set of variables $k_i, \theta_i, \phi_i$).

- Factors $f$ must be integrable over tangent space. Regulators of singularities necessary.

- If we request that

$$\sigma_{tangent} = 1 =$$

$$\sum_{l=0}^{\infty} \exp(-F) \frac{1}{l!} \prod_{i=1}^{l} \left[ f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} \right.$$

and that sum rules originating from perturbative approach will not change an overall normalization of the cross section, we will get Monte Carlo solution of PHOTOS type.

- For that to work, real emission and virtual corrections need to be calculated and their factorization properties analyzed.

- Choice of $f$ must be synchronized with those results.

- If such conditions are fulfilled construction of Monte Carlo algorithm is possible

- PHOTOS can be used as prototype.

*Heuristic CW complexes*

We define our crude distribution over yellow space (surface=1) (reprersented by sum of: red point, green lines and flat yellow square)
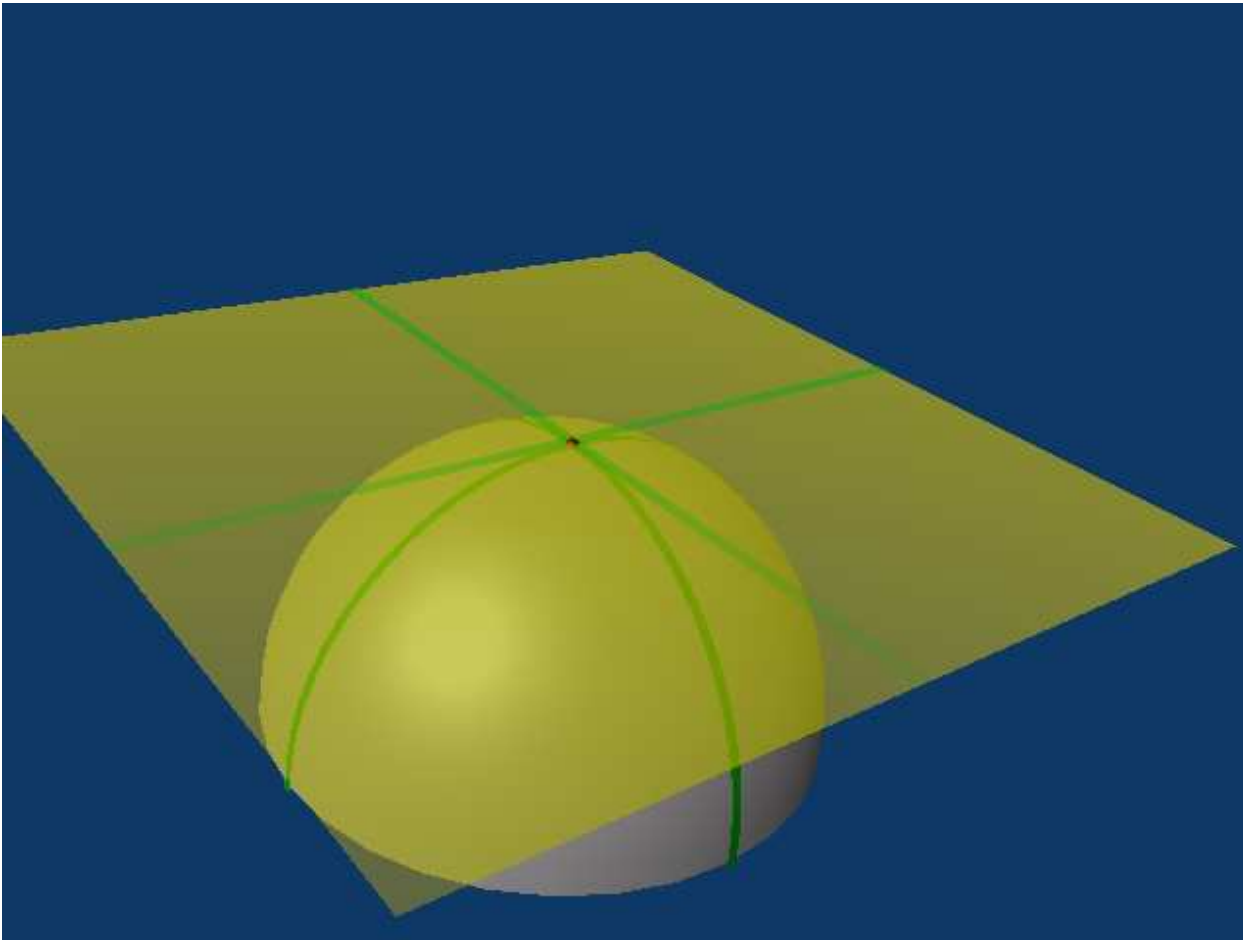
*Heuristic CW complexes projection step 1*

We project in steps,

relative measure of point and lines on cylinder is larger than in previous step, overall

measure remain 1.
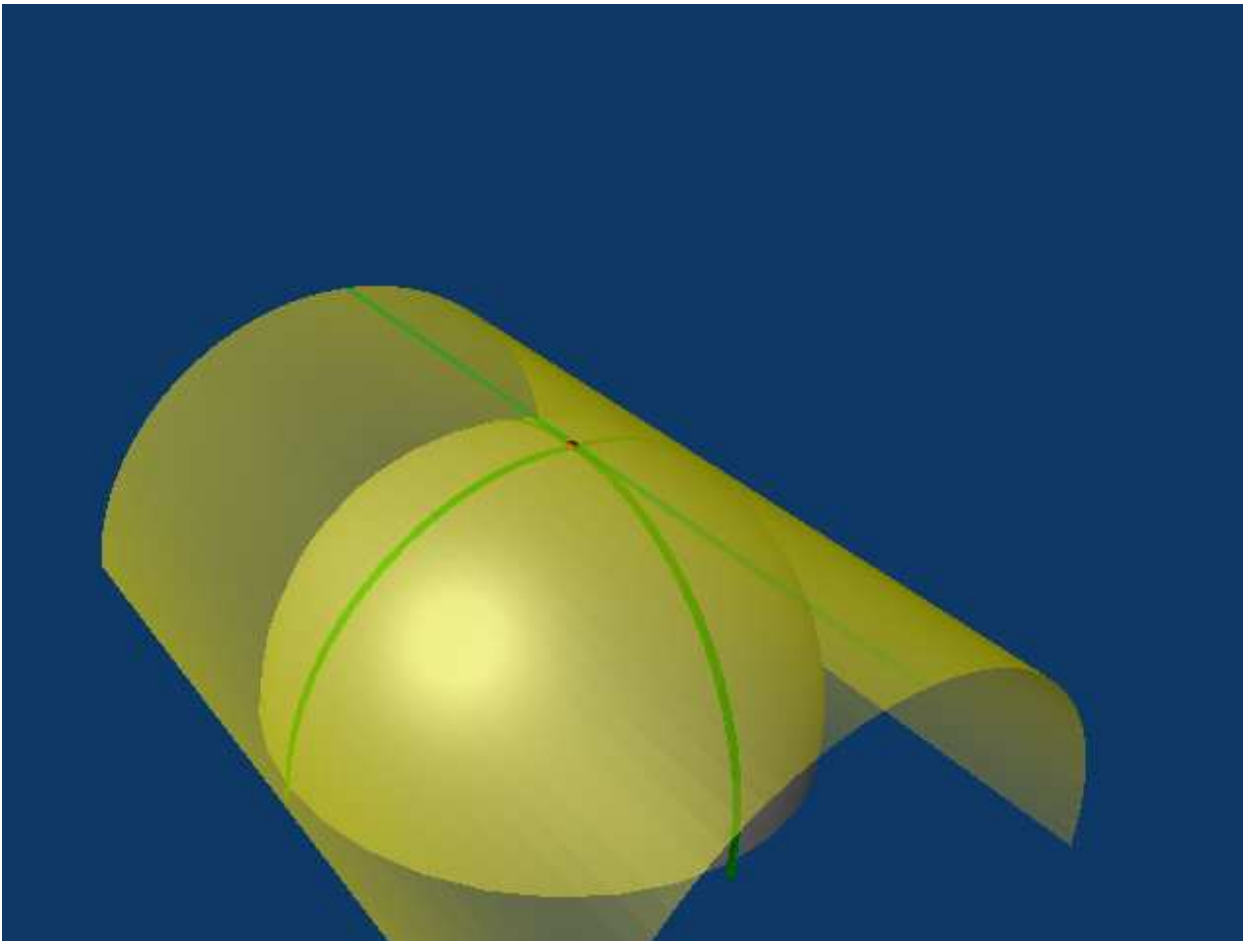
*Heuristic CW complexes projection step 2*

Final distribution does not match the exact one, solely because approximation in matrix elements, phase space is exact.
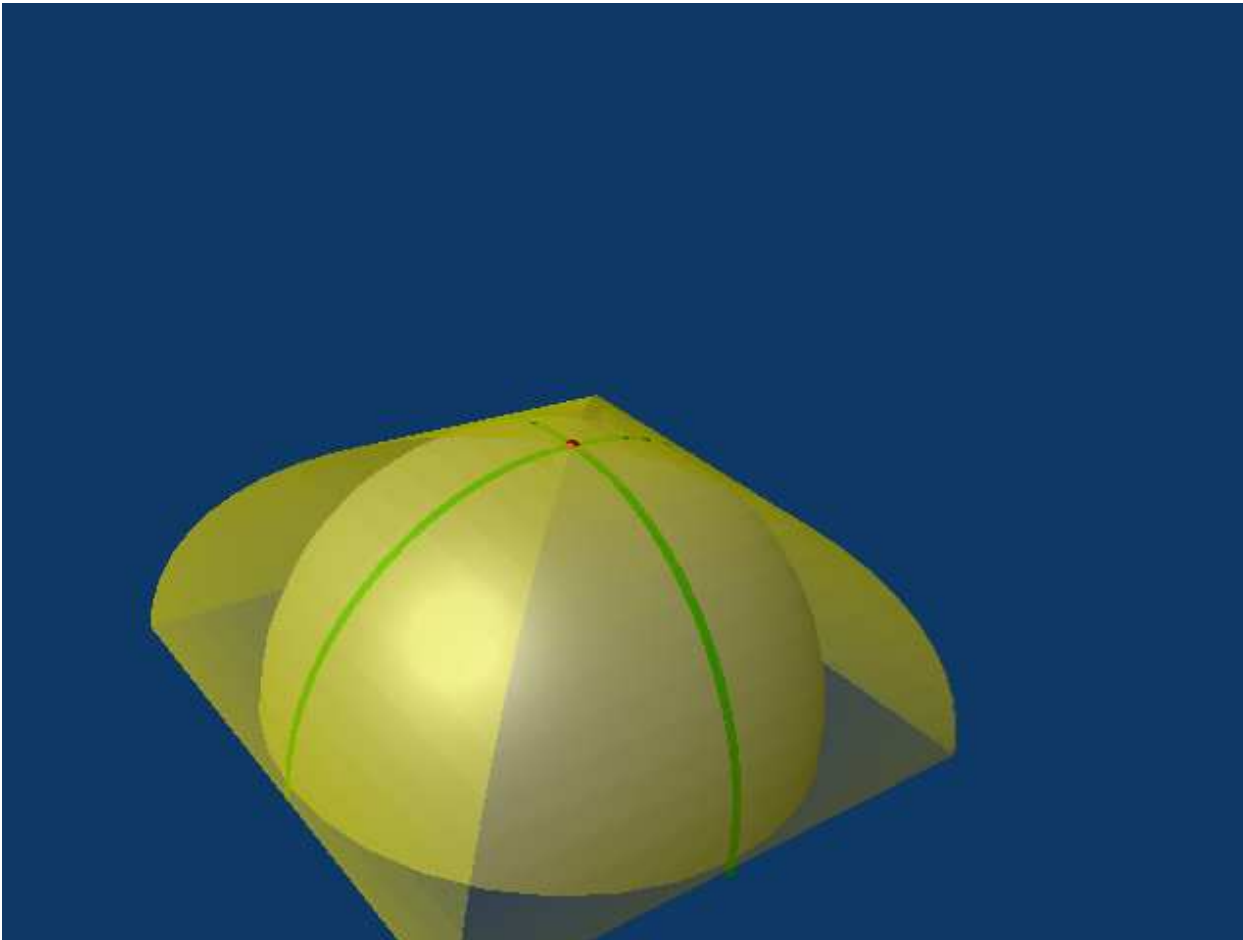
**Why it could work?**

- because we could use our multibody phase space parametrization

- we could measure a 'distance' between points from $n$- and $(n + l)$-body configurations

- we could construct triangulation(s) (better to say CW-complexes) matching structures of singularities.

- such CW-complexes for exact space and tangent space were identical

- to achieve that we could use properties of factorization as known since ever

- infrared singularity being within perturbative domain was a bonus.

- to optimize we studied spin amplitudes (a lot and by naked eye!)

- this is also a 'to do' list if extension to QCD are attempted

*Problems With Phase Space (today we skip details)*

On the pictures just shown:

• points, lines and surfaces represented increase number of lorentz group representations multiplied to give the particular phase space multiplicity.

• In reality life is worse, these are all sets of double precision computer words. **all these objects have unphysical extra dimensions due to rounding errors**!

• This is potentially serious, eg. for 5 TeV electrons

• Unstable particles resonances have widths or even complex lineshapes.

• Another substantial source of miseries

## *Problems With Event Record (we skip today)*
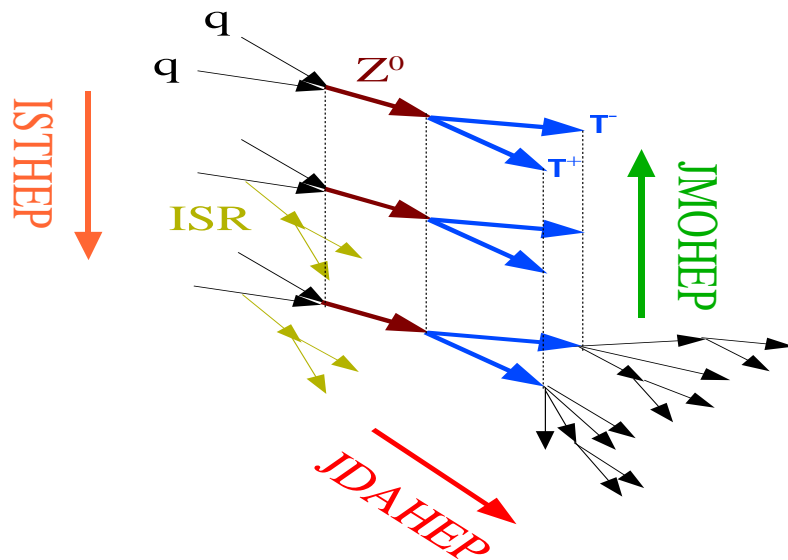


1. Hard process

2. with shower

3. after hadronization

4. Event record overloaded with physics beyond design $\rightarrow$ gramar problems.

5. Here we have basically $LL$ phenomenology only.

## *This Is Physics Not F77!*

Similar problems are in any use of full scale Monte Carlos, lots of complaints at MC4LHC workshop, `HEPEVTrepair` utility (C. Biscarat and ZW) being probed in D0.

Design of event structure WITH some grammar requirements AND WITHOUT neglecting possible physics is needed NOW to avoid large problems later.

*Z. Was* *CERN October 19, 2007*

- The fully differential distribution from MUSTRAAL (used also in KORALZ for single photon mode) reads:

$$X_f = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s}s^2 \quad \left\{ \frac{1}{(k'_+ k'_-)}\left[ \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u)\right]\right\}$$

- Here:

$$s = 2p_+ \cdot p_-, \quad s' = 2q_+ \cdot q_-,$$
$$t = 2p_+ \cdot q_+, \quad t' = 2p_+ \cdot q_-,$$
$$u = 2p_+ \cdot q_-, \quad u' = 2_- \cdot q_+,$$
$$k'_\pm = q_\pm \cdot k, \quad x_k = 2E_\gamma/\sqrt{s}$$

- The $\Delta$ term is responsable for final state mass dependent terms, $p_+$, $p_-$, $q_+$, $q_-$, $k$ denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

- after trivial manipulation it can be written as:

$$X_f = \frac{Q'^2 \alpha (1 - \Delta)}{4\pi^2 s} s^2 \left\{ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_-} \left[ \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t', u) \right] \right.$$

$$\left. + \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_+} \left[ \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t', u) \right] \right\}$$

- In PHOTOS the following expression is used in universal application (AP adj.):

$$X_f^{PHOTOS} = \frac{Q'^2 \alpha (1 - \Delta)}{4\pi^2 s} s^2 \left\{ \right.$$

$$\frac{1}{k'_+ + k'_-} \frac{1}{k'_-} \left[ (1 + (1 - x_k)^2) \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega} \left( s, \frac{s(1 - \cos \Theta_+)}{2}, \frac{s(1 + \cos \Theta_+)}{2} \right) \right] \frac{(1 + \beta \cos \Theta_\gamma)}{2}$$

$$+ \frac{1}{k'_+ + k'_-} \frac{1}{k'_+} \left[ (1 + (1 - x_k)^2) \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega} \left( s, \frac{s(1 - \cos \Theta_-)}{2}, \frac{s(1 + \cos \Theta_-)}{2} \right) \right] \frac{(1 - \beta \cos \Theta_\gamma)}{2} \left. \right\}$$

$$\text{where}: \Theta_+ = \angle(p_+, q_+), \ \Theta_- = \angle(p_-, q_-)$$

$$\Theta_\gamma = \angle(\gamma, \mu^-) \text{ are defined in } (\mu^+, \mu^-)\text{-pair rest frame}$$

- also factor $\Gamma^{total}/\Gamma^{Born} = 1 + 3/4\alpha/\pi$ defines first order weight.

**The differences are important**

- The two expressions define weight to make out of PHOTOS complete first order.

- The PHOTOS expression separates (i) Final state bremsstrahlung (ii) electroweak parameters of the Born Cross section (iii) Initial state bremsstrahlung that is orientation of the spin quantization axix for Z.

- That would be heavy burden for managing PHOTOS interfaces. I know, because we encounter such difficulties for universal interface for TAUOLA.

- It is possible but extremenly inconvenient. Parts of generation managed by distinct authors.

- Of course all this has to be understood in context of Leading Pole approximaition. For example initial-fi nal state interference breaks the simplifi cation. Limitations need to be controlled: Phys. Lett. B219:103,1989.

*Scalar QED for matrix elements in B decays*

- The one-loop QED correction to the decay width can be represented as the sum of the Born contribution with the contributions due to virtual loop diagrams and soft and hard photon emissions.

$$d\Gamma^{\text{Total}} = d\Gamma^{\text{Born}} \left\{ 1 + \frac{\alpha}{\pi} \left[ \delta^{\text{Soft}}(m_\gamma, \omega) + \delta^{\text{Virt}}(m_\gamma, \mu_{UV}) \right] \right\} + d\Gamma^{\text{Hard}}(\omega)$$

- where for **Neutral meson decay channels**, hard photon contribution:

$$d\Gamma^{\text{Hard}} = |A^{\text{Born}}|^2 4\pi\alpha \left( q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_2 \frac{k_2.\epsilon}{k_2.k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

- for **Charged meson decay channels**, hard photon contribution:

$$d\Gamma^{\text{Hard}} = |A^{\text{Born}}|^2 4\pi\alpha \left( q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q \frac{P.\epsilon}{P.k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

**$\mathcal{M}$atrix $\mathcal{E}$lement (anything in common?):**

- We have seen nice properties of matrix element squared which were factorizing into Born-like distribution and photon factor.

- It was shown many years ago by Ronald Kleiss that such property does not hold beyond first order!

- Dead end? Let's verify.

- single photon/gluon (momentum $k_1$ polarization $e_1$ fermion spinors $u(p)$ and $v(q)$ and color $T^A$ dropped ) emisson amplitude can be written as:

$$I_1^{\{1,2\}} = \left[ J \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{q \cdot e_1}{q \cdot k_1} \right) \right] - \left[ \frac{1}{2} \frac{\not{e}_1 \not{k}_1}{p \cdot k_1} J \right] + \left[ \frac{1}{2} J \frac{\not{e}_1 \not{k}_1}{q \cdot k_1} \right]$$

- note three gauge invariant segments, and coincidence of eikonal segment with sacalar QED amplitude!

## *Matrix Element (double emission):*

- For our program to work for FSR QED, it was necessary to understand all points of a **to do list** given in transparency 14

- The structure of exact spin amplitudes of as high order of pertubation expansion as only available was a high priority.

- We will present first, such properties of QED spin amplitudes which were useful for solutions used in PHOTOS and KKMC Monte Carlos.

- Later we will check if similar properties hold for QCD as well.

- To identify the building blocks we have used gauge invariance, and we have used also segments localized at lower order.

- For tree diagrams gauge invariance mean in practice that replacement $k \to e$ set experession to zero

**Matrix Element:** $e^+e^- \rightarrow \nu_e \bar{\nu}_e \gamma\gamma$

- This case is rich with triple and quartic gauge couplings ($WW\gamma$ and $WW\gamma\gamma$)

- The gauge invariance was used in this case to separate complete amplitudes into parts.

- Semi automaticmethod was used. Terms of some properties were identified, all diagrams with such terms were analyzed.

- Gauge invariant groups of terms was set aside, and remnant was searched for pecularity. Further diagrams, sharing properties were taken.

- For tree diagrams gauge invariance means that if $e \rightarrow k$ amplitude equals zero.

- This semi-automated method helped to separate exact spin amplitude into (at least) $7_{-Z} + 11_{-W} = 18$ individually gauge invariant parts.

- **More: separation match structure of singularities**, it was extensively used in KKMC.

*Exact Matrix Element:* $e^+e^- \to \nu_\mu \bar{\nu}_\mu \gamma\gamma$ *can be written explicitly*

● We use conventions from recent paper with A. van Hameren. Expressions are valid also for QCD and any current $J$, part proportional to $\{T^A T^B\}$, $T^A$ is for fi rst $T^B$ for second gluon.

● To get complete amplitude we sum the expressions below and place them between spinors, eg. $\bar{u}(p)$ and $v(q)$; 1-st/2-nd photon/gluon momenta/polarizations are: $k_1/k_2\ e_1/e_2$.

$$I_1^{\{1,2\}} = \frac{1}{2}\, J\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right) \qquad\qquad eikonal$$

$$I_{2l}^{\{1,2\}} = -\frac{1}{4}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\frac{\not{e}_2 \not{k}_2}{p\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right)\frac{\not{e}_1 \not{k}_1}{p\cdot k_1}\right] J \qquad \beta_1$$

$$I_{2r}^{\{1,2\}} = \frac{1}{4}\, J\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\frac{\not{k}_2 \not{e}_2}{q\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right)\frac{\not{k}_1 \not{e}_1}{q\cdot k_1}\right] \qquad \beta_1$$

$$I_3^{\{1,2\}} = -\frac{1}{8}\left(\frac{\not{e}_1 \not{k}_1}{p \cdot k_1} J \frac{\not{k}_2 \not{e}_2}{q \cdot k_2} + \frac{\not{e}_2 \not{k}_2}{p \cdot k_2} J \frac{\not{k}_1 \not{e}_1}{q \cdot k_1}\right) \qquad start\,for\,\beta_2...$$

$$I_{4p}^{\{1,2\}} = \frac{1}{8}\frac{1}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2}\left(\frac{\not{e}_1 \not{k}_1 \not{e}_2 \not{k}_2}{p \cdot k_1} + \frac{\not{e}_2 \not{k}_2 \not{e}_1 \not{k}_1}{p \cdot k_2}\right) J$$

$$I_{4q}^{\{1,2\}} = \frac{1}{8} J \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2}\left(\frac{\not{k}_2 \not{e}_2 \not{k}_1 \not{e}_1}{q \cdot k_1} + \frac{\not{k}_1 \not{e}_1 \not{k}_2 \not{e}_2}{q \cdot k_2}\right)$$

$$I_{5pA}^{\{1,2\}} = \frac{1}{2} J \frac{k_1 \cdot k_2}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2}\left(\frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1}\right)\left(\frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2}\right)$$

$$I_{5pB}^{\{1,2\}} = -\frac{1}{2} J \frac{1}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2}\left(\frac{k_1 \cdot e_2 k_2 \cdot e_1}{k_1 \cdot k_2} - e_1 \cdot e_2\right)$$

$$I_{5qA}^{\{1,2\}} = \frac{1}{2} J \frac{k_1 \cdot k_2}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2}\left(\frac{q \cdot e_1}{q \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1}\right)\left(\frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2}\right)$$

$$I_{5qB}^{\{1,2\}} = -\frac{1}{2} J \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2}\left(\frac{k_1 \cdot e_2 k_2 \cdot e_1}{k_1 \cdot k_2} - e_1 \cdot e_2\right)$$

$$I_{6B}^{\{1,2\}} = -\frac{1}{4}\frac{k_1 \cdot k_2}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2}\left[+\left(\frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_1 \cdot k_2}\right)\frac{\not{e}_2 \not{k}_2}{p \cdot k_2} + \left(\frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2}\right)\frac{\not{e}_1 \not{k}_1}{p \cdot k_1}\right]\not{J}$$

$$I_{7B}^{\{1,2\}} = -\frac{1}{4}\not{J}\frac{k_1 \cdot k_2}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2}\left[+\left(\frac{q \cdot e_1}{q \cdot k_1} - \frac{k_2 \cdot e_1}{k_1 \cdot k_2}\right)\frac{\not{k}_2 \not{e}_2}{q \cdot k_2} + \left(\frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2}\right)\frac{\not{k}_1 \not{e}_1}{q \cdot k_1}\right]$$

- for the **exponentiation** we have used **separation** into 3 parts only. It is **crystal clear**, also in case of contributions with $t$-channel $W$, excellent for KKMC,

- $I_3^{\{1,2\}}$, $I_{4p}^{\{1,2\}}$, $I_{4q}^{\{1,2\}}$ is studied separately to improve options for PHOTOS kernel iteration. Things are less easy, concept of effective fermionic momenta must be used eg. $u((p - k_1)_{long})$, it make sense in some limits ony. Having spin amlitude level 'proto kernels' and jacobian cancelling factors, is useful nonetheless.

- We could avoid phase space ordering, assure full phase space coverage and proper LL contributions to lepton spectra (once phase is partly integrated by MC).

- Clearly visible, further separation of $\beta_2$ terms, seem to be of no use/misleading.

$\mathcal{M}$atrix $\mathcal{E}$lement: $q\bar{q} \to Jgg$ - *part proportional to* $T^A T^B$ *fermion spinors dropped*

$$I_{lr}^{(1,2)} = \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{e}_1 \not{k}_1}{2p \cdot k_1} \right) J \left( \frac{\not{k}_2 \not{e}_2}{2q \cdot k_2} + \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{q \cdot e_2}{q \cdot k_2} \right)$$

$$I_{ll}^{(1,2)} = \frac{p \cdot k_2}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{e}_1 \not{k}_1}{2p \cdot k_1} \right) \left( \frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{\not{e}_2 \not{k}_2}{2p \cdot k_2} \right) J$$

$$I_{rr}^{(1,2)} = J \frac{q \cdot k_1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \left( \frac{q \cdot e_1}{q \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{k}_1 \not{e}_1}{2q \cdot k_1} \right) \left( \frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{\not{k}_2 \not{e}_2}{2q \cdot k_2} \right)$$

$$I_{e}^{(1,2)} = J \left( 1 - \frac{p \cdot k_2}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} - \frac{q \cdot k_1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \right) \left( \frac{k_1 \cdot e_2}{k_1 \cdot k_2} \frac{k_2 \cdot e_1}{k_1 \cdot k_2} - \frac{e_1 \cdot e_2}{k_1 \cdot k_2} \right)$$

Remainder:

$$I_{p}^{(1,2)} = -\frac{1}{4} \frac{1}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} \left( \frac{\not{e}_1 \not{k}_1 \not{e}_2 \not{k}_2 - \not{e}_2 \not{k}_2 \not{e}_1 \not{k}_1}{k_1 \cdot k_2} \right) J$$

$$I_{q}^{(1,2)} = -\frac{1}{4} J \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \left( \frac{\not{k}_1 \not{e}_1 \not{k}_2 \not{e}_2 - \not{k}_2 \not{e}_2 \not{k}_1 \not{e}_1}{k_1 \cdot k_2} \right)$$

*$\mathcal{M}$atrix $\mathcal{E}$lement:* $q\bar{q} \to Jgg$ *- part proportional to* $T^B T^A$ *fermion spinors dropped*

$$I_{lr}^{(2,1)} = \left( \frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{\not{e}_2 \not{k}_2}{2p \cdot k_2} \right) J \left( \frac{\not{k}_1 \not{e}_1}{2q \cdot k_1} + \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{q \cdot e_1}{q \cdot k_1} \right)$$

$$I_{ll}^{(2,1)} = \frac{p \cdot k_1}{p \cdot k_2 + p \cdot k_1 - k_2 \cdot k_1} \left( \frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{\not{e}_2 \not{k}_2}{2p \cdot k_2} \right) \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{e}_1 \not{k}_1}{2p \cdot k_1} \right) J$$

$$I_{rr}^{(2,1)} = J \frac{q \cdot k_2}{q \cdot k_2 + q \cdot k_1 - k_2 \cdot k_1} \left( \frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{\not{k}_2 \not{e}_2}{2q \cdot k_2} \right) \left( \frac{q \cdot e_1}{q \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{k}_1 \not{e}_1}{2q \cdot k_1} \right)$$

$$I_e^{(2,1)} = J \left( 1 - \frac{p \cdot k_1}{p \cdot k_2 + p \cdot k_1 - k_2 \cdot k_1} - \frac{q \cdot k_2}{q \cdot k_2 + q \cdot k_1 - k_2 \cdot k_1} \right) \left( \frac{k_2 \cdot e_1}{k_2 \cdot k_1} \frac{k_1 \cdot e_2}{k_2 \cdot k_1} - \frac{e_2 \cdot e_1}{k_2 \cdot k_1} \right)$$

$$I_p^{(2,1)} = -\frac{1}{4} \frac{1}{p \cdot k_2 + p \cdot k_1 - k_2 \cdot k_1} \left( \frac{\not{e}_2 \not{k}_2 \not{e}_1 \not{k}_1 - \not{e}_1 \not{k}_1 \not{e}_2 \not{k}_2}{k_2 \cdot k_1} \right) J$$

$$I_q^{(2,1)} = -\frac{1}{4} J \frac{1}{q \cdot k_2 + q \cdot k_1 - k_2 \cdot k_1} \left( \frac{\not{k}_2 \not{e}_2 \not{k}_1 \not{e}_1 - \not{k}_1 \not{e}_1 \not{k}_2 \not{e}_2}{k_2 \cdot k_1} \right)$$

*For QCD we have separation too; 12 gauge invariant parts*

- Is this compact form, for exact massive QCD spin amplitudes, of any use?

- What is a use for terms like

$$\left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{e}_1 \not{k}_1}{2p \cdot k_1} \right) \qquad A$$

- or of

$$\frac{q \cdot k_1}{q \cdot k_1 + q \cdot k_2 - k_2 \cdot k_1} \qquad B$$

- I have not explored in full. Not even prototype use for QCD was tried by me so far.

- Instead lets look at least at some limits and corners of phase space for pedagogy and fun.

- Terms like A once integrated over part of phase space give Atarelli-Parisi kernel

- Terms like B if combined with phase space Jacobians help to redefine can be merged with $v(q)$ to get $v(q - k_2)$

*As in QED, case of soft (ordered) gluons is straightforward and easy*

● In this case we assumed that $\sqrt{s} \gg k_1^0 \gg k_2^0$

$$\mathcal{M}_{BFKL} = \bar{u}(p)\slashed{J}v(q) \cdot$$
$$\left[ \frac{1}{2}T^a T^b \left( \frac{q \cdot e_1}{q \cdot k_1} - \frac{p \cdot e_1}{p \cdot k_1} \right) \left( \frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} \right) \right.$$
$$\left. + \frac{1}{2}T^b T^a \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{q \cdot e_1}{q \cdot k_1} \right) \left( \frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} \right) \right]$$

● Colour factors and spinors are given explicitly now.

● Use of BFKL subscript is may be an abuse.

● We got something which clearly show expressions for consecutive emissions from dipoles which is at the same time valid all over phase space

● Dropped out parts of amplitudes can be restored, no loss of precision!

As in QED, case of soft (not ordered) gluons is straightforward and easy

$$\mathcal{M}_{BFKL'} = \bar{u}(p)\not{J}v(q) \cdot \Bigg\{$$

$$\frac{1}{4}[T^a T^b] \left(\frac{p\cdot k_1 - p\cdot k_2}{p\cdot k_1 + p\cdot k_2} + \frac{q\cdot k_2 - q\cdot k_1}{q\cdot k_1 + q\cdot k_2}\right)$$

$$\left[\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)\left(\frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right) + \left(\frac{k_2\cdot e_1}{k_2\cdot k_1}\frac{k_1\cdot e_2}{k_2\cdot k_1} - \frac{e_2\cdot e_1}{k_2\cdot k_1}\right)\right]$$

$$+\frac{1}{2}T^a T^b \left[\frac{p\cdot k_2}{p\cdot k_1 + p\cdot k_2}\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right)\right.$$

$$\left.+\frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2}\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{p\cdot e_1}{p\cdot k_1}\right)\left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)\right]$$

$$+\frac{1}{2}T^b T^a \left[\frac{p\cdot k_1}{p\cdot k_2 + p\cdot k_1}\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\right.$$

$$\left.+\frac{q\cdot k_2}{q\cdot k_2 + q\cdot k_1}\left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{p\cdot e_2}{p\cdot k_2}\right)\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1}\right)\right]$$

$$\Bigg\}$$

- In this case we assumed that $\sqrt{s} \gg k_1^0, \; k_2^0 \;, k_1^0, \simeq k_2^0$ allowed

- This time, we got twice more of dipole like terms, which are weighted by scalar factors.

- We got also group of terms proportional to comutator of color generators and proportional to virtuality of intermediate gluon.

- This is a lear sign that we might have separated the contribution to running of the coupling constant.

- As in the previous case one can easily write dropped out parts of the amplitude.

**Collinear limit**

- We use $pk_1 \gg pk_2$ or $qk_1 \gg qk_2$ to drop terms (we allow $pk_1 \simeq k_1 k_2$ and/or $qk_1 \simeq k_1 k_2$) to get

$$T^a T^b \bar{u}(p) \left\{ \frac{q \cdot k_1}{q \cdot k_1 - k_1 \cdot k_2} \not{J} \left( \frac{q \cdot e_1}{q \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{k_1}\not{e_1}}{2q \cdot k_1} \right) - \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{e_1}\not{k_1}}{2p \cdot k_1} \right) \not{J} \right\}$$
$$\left( \frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{\not{k_2}\not{e_2}}{2q \cdot k_2} \right) v(q)$$

$$T^b T^a \bar{u}(p) \left( \frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{\not{e_2}\not{k_2}}{2p \cdot k_2} \right)$$
$$\left\{ \frac{p \cdot k_1}{p \cdot k_1 - k_2 \cdot k_1} \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{\not{e_1}\not{k_1}}{2p \cdot k_1} \right) \not{J} + \not{J} \left( \frac{\not{k_1}\not{e_1}}{2q \cdot k_1} + \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{q \cdot e_1}{q \cdot k_1} \right) \right\} v(q)$$

The remnant, hopefully partly to be hidden into running of the coupling constatn is

not that short, it is importan only if $k_1 k_2$ is small:

$$-\bar{u}(p)\not{J}v(q)\frac{1}{2}\left(T^aT^b\frac{k_1\cdot k_2}{q\cdot k_1 - k_1\cdot k_2} + T^bT^a\frac{k_1\cdot k_2}{p\cdot k_1 - k_1\cdot k_2}\right)\left(\frac{k_1\cdot e_2}{k_1\cdot k_2}\frac{k_2\cdot e_1}{k_1\cdot k_2} - \frac{e_1\cdot e_2}{k_1\cdot k_2}\right)$$

$$+\bar{u}(p)\left(\not{k_2}\not{e_2}\not{k_1}\not{e_1} - \not{k_1}\not{e_1}\not{k_2}\not{e_2}\right)\not{J}v(q)\frac{1}{8}(T^aT^b - T^bT^a)\frac{1}{k_2\cdot k_1}\frac{1}{p\cdot k_1 - k_2\cdot k_1}$$

$$+\bar{u}(p)\not{J}\left(\not{k_2}\not{e_2}\not{k_1}\not{e_1} - \not{k_1}\not{e_1}\not{k_2}\not{e_2}\right)v(q)\frac{1}{8}(T^aT^b - T^bT^a)\frac{1}{k_2\cdot k_1}\frac{1}{q\cdot k_1 - k_2\cdot k_1} \qquad (8)$$

We get too many options to discuss, language of amplitudes with dropped terms seem to be less convenient!

Nest step(s) of analysis, with kinematical cases are discussed individually seem to be natural.

This is however out of scope of the talk when we searched for truncated amplitudes of some nice form, valid all over the phase space and with dropped terms easy to recover.

## *Summary*

* **We have presented phase-space Monte Carlo context, where parts of spin amplitudes are to be used.**

* **We have presented first order ME and how parts appear.**

* **Case of double bremsstrahlung in QED was studied**

* **and followed with discussion of double gluon emission.**

* **Nice properties of spin amplitudes parts, also if some limits were used to drop some terms were presented**

* **application to QCD phenomenology is left to 'hopefully in near future'**

* **that is definite progress,**

* **some common ground with work cooridnated by Staszek Jadach become visible for the first time.**
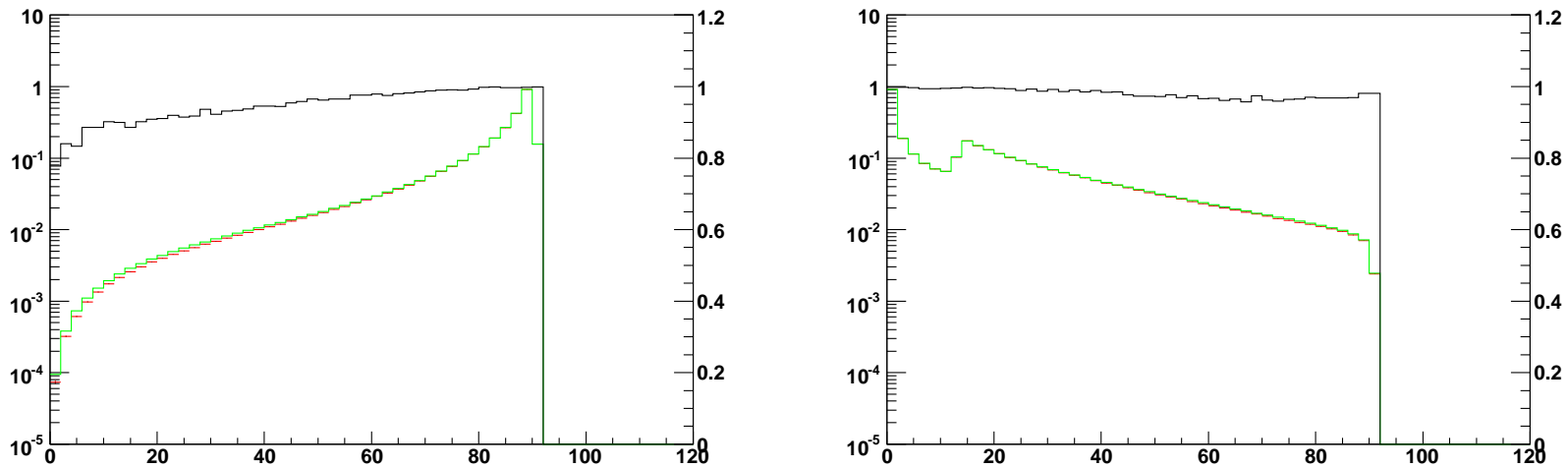
*Figure 1: Comparison of standard PHOTOS and KORALZ for single photon emission. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00534. In the right frame the invariant mass of $\mu^-\gamma$; SDP=0.00296. The histograms produced by the two programs (logarithmic scale) and their ratio (linear scale, black line) are plotted in both frames. The fraction of events with hard photon was 17.4863 $\pm$ 0.0042% for KORALZ and 17.6378 $\pm$ 0.0042% for PHOTOS.*
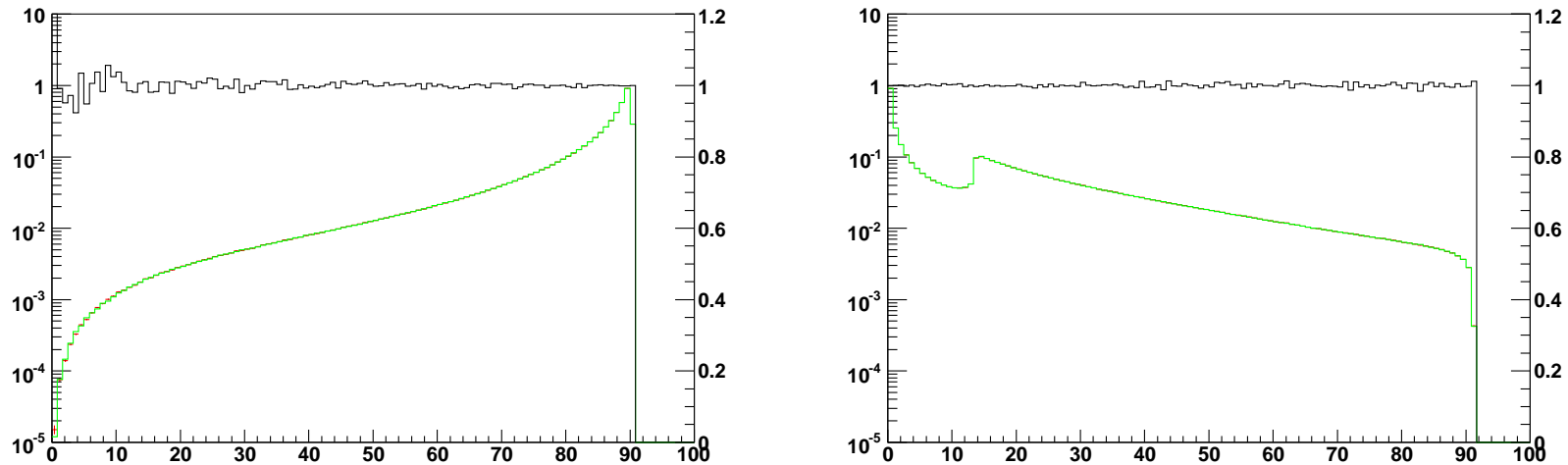
*Figure 2: Comparisons of improved PHOTOS and KORALZ for single photon emission. In the left frame the invariant mass of the $\mu^+\mu^-$ pair. In the right frame the invariant mass of $\mu^-\gamma$ pair is shown. In both cases differences between PHOTOS and KORALZ are below statistical error. The fraction of events with hard photon was $17.4890 \pm 0.0042\%$ for KORALZ and $17.4926 \pm 0.0042\%$ for PHOTOS.*

$B^- \rightarrow \pi^0 K^-$ *: standard PHOTOS* looks good, but ...



**Photon Energy**

PHOTOS (Standard)

SANC (Scalar QED)

**Kaon Energy**

**Photon angle with res. $K^-$**

$B^- \rightarrow \pi^0 K^- \gamma$

$\pi^0 K^-$ acoll. angle
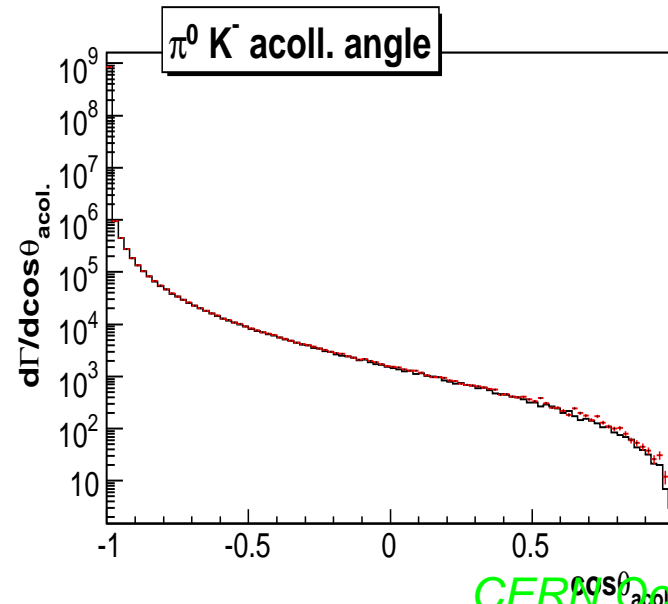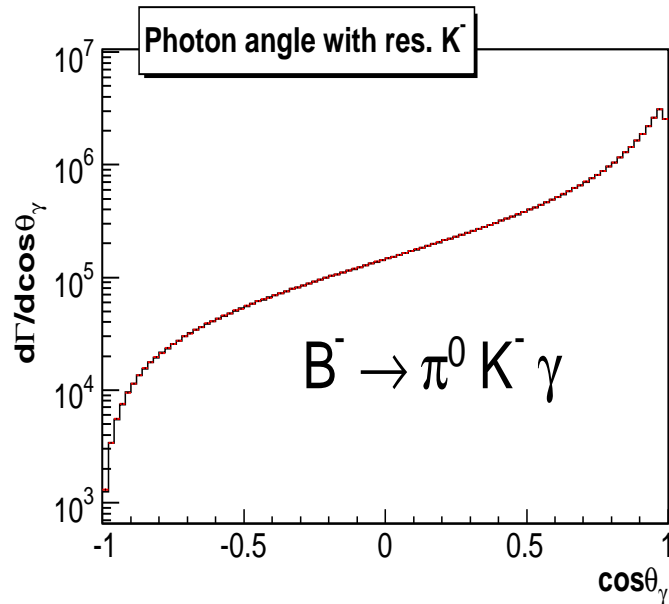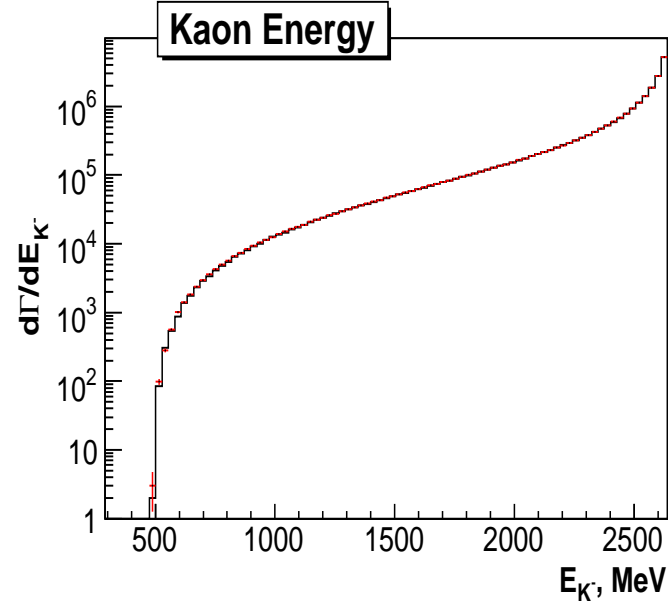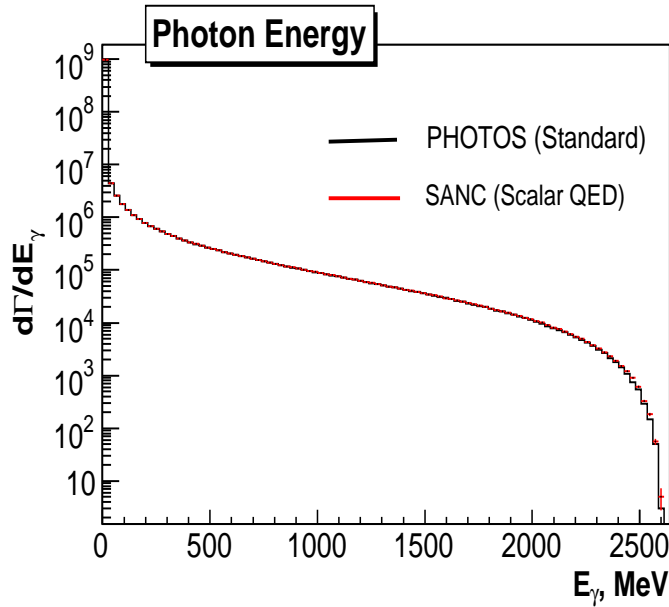
$B^- \to \pi^0 K^-$ : *standard PHOTOS*    *not perfect*

$B^- \to \pi^0 K^-$ : NLO improved PHOTOS Looks good



**Photon Energy**

PHOTOS (Corrected)

SANC (Scalar QED)

$\frac{d\Gamma}{dE_\gamma}$ — $E_\gamma$, MeV

**Kaon Energy**

$\frac{d\Gamma}{dE_{K^-}}$ — $E_{K^-}$, MeV

$M_{B^-} = 5279$ MeV

$M_{K^-} = 494$ MeV

$M_{\pi^0} = 139$ MeV

**Photon angle with res. $K^-$**

$\frac{d\Gamma}{d\cos\theta_\gamma}$ — $\cos\theta_\gamma$

$B^- \to K^- \pi^0 \gamma$

**$\pi^0 K^-$ acoll. angle**

$\frac{d\Gamma}{d\cos\theta_{acol.}}$ — $\cos\theta_{acol.}$

$\Gamma^{Total}/\Gamma^{Born} = 1.00005(3)$

$\mu_{UV} = 2500$ MeV
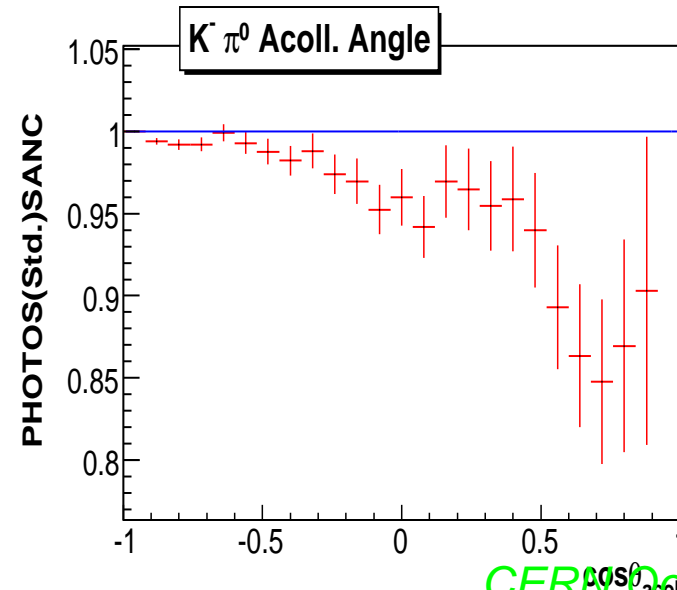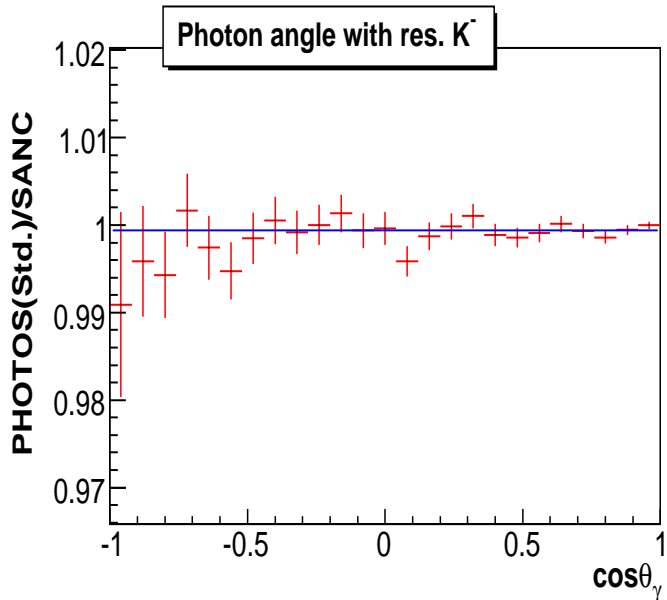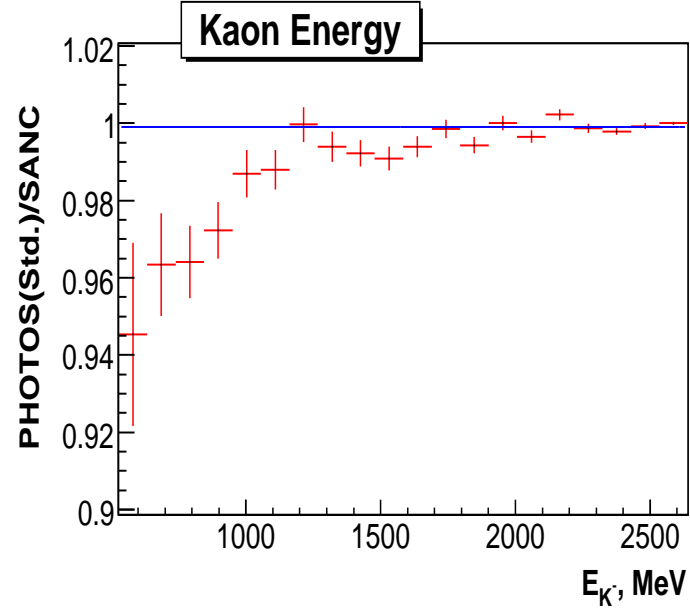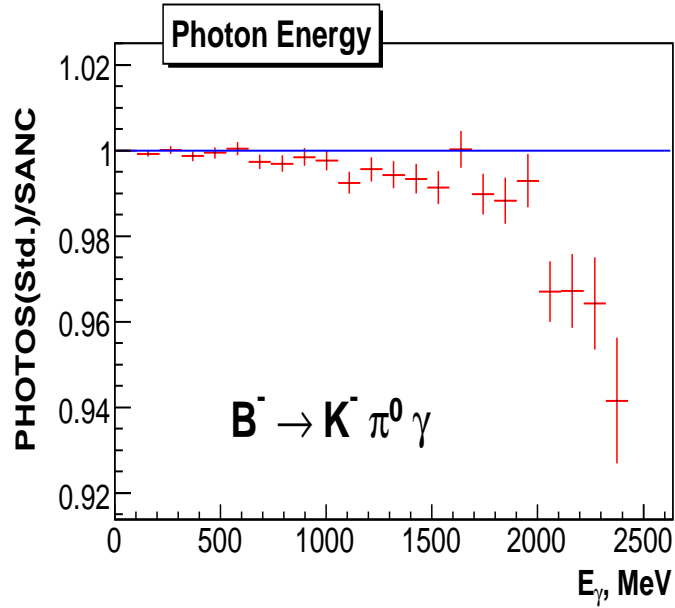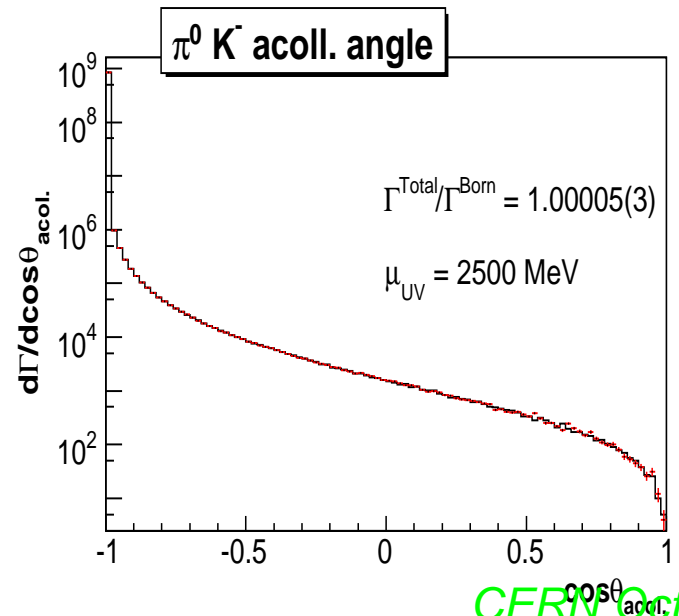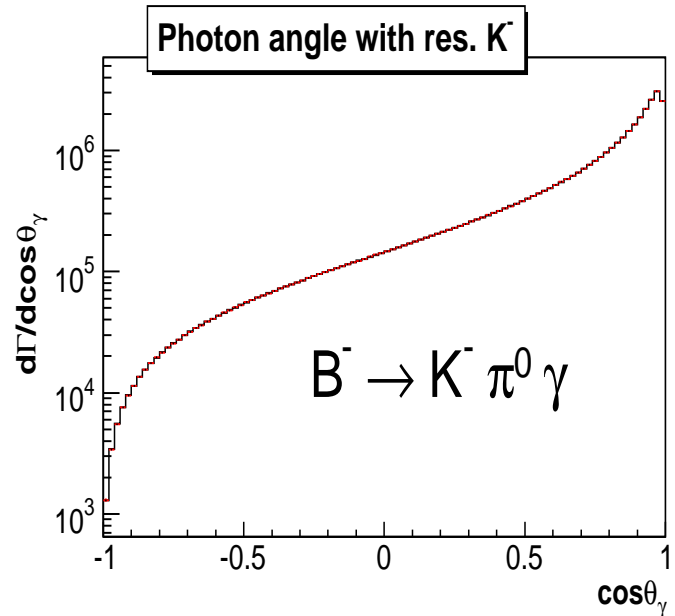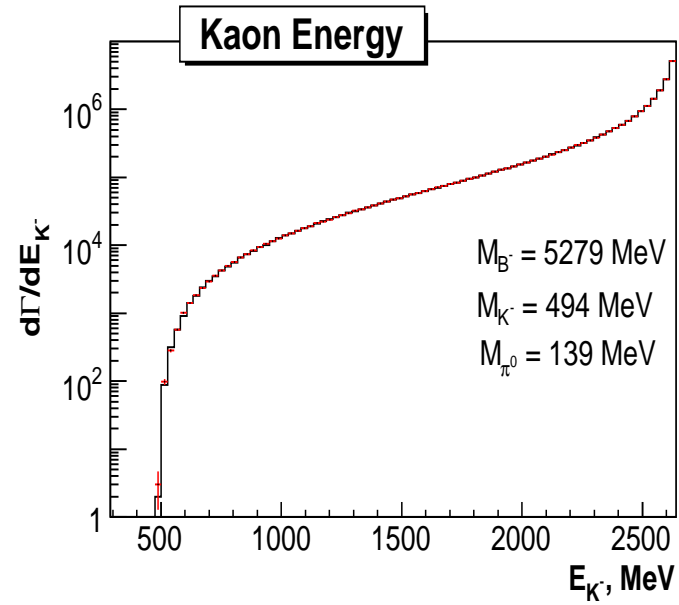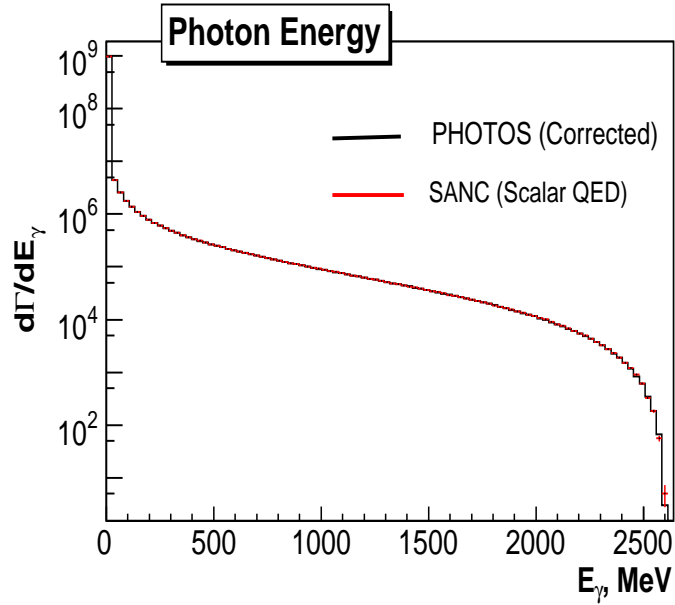
$B^- \to \pi^0 K^-$ *: NLO improved PHOTOS ... and is good.*
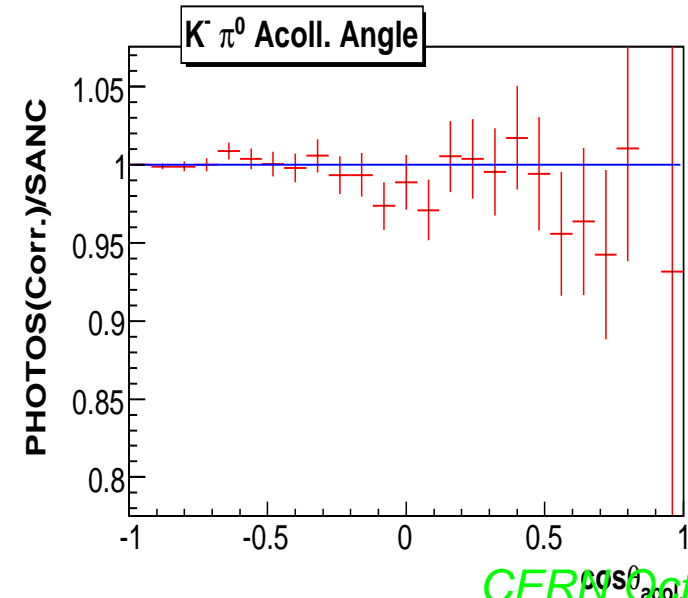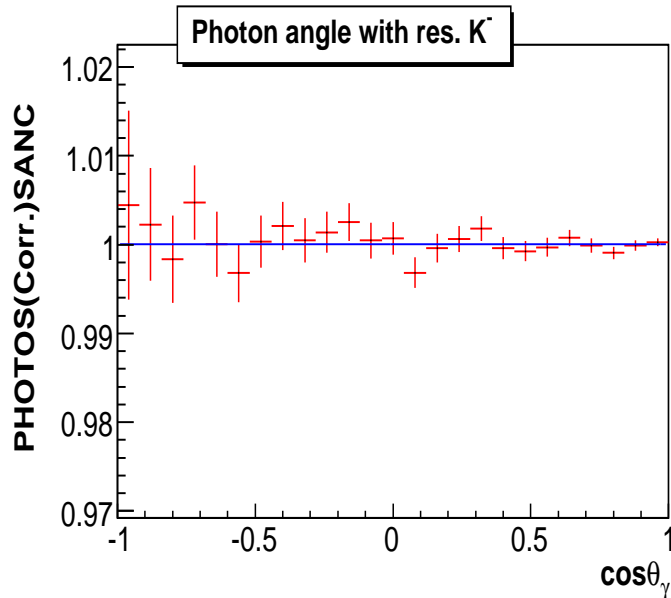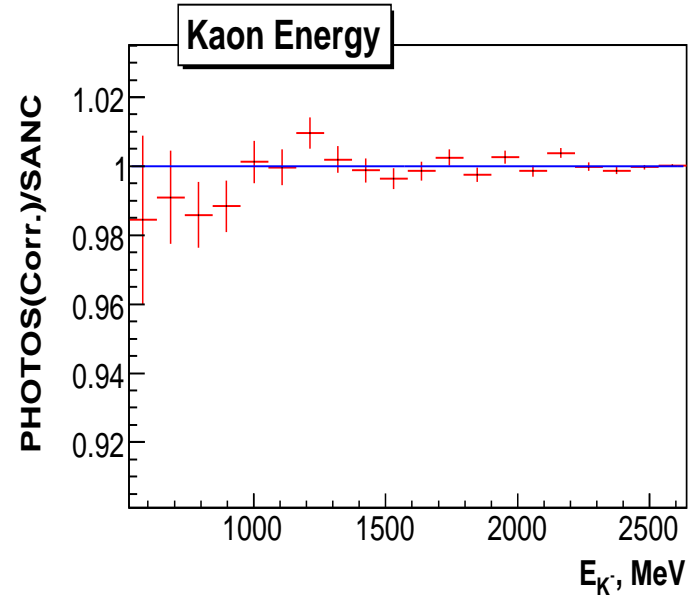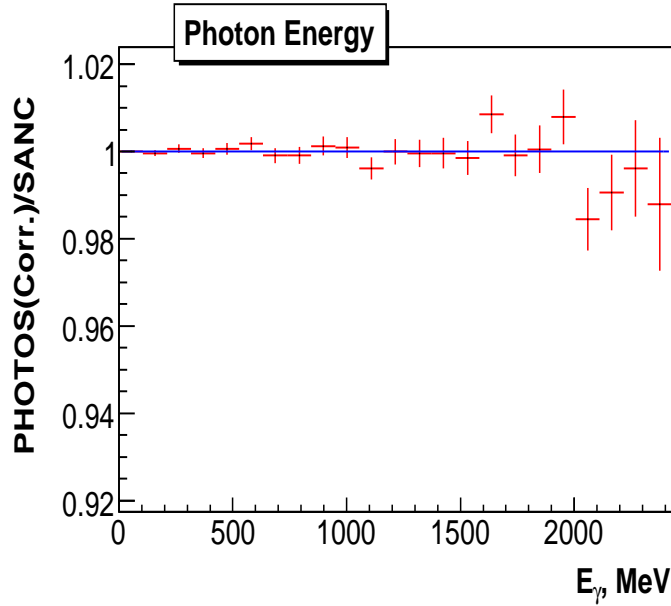
$B^0 \to \pi^- K^+$ : *standard PHOTOS* Looks good ...
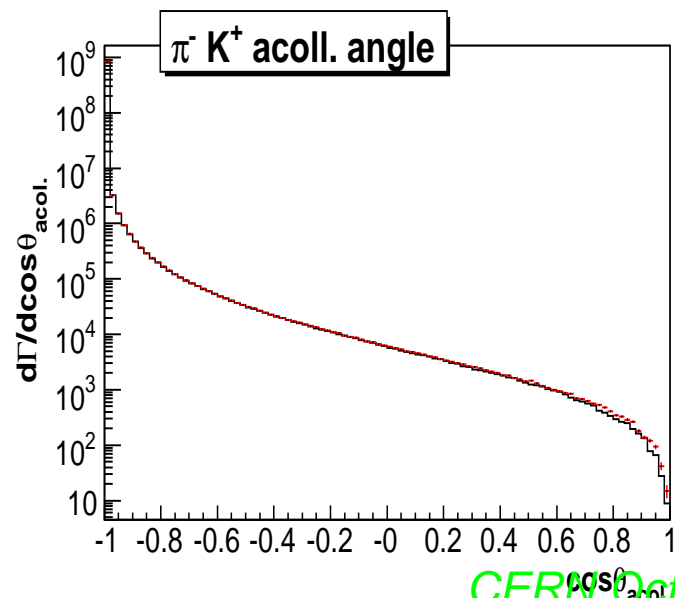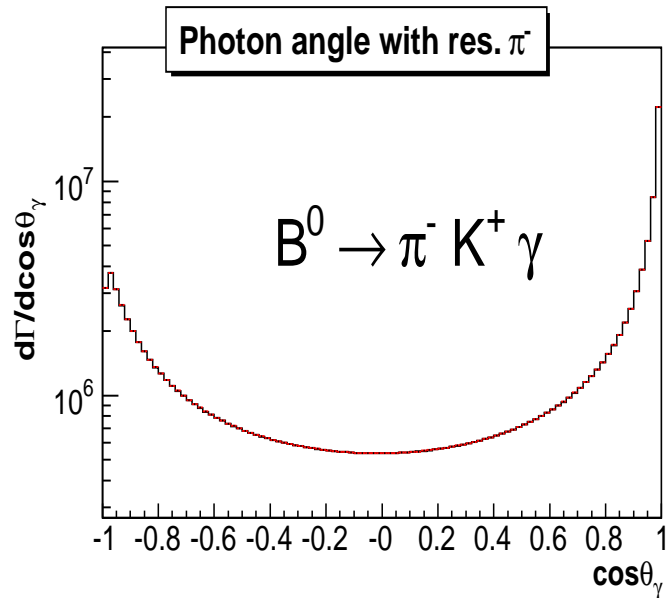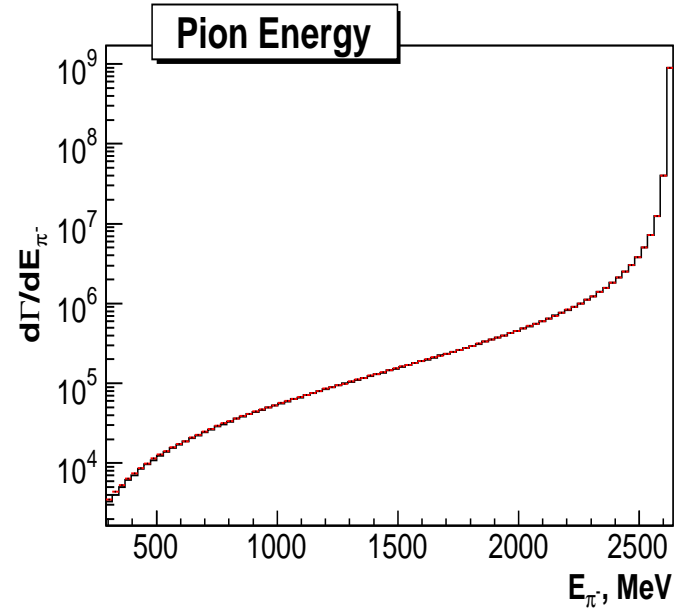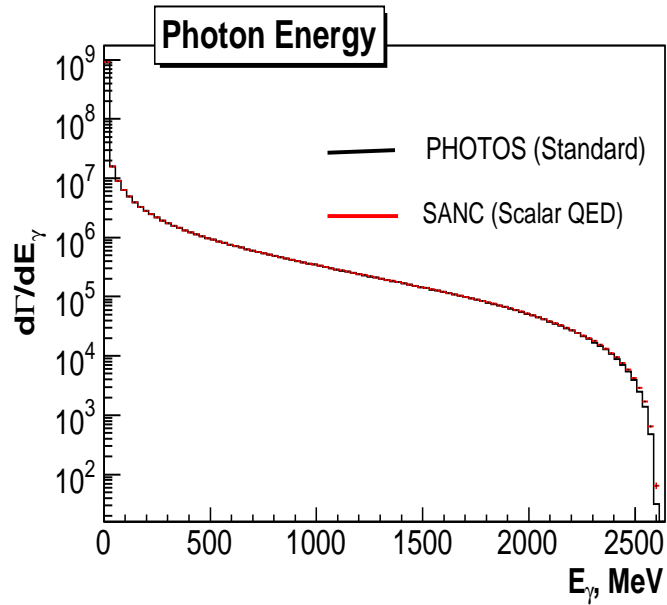
$B^0 \to \pi^- K^+$ *: standard PHOTOS* ... but not perfect.

$B^0 \to \pi^- K^+$ *: NLO improved PHOTOS* Looks good ...



**Photon Energy**

PHOTOS (Corrected)

SANC (Scalar QED)

$\frac{d\Gamma}{dE_\gamma}$

$E_\gamma$, MeV

**Pion Energy**

$M_{K^+}$ = 494 MeV

$M_{\pi^-}$ = 139 MeV

$M_{B^0}$ = 5279 MeV

$\frac{d\Gamma}{dE_{\pi^-}}$

$E_{\pi^-}$, MeV

**Photon angle with res. $\pi^-$**

$B^0 \to \pi^- K^+ \gamma$

$\frac{d\Gamma}{d\cos\theta_\gamma}$

$\cos\theta_\gamma$

**$\pi^- K^+$ acoll. angle**

$\Gamma^{Total}/\Gamma^{Born}$ = 1.0000(3)

$\mu_{UV}$ = 900 MeV

$\frac{d\Gamma}{d\cos\theta_{acol.}}$

$\cos\theta_{acol.}$

Results from: G. Nanava, Z. Was, hep-ph/0607019
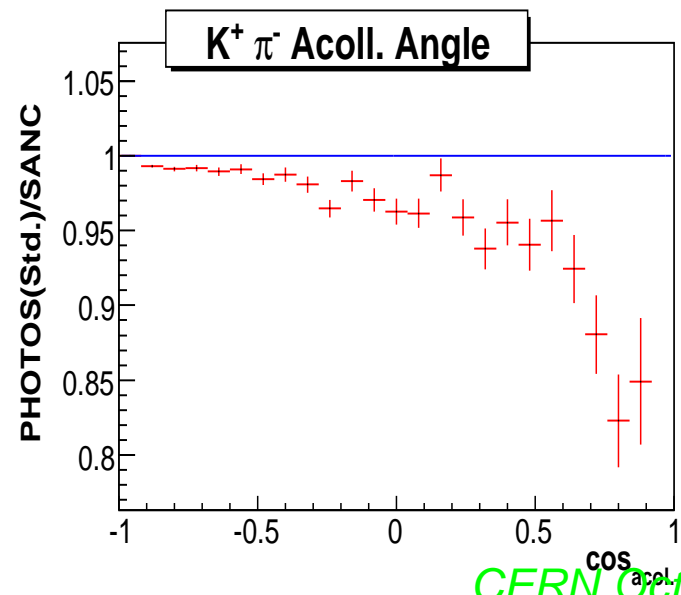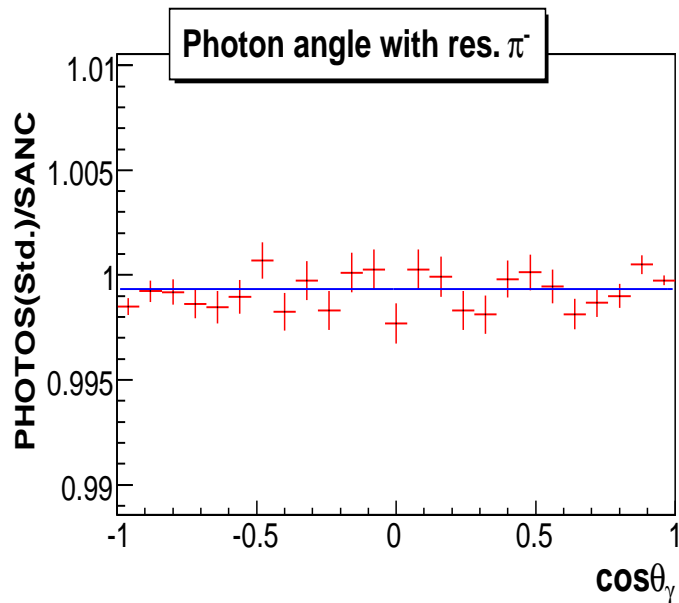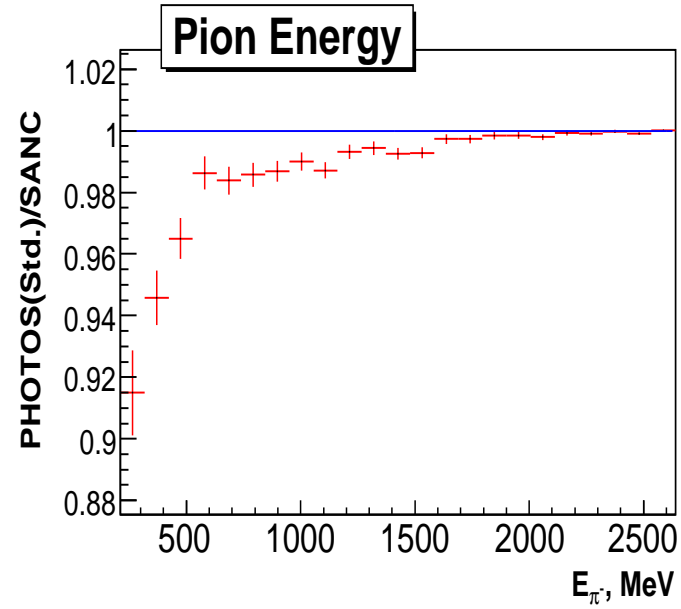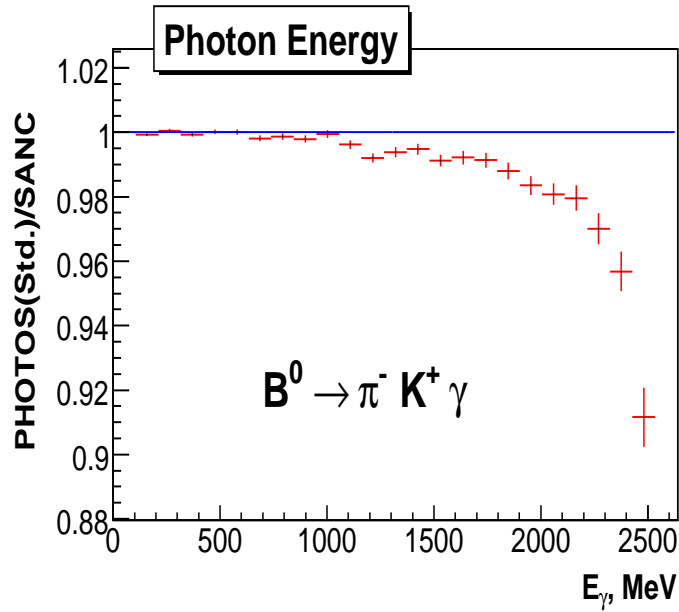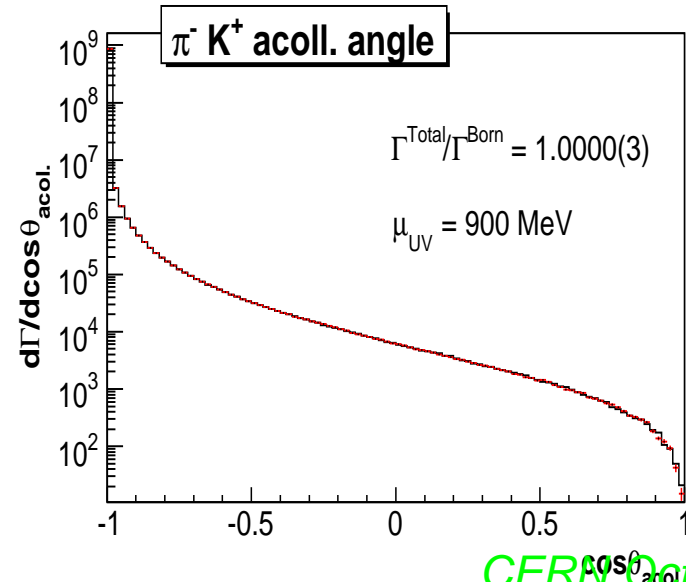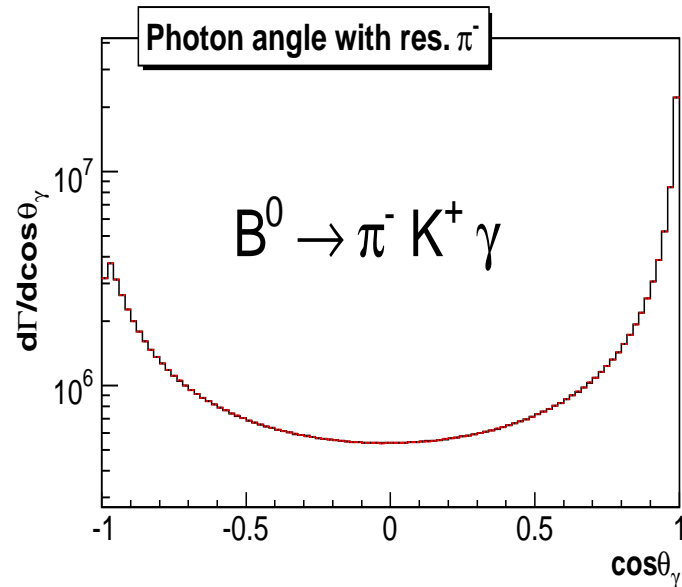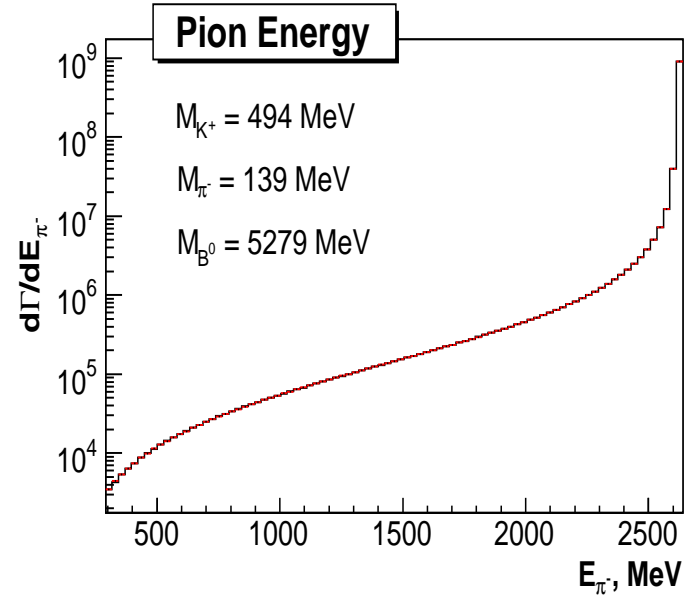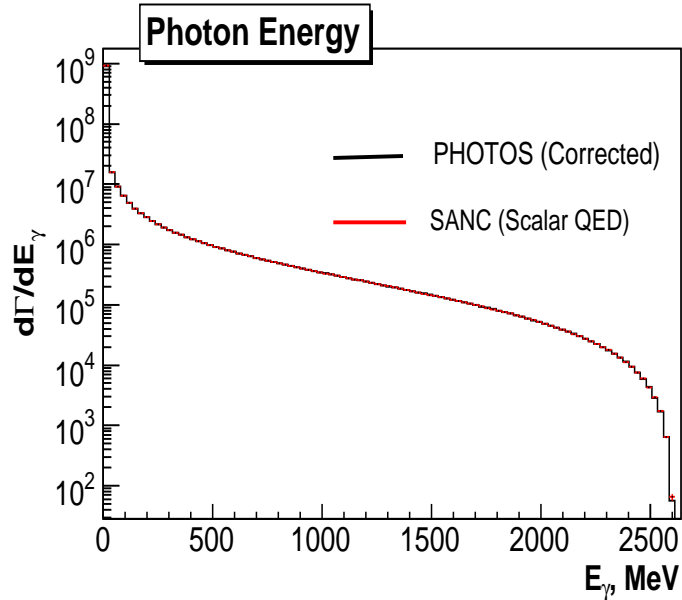
$B^0 \to \pi^- K^+$; NLO improved PHOTOS ... also perfect !
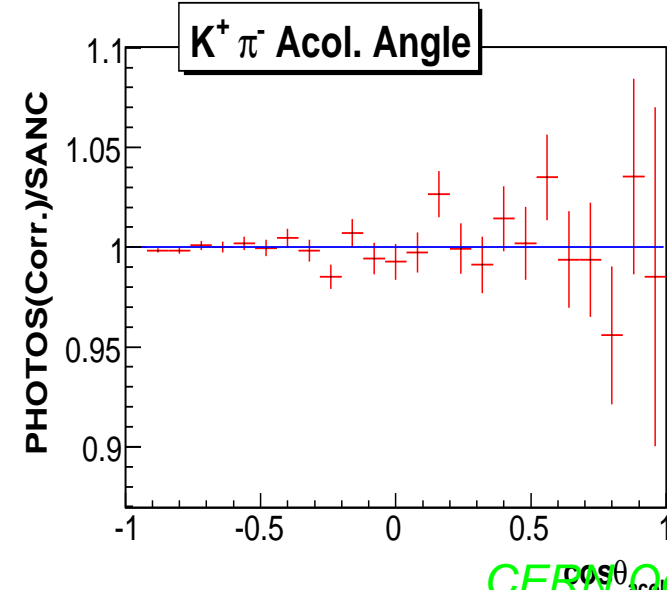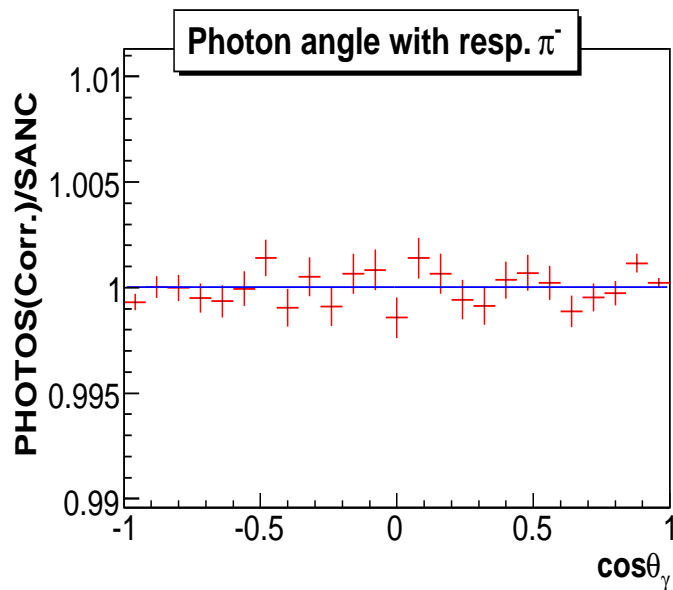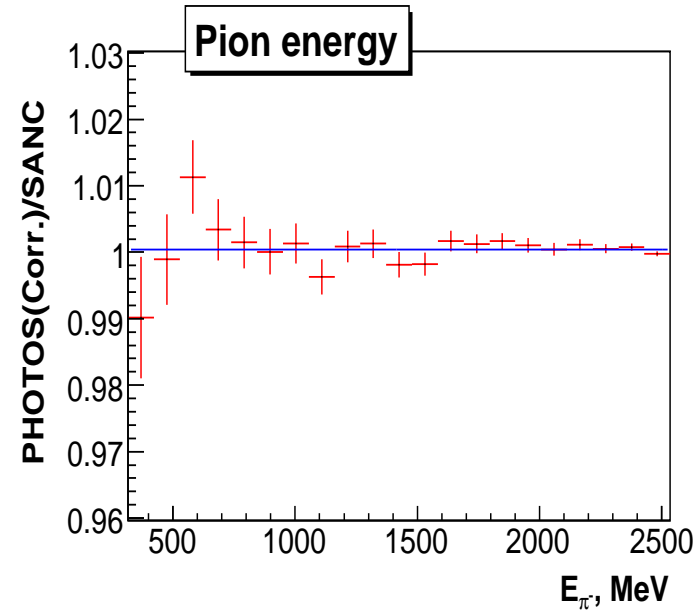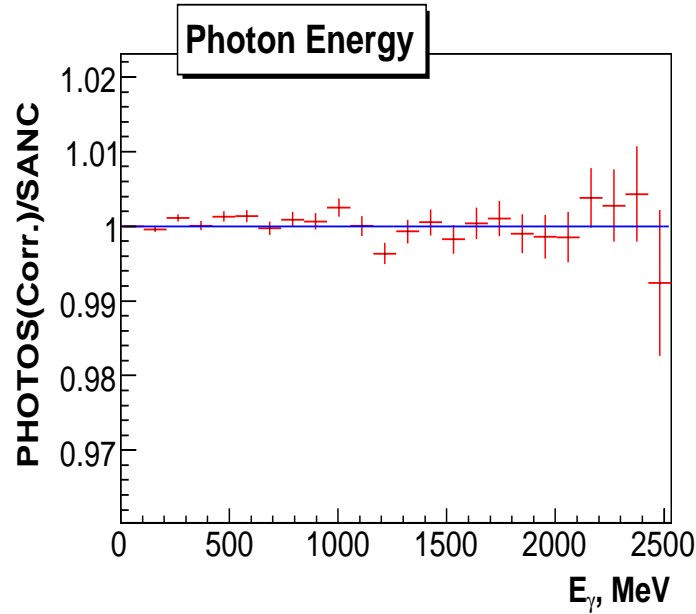
$\tau \rightarrow l\nu\bar{\nu}(\gamma)$ **PHOTOS vs TAUOLA**

Plot of worst agreement for the channel. Distribution of $\gamma\nu_\tau\nu_\mu$ system mass is shown .



Comparison of Mass(1) of gamma nu_tau nu_mu~ in channel tau- => gamma nu_tau nu_mu~ mu-    SDP
**0.00444**
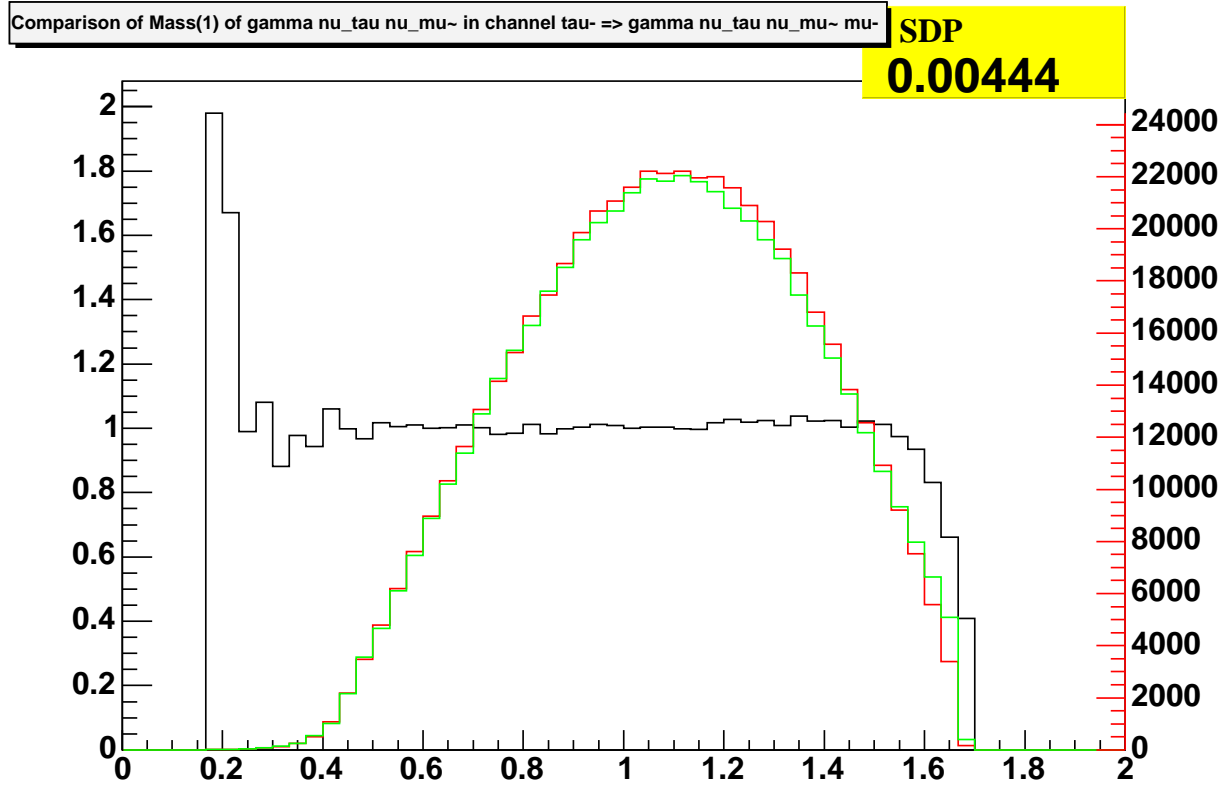
Also the fraction of events with photon above threshold agrees better than permille level.

In TAUOLA complete matrix element, comparison test PHOTOS approximations and design.

*Phys. Lett, B 303 (1993) 163-169*
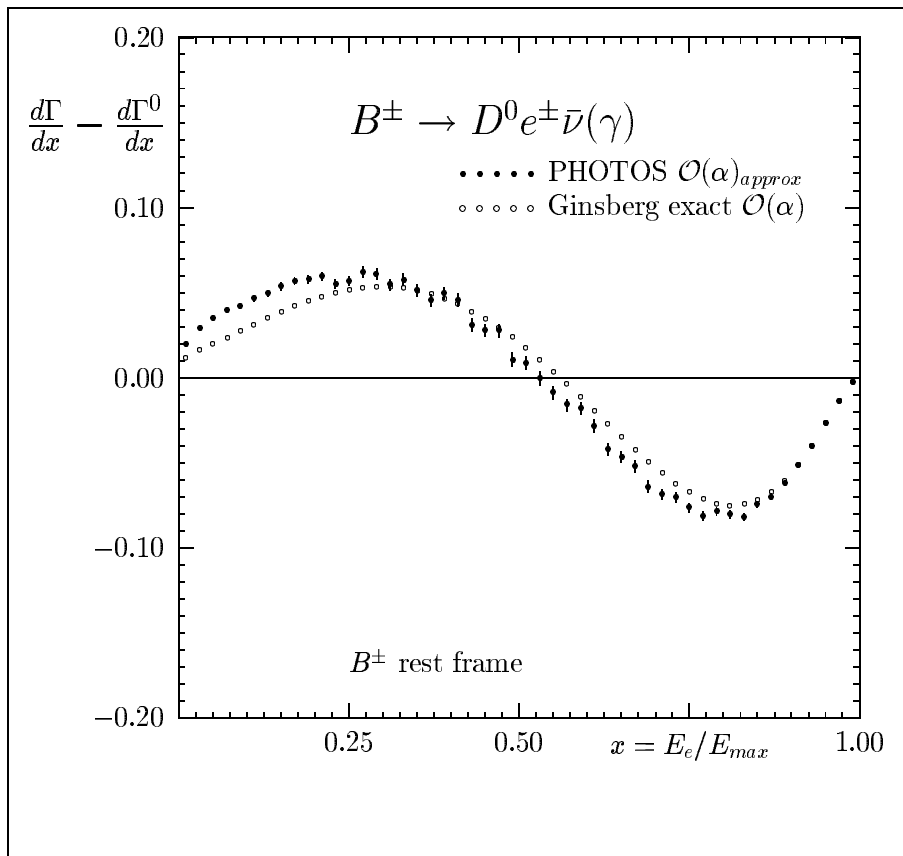


Radiative correction to the decay rate $(d\Gamma/dx - d\Gamma^0/dx)$ for $B^\pm \to D^0 e^\pm \bar\nu(\gamma)$ in the $B^\pm$ rest frame. Open circles are from the exact analytical formula [2], points with the marked statistical errors from PHOTOS applied to JETSET 7.3. A total of $10^7$ events have been generated. The results are given in units of $(G_\mu^2 m_B^5/32\pi^3)N_\eta|V_{cb}|^2|f_+^D|^2$, where $N_\eta = \eta^5 \int_0^1 x^2(1-x)^2/(1-\eta x)dx$ and $\eta = 1 - m_D^2/m_B^2$.

- *"QED bremsstrahlung in semileptonic $B$ and leptonic $\tau$ decays"* by E. Richter-Was.

- agreement up to 1%

- disagreement in the low-$x$ region due to missing sub-leading terms

- study performed in 1993.

$K \to \pi e \nu (\gamma)$ **PHOTOS w/Interf vs Gasser**
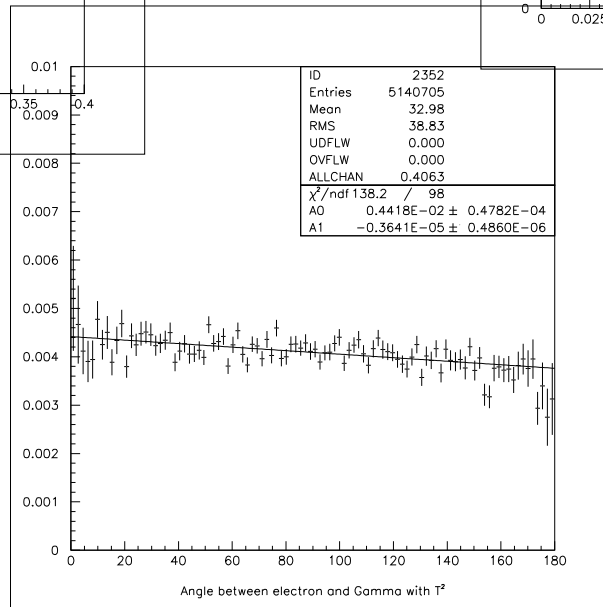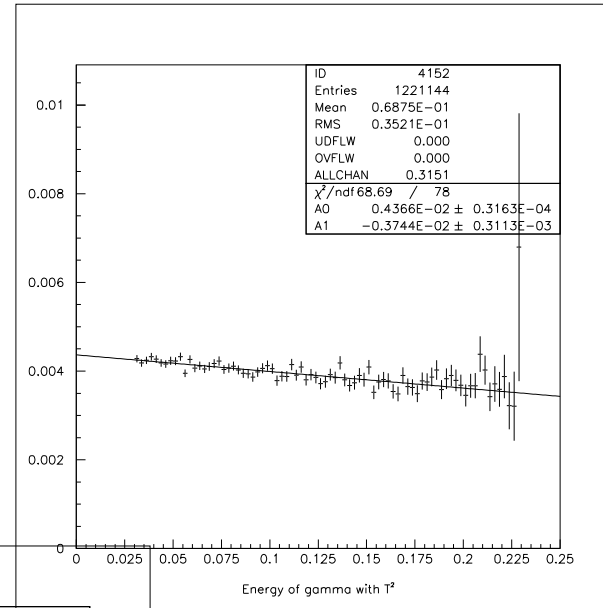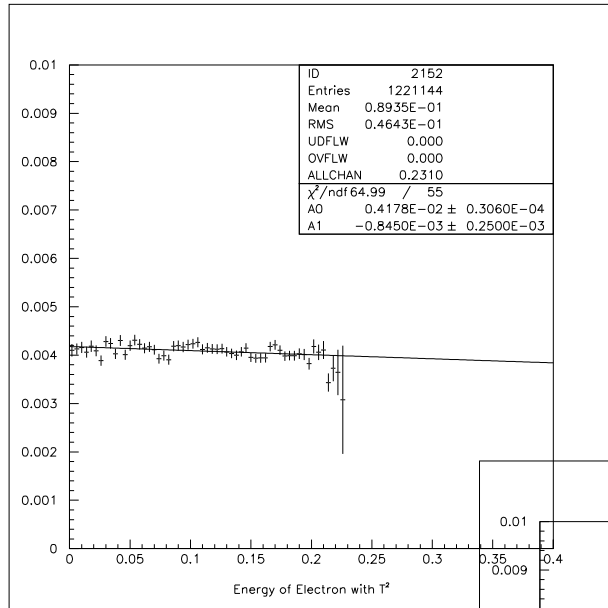


This was OK in 2005                                      but it is not systematic work.

Events with and without photon:

| $R = \dfrac{\Gamma_{K_{e3\gamma}}}{\Gamma_{K_{e3}}}$ | PHOTOS % | GASSER % |
|---|---|---|
| $5 < E_\gamma < 15\,MeV$ | 2.38 | 2.42 |
| $15 < E_\gamma < 45\,MeV$ | 2.03 | 2.07 |
| $\Theta_{e,\gamma} > 20$ | 0.876 | 0.96 |

courtesy of NA48 and Prof. L.Litov

This results can be obtained starting from PHOTOS version 2.13.
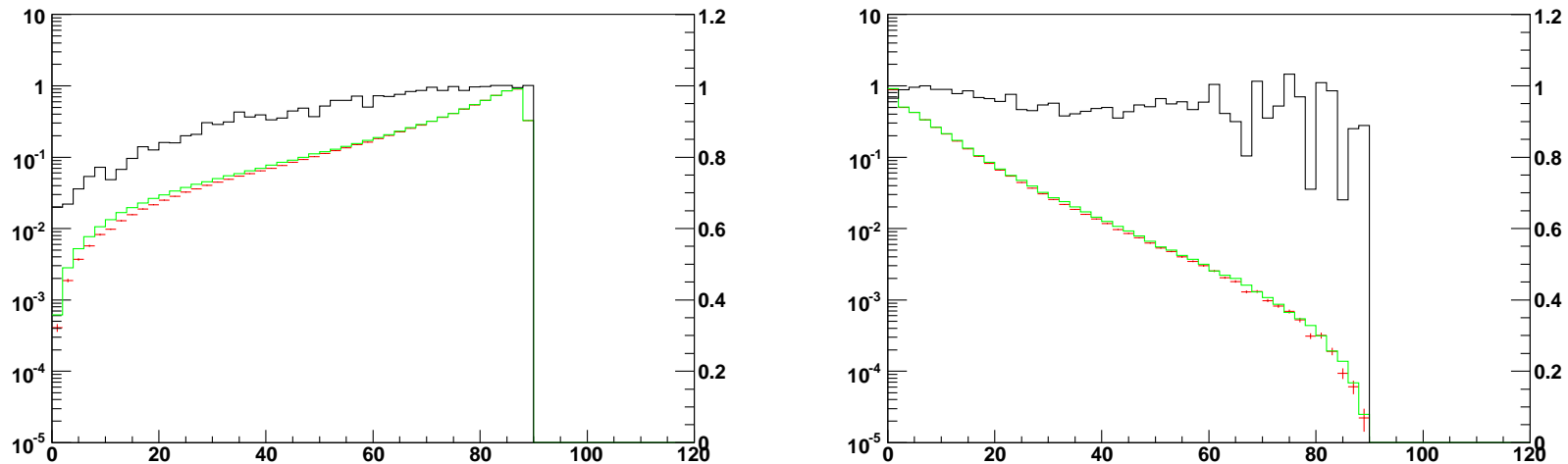
## Multiphoton radiation
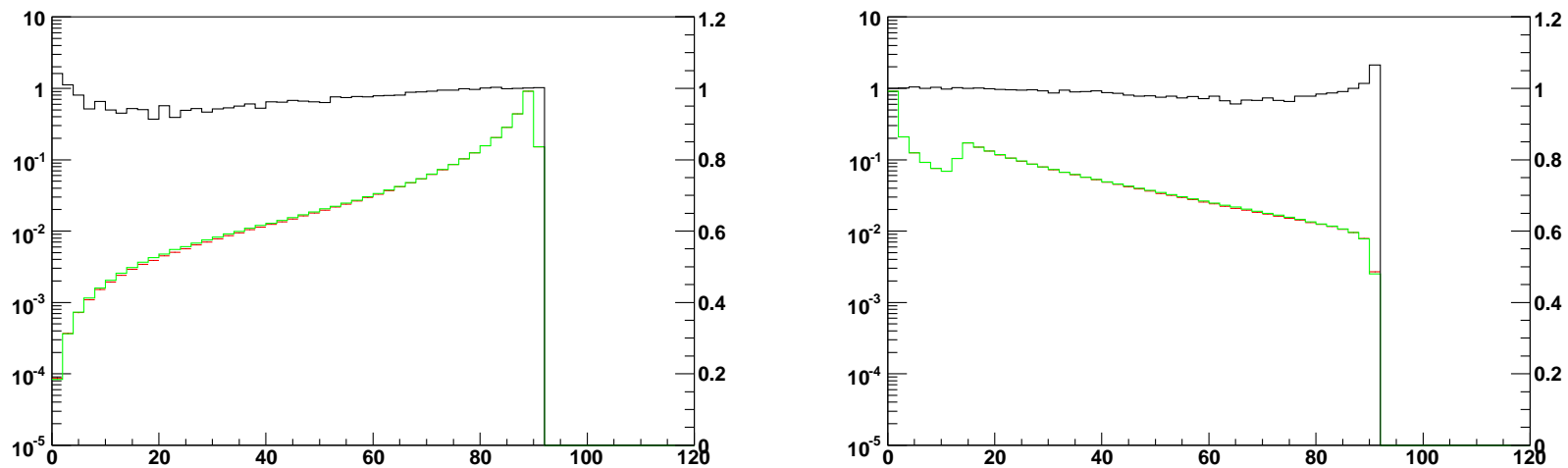
*Figure 3: Comparison of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00409. In right frame the invariant mass of the $\mu^-\gamma$ pair; SDP=0.0025. The pattern of differences between PHOTOS and KKMC is similar to the one of Fig 1. The fraction of events with hard photon was 16.0824 $\pm$ 0.0040% for KKMC and 16.1628 $\pm$ 0.0040% for PHOTOS.*
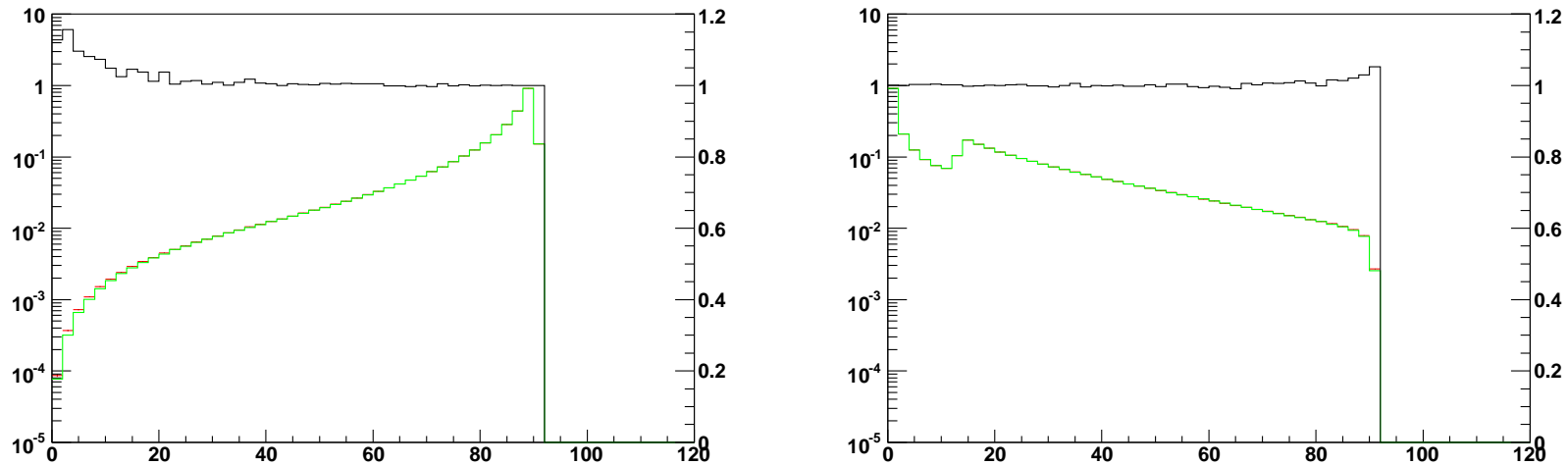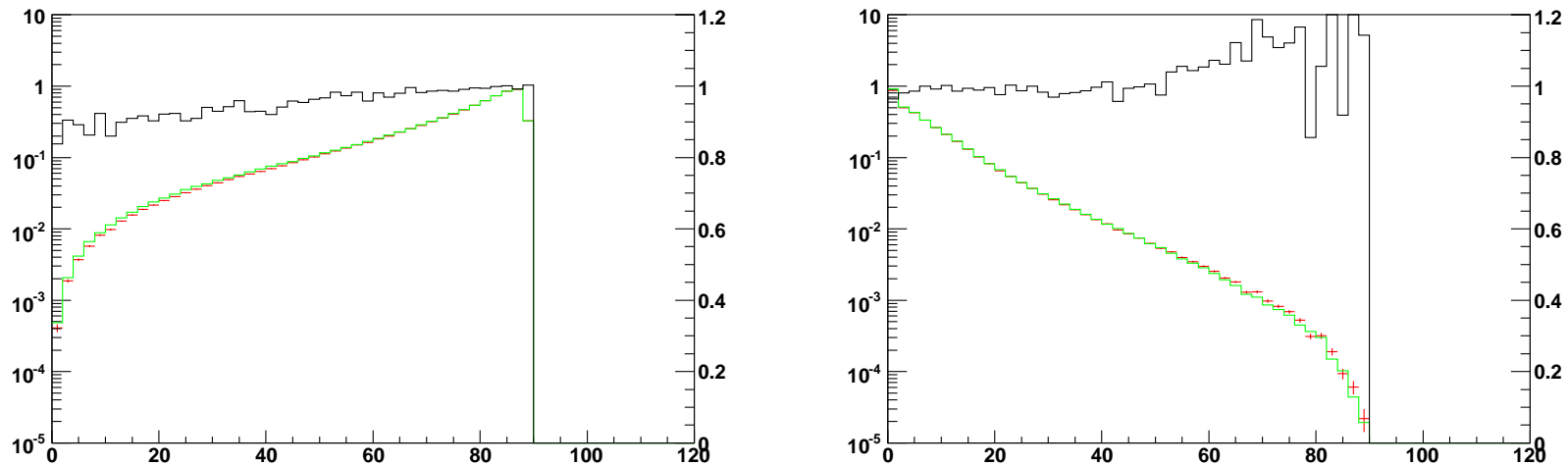
*Figure 4: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.0000249. In the right frame the invariant mass of the $\mu^-\gamma$ pair; SDP=0.0000203. The fraction of events with hard photon was 16.0824 $\pm$ 0.004% for KKMC and 16.0688 $\pm$ 0.004% for PHOTOS.*

*Figure 5: Comparisons of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP= 0.00918. In the right frame the invariant mass of the $\gamma\gamma$ pair; SDP=0.00268. The fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KKMC and 1.2952 $\pm$ 0.0011% for PHOTOS.*

*Figure 6: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP= 0.00142. In the right frame the invariant mass of the $\gamma\gamma$; SDP=0.00293. The fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KKMC and 1.2868 $\pm$ 0.0011% for PHOTOS.*

*Acoplanarity distribution – Looks good*

Acoplanarity

**KKMC**
**KKMC+PHOTOS EXP**
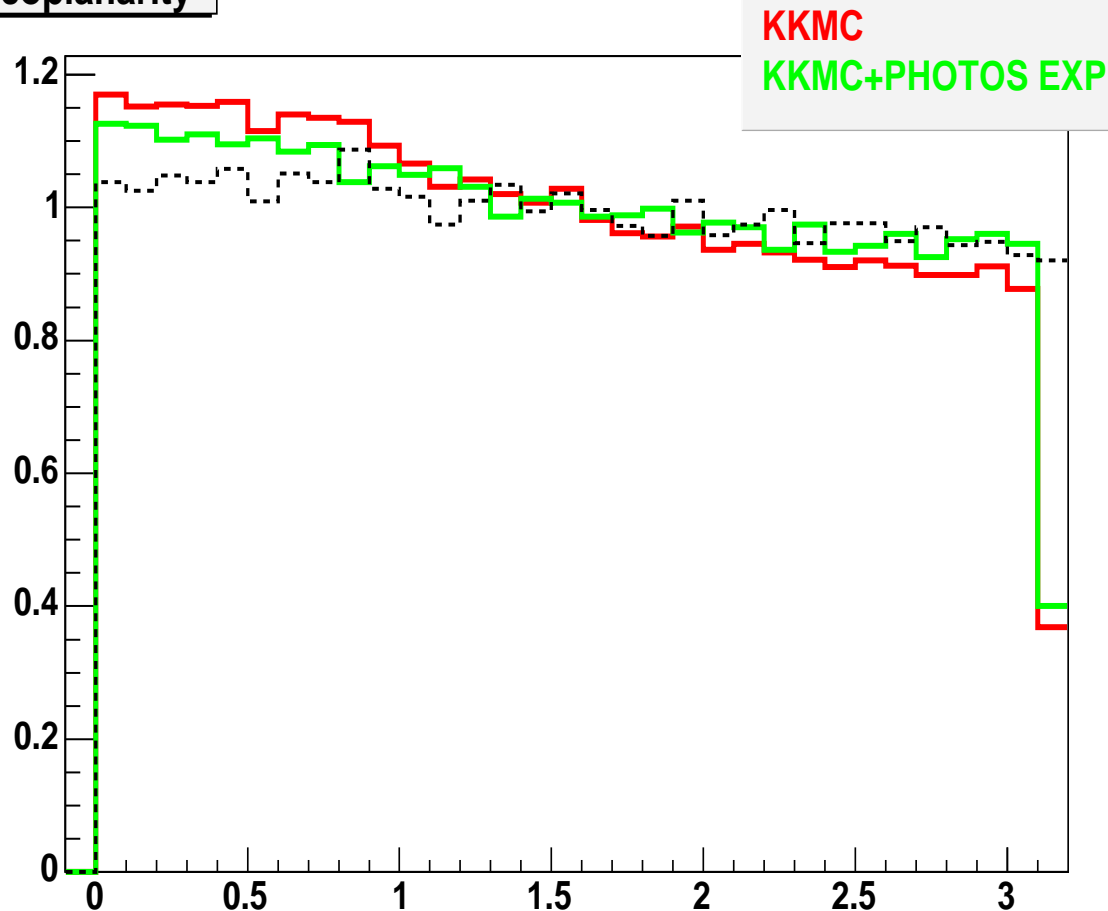


Two plane spanned on $\mu^+$ and respectively two hardest photons localized in the same hemisphere as $\mu^+$. Why PHOTOS works so good?
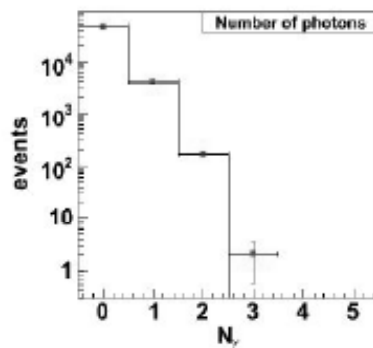
# A successful validation example..

- Comparison between PHOTOS (supposed to be an approximate algorithm in principle) and HORACE (exact QED DGLAP solution):
  - Turns out that PHOTOS is doing an excellent job!

## HORACE vs Photos (3)

- Photon multiplicity and transverse momentum spectrum done with standalone generators (outside Athena)

perfect agreement for all $p_T$ range
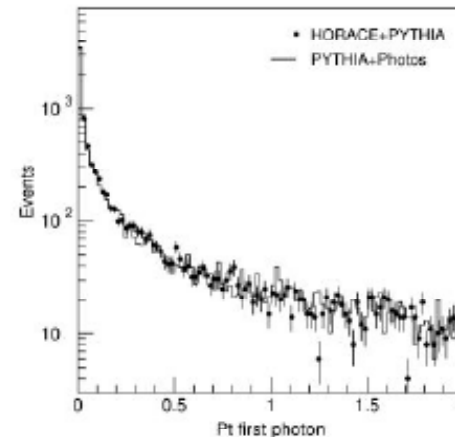


with cut $p_T(\gamma) > 500$ MeV perfect agreement also in Athena iterfaced version to third hard photon
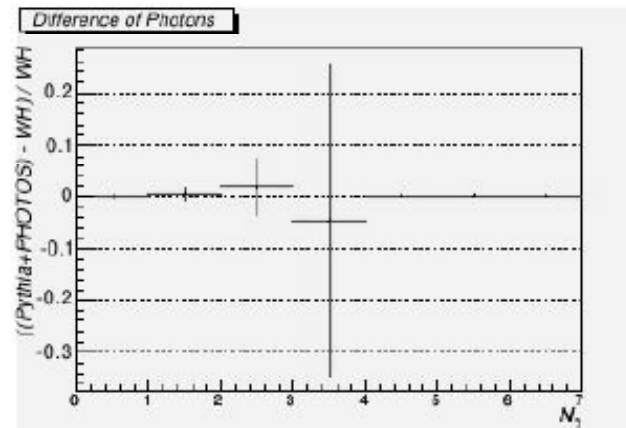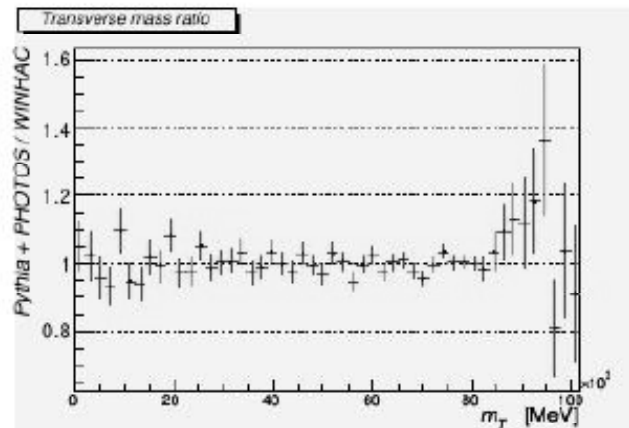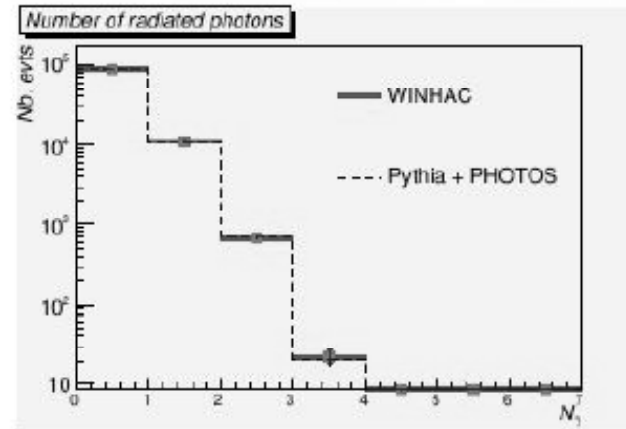
- ▪ Pythia + HORACE
- — Pythia + Photos

29/05/06    M. Bellomo - ATLAS Monte Carlo    15

This is for Z production at LHC.

21

# And another one.. Our Winhac effort
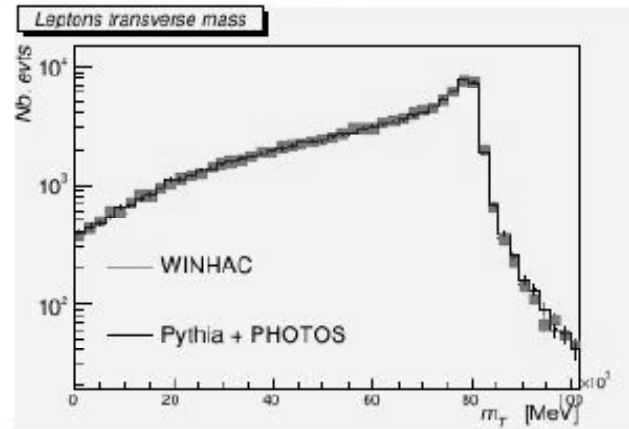


This is for W production at LHC.

## MC Generators for LHC at ATLAS

ATLAS Overview Week (February 2007)

Borut Kersevan
Jozef Stefan Inst.
Univ. of Ljubljana

ATLAS experience:
- Generators used
- Validation procedures
- Interesting examples

Not systematic work on algorithm, but program validation for ATLAS. From one day talk at CERN main auditorium 11 am.