PHOTOS and **NLO**

Z. Was

Institute of Nuclear Physics, Krakow and CERN-PH, Geneva

talk include recent contributions obtained with:

P. Golonka

CERN IT/CO-BE , Geneva, Institute of Nuclear Physics, Krakow

G. Nanava

JINR, Dubna, Russia, Institute of Nuclear Physics, Krakow

and:

Members of BELLE and NA48 Collab.

Some misprints found, plots: transparencies 36-43 replaced/improved (25.05).

Web pages: http://wasm.home.cern.ch/wasm/goodies.html

http://piters.home.cern.ch/piters/MC/PHOTOS-MCTESTER/

Supported in part by the EU grant MTKD-CT-2004-510126, in partnership with the CERN Physics Department

$\mathcal W$ hy worry?

Because QED corrections affect interpretation of measured quantities: cut off induced corrections to the rates, to parity sensitive asymmetries ...

Effects appear sometimes in unexpected ways, misidentifications, radiative corrections to backgrounds etc.

Expertise in experimental side is needed. I have to skip examples, they take too long to explain.

Some examples were included in my pheno-club presentation; march 2006.

\mathcal{M} otivation

- PHOTOS Monte is used by eg. Belle/Babar/TEVATRON/LHC for simulation of QED bremsstrahlung in decays.
- For many years the precison of the program prediction was not an issue.
- In recent years things started to change. Excellent news for me.
- Tests demonstrating precision and flexibility became public in 2005: hep-ph/0508015 and Eur.Phys.J.C45:97-107,2006. Precision level 0.1%.
- Recently implementation of NLO corrections into PHOTOS for Z (hep-ph/0604232) and B (my talk on May 19) decay was completed.
- Effort in explaining program foundations is justified now.
- I hope of possible future extensions into QCD, it is my motivation as well.
- I want to use PHOTOS project as a learning vehicle.



Plan

- Brief presentation of PHOTOS.
- Phase space and algorithm.
- Single photon bremsstrahlung kernels Z and B decays.
- Numerical tests.
- Multiple photon generation case of Z decay.
- Numerical tests.
- Summary and outlook.

Presentation

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P.Golonka) is used to simulate the effect of radiatiative corrections in decays, since 1989.
- Full events combining complicated tree structure of production and subsequent decays have to be fed in, e.g. help of F77 HEPEVT event record.
- This is often source of technical difficulties as standard is often overruled.
- At every event decay branching, PHOTOS intervene. With certain probability extra photon may be added and kinematics of other particles adjusted.
- PHOTOS works on four-momenta; watch numerical stability.
- I will not talk about those time consuming aspects but about relation of PHOTOS with explicit field theory calculations, n-body phase space, and expansions with respect to leadin-log truncation and eikonal-truncation.
- Program can provide precise results if one invest in process dependent ME.

CERN, May 2006

\mathcal{M} ain \mathcal{R} eferences

- E. Barberio, B. van Eijk and Z. Was, Comput. Phys. Commun. 66, 115 (1991).
- E. Barberio and Z. Was, Comput. Phys. Commun. 79, 291 (1994).
- P. Golonka, B. Kersevan, T. Pierzchala, E. Richter-Was, Z. Was and M. Worek, arXiv:hep-ph/0312240, Comput. Phys. Commun.in print.
- P. Golonka, T. Pierzchala and Z. Was, Comput. Phys. Commun. 157 (2004) 39
- Z. Was, Eur. Phys. J. C 44 (2005) 489
- P. Golonka and Z. Was, arXiv:hep-ph/0506026, Eur. Phys. J. C 45 (2006) 97 and arXiv:hep-ph/0508015, arXiv:hep-ph/0604232
- G. Nanava, Z. Was, in pereparation.

Phase Space: (trivialities)

Let us recall the element of Lorentz-invariant

phase space (Lips):

$$dLips_{n+1}(P) = \frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3} \frac{d^3q}{2q^0(2\pi)^3} (2\pi)^4 \delta^4 \left(P - \sum_{1}^{n} k_i - q\right)$$

$$= d^4p \delta^4 (P - p - q) \frac{d^3q}{2q^0(2\pi)^3} \frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3} (2\pi)^4 \delta^4 \left(p - \sum_{1}^{n} k_i\right)$$

$$= d^4p \delta^4 (P - p - q) \frac{d^3q}{2q^0(2\pi)^3} dLips_n(p \to k_1 \dots k_n).$$

Integration variables, the four-vector p, compensated with $\delta^4 (p - \sum_{1}^{n} k_i)$, and another integration variable M_1 compensated with $\delta (p^2 - M_1^2)$ are introduced.

CERN, May 2006

Phase Space: (cont.)

$$dLips_{n+1}(P) = \begin{bmatrix} k_{\gamma}dk_{\gamma}d\cos\theta d\phi \frac{1}{2(2\pi)^3} \end{bmatrix} \times dLips_n(p \to k_1...k_n)$$

If we had l photons accompanying n other particles, the factor in square brackets would be iterated. A statistical factor $\frac{1}{l!}$ would complete the formula for the phase-space parametrization, which is quite similar to the formal expansion of the exponent.

Replace $dLips_n(p \rightarrow k_1...k_n)$ in above formula by $dLips_n(P \rightarrow k_1...k_n)$ and we obtain a **tangent space**. Photons do not affect other particles' momenta at all. Have no boundaries on energy and are independent one from another.

This expression would be only slightly more complicated if instead of photon massive particle was to be added.

9

CERN, May 2006

Phase Space: (cont.)

$$dLips_{n+1}(P) = \left[4dk_{\gamma}k_{\gamma}d\cos\theta d\phi \frac{1}{8(2\pi)^3} \times \frac{\lambda^{1/2}(1,m_1^2/p^2,M_{2...n}^2/p^2)}{\lambda^{1/2}(1,m_1^2/P^2,M_{2...n}^2/P^2)} \right] \\ \times dLips_n(P \to \bar{k}_1...\bar{k}_n)$$

The formula should read as follow:

- 1. Take the distribution of n-body phase space
- 2. Turn it back into some coordinate variables; choose two sub-groups: here 1 and 2...n.
- 3. add newly generated variables for photon accordingly to expr. in sqare bracket.
- 4. construct new kinematical configuration from all variables. If we had l photons accompanying n other particles, the factor in square brackets would be iterated. A statistical factor $\frac{1}{l!}$ would complete the formula for the phase-space parametrization, which is quite similar to the formal expansion of the exponent.

Phase Space: (cont.)

$$dLips_{n+l}(P) = \frac{1}{l!} \prod_{i=1}^{l} \left[k_{\gamma_i} dk_{\gamma} d\cos \theta_i d\phi_i \frac{1}{2(2\pi)^3} \right] \times dLips_n(P \to k_1 \dots k_n).$$

• We defined **tangent space**. Photons do not affect other particles' momenta. Also, have no boundaries on energy and are independent one from another.

• It is important to realize that one has to control matrix element on the tangent space to define transformation to the real space. Rejection diminish photon mutiplicity.

- Rejection implements changes in phase space density and properties of matrix element.
- Rejection is performed photon after photon; phase space is in (principle) trivial.
- It remain to work out how the matrix element for original n particles plus photon(s) should look.

10

CERN, May 2006

- 1. Accordingly to Poissonian distribution $P(n) = e^{-\lambda} \frac{\lambda^n}{n!}$ with its λ sufficiently large and otherwise arbitrary multiplicity for photons in tangent space is generated. For fixed maximum multiplicity crude distribution is a combination of binomial distributions, then $\lambda < 1$.
- 2. For each photon energy and angular orientation is generated and Jacobians are calculated.
- 3. New configuration from n'+1 body phase space can be now constructed or rejection, individually on every photon candidate, can be performed.
- 4. We skip refinements necessary due to multibranching. It is similar like in: TAUOLA paper: CPC 76 (1993) 361.
- 5. Note also that in this way correlations between photons are introduced.
- 6. I skip essential point of choice of frames for angular orientation.
- 7. But I do not skip matrix element: real emissions and virtual corrections.





Heuristic plots

Difference between yellow surface and underlying hemisphere represent missing

parts of e.g. second order matrix element.





- In my construction I rely on properties of factorization, limits of my personal experience are summarized in paper on $e^+e^- \rightarrow \nu_e \bar{\nu}_e \gamma \gamma$, EPJC C44 (2005) 489.
- Matching NLO kernels for iterations lead to many options!
- In tangent space construction of ME was trivial, because photons were independent.
- In fact we went to construct even more primitive space than tangent space based on eikonal approximation.
- We have piled up. emissions from all possible fi nal states together; just one common direction with respect to which photons are generated. At this step interferences between emissions from different photons were missing of course.
- In this way we obtained poissonian distribution for number of photon candidates $P(n) = e^{-\lambda} \frac{\lambda^n}{n!}$ with nearly arbitrary λ .
- I checked that with the precision of better than 10^{-4} results remain unchanged, even with drastic changes of crude level photon multiplicity.

• The three spaces:

- (i) of normal phase space configurations with arbitrary number of photons
- (ii) tangent spaces with arbitrary number of photons
- (iii) degenerate tangent space also with arbitrary number of photons,
- all spaces have volumes normalized to unity (watch: real photon and virtual corrections),
- residual parts of Sudakov form-factors not matching that constraint have to be shifted to overall normalization.
- there are precisely defined transformation between those spaces; Condition of decreasing photon multiplicity when we go from space to space is useful for MC construction.
- Instead of providing details of organization of these spaces, let me show some numerical results.

Back to starting point

- From the point of view of matrix element and choice of internal angular variables *in case* of Z decays, PHOTOS is very close to MUSTRAAL MC by F. Berends S. Jadach and R. Kleiss, CPC 29 (1983) 185.
- The first step to re-introduce NLO terms for Z decay into PHOTOS was to check relations between 4-vectors and angles used in different version of phase space parametrization.
- We had to separate the parts of weight responsable for phase space from this of matrix element.
- Some factors of the type $\lambda^{1/2}(1,m_1^2/M^2,m_2^2/M^2)$ had to be reinstalled.

• The fully differential distribution from MUSTRAAL (used also in KORALZ for single photon mode) reads:

$$X_f = \frac{Q'^2 \alpha (1 - \Delta)}{4\pi^2 s} s^2 \left\{ \frac{1}{(k'_+ k'_-)} \left[\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s, t', u) \right] \right\}$$

• Here:

$$s = 2p_{+} \cdot p_{-}, \quad s' = 2q_{+} \cdot q_{-},$$

$$t = 2p_{+} \cdot q_{+}, \quad t' = 2p_{+} \cdot q_{-},$$

$$u = 2p_{+} \cdot q_{-}, \quad u' = 2_{-} \cdot q_{+},$$

$$k'_{\pm} = q_{\pm} \cdot k, \quad x_{k} = 2E_{\gamma}/\sqrt{s}$$

• The Δ term is responsable for final state mass dependent terms, p_+ , p_- , q_+ , q_- , k denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

CERN, May 2006

• after trivial manipulation it can be written as:

$$X_{f} = \frac{Q'^{2}\alpha(1-\Delta)}{4\pi^{2}s}s^{2} \left\{ \frac{1}{(k'_{+}+k'_{-})}\frac{1}{k'_{-}} \left[\frac{\mathrm{d}\sigma_{B}}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_{B}}{\mathrm{d}\Omega}(s,t',u) \right] + \frac{1}{(k'_{+}+k'_{-})}\frac{1}{k'_{+}} \left[\frac{\mathrm{d}\sigma_{B}}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_{B}}{\mathrm{d}\Omega}(s,t',u) \right] \right\}$$

• In PHOTOS the following expression is used:

$$\begin{split} X_f^{PHOTOS} &= \frac{Q'^2 \alpha (1-\Delta)}{4\pi^2 s} s^2 \Biggl\{ \\ & \frac{1}{k'_+ + k'_-} \frac{1}{k'_-} \quad \left[(1+(1-x_k)^2) \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega} \left(s, \frac{s(1-\cos\Theta_+)}{2}, \frac{s(1+\cos\Theta_+)}{2}\right) \right] \frac{(1+\beta\cos\Theta_+)}{2} \\ & + \frac{1}{k'_+ + k'_-} \frac{1}{k'_+} \quad \left[(1+(1-x_k)^2) \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega} \left(s, \frac{s(1-\cos\Theta_-)}{2}, \frac{s(1+\cos\Theta_-)}{2}\right) \right] \frac{(1-\beta\cos\Theta_+)}{2} \Biggr\} \\ & \text{ where } : \Theta_+ = \angle (p_+, q_+), \ \Theta_- = \angle (p_-, q_-) \\ \Theta_\gamma = \angle (\gamma, \mu^-) \text{ are defined in } (\mu^+, \mu^-) \text{-pair rest frame} \end{aligned}$$

$$\bullet \text{ also factor } \Gamma^{total} / \Gamma^{Born} = 1 + 3/4\alpha / \pi \text{ defines first order weight.} \end{split}$$

1

CERN, May 2006

The differences are important

- The two expressions define weight to make out of PHOTOS complete first order.
- The PHOTOS expression separates (i) Final state bremsstrahlung (ii) electroweak parameters of the Born Cross section (iii) Initial state bremsstrahlung that is orientation of the spin quantization axix for Z.
- That would be heavy burden for managing PHOTOS interfaces. I know, because we encounter such difficulties for universal interface for TAUOLA.
- It is possible but extremenly inconvenient. Parts of generation managed by distinct authors.
- Of course all this has to be understood in context of Leading Pole approximation. For example initial-fi nal state interference breaks the simplification. Limitations need to be controlled: Phys. Lett. B219:103,1989.

Scalar QED for matrix elements in B decays

- Scalar QED is not an ultimate theory in the case of decays like $B^-\to\pi^0 K^-$ or $B^0\to\pi^+K^-$
- Nonetheless matrix elements can be calculated and provie good input for tests.
- Massive final states, $m_{\pi}/m_B \neq m_K/m_B \simeq 0.1$.
- Scalar particles.
- In fact much simpler matrix element than in case of Z decay.
- The one-loop QED correction to the decay width can be represented as the sum of the Born contribution with the contributions due to virtual loop diagrams and soft and hard photon emissions.

$$d\Gamma^{\text{Total}} = d\Gamma^{\text{Born}} \left\{ 1 + \frac{\alpha}{\pi} \left[\delta^{\text{Soft}}(\boldsymbol{m}_{\gamma}, \boldsymbol{\omega}) + \delta^{\text{Virt}}(\boldsymbol{m}_{\gamma}, \boldsymbol{\mu}_{\boldsymbol{U}\boldsymbol{V}}) \right] \right\} + d\Gamma^{\text{Hard}}(\boldsymbol{\omega})$$

CERN. May 2006

where for **Neutral meson decay channels** we have: – Virtual photon contribution

$$\begin{split} \delta^{\text{Virt}}(m_{\gamma}, \mu_{UV}) &= \left[1 + \frac{M^2 - m_1^2 - m_2^2}{\Lambda} \ln \frac{2m_1m_2}{M^2 - m_1^2 - m_2^2 + \Lambda} \right] \ln \frac{M^2}{m_{\gamma}^2} + \frac{3}{2} \ln \frac{\mu_{UV}^2}{M^2} \\ &+ \frac{M^2 - m_1^2 - m_2^2}{2\Lambda} \left[\text{Li}_2 \left(\frac{M^2 + m_1^2 - m_2^2 + \Lambda}{2\Lambda} \right) - \text{Li}_2 \left(\frac{-M^2 + m_2^2 - m_1^2 + \Lambda}{2\Lambda} \right) \right] \\ &+ 2 \ln \frac{2Mm_1}{M^2 + m_1^2 - m_2^2 + \Lambda} \ln \frac{m_1\Lambda}{M^3} + (1 \leftrightarrow 2) + \pi^2 \right] \\ &- \frac{\Lambda}{2M^2} \ln \frac{2m_1m_2}{M^2 - m_1^2 - m_2^2 + \Lambda} + \frac{m_2^2 - m_1^2}{4M^2} \ln \frac{m_2^2}{m_1^2} - \frac{1}{2} \ln \frac{m_1m_2}{M^2} + 1 \end{split}$$

– Soft photon contribution

$$\delta^{\text{Soft}}(m_{\gamma}, \omega) = \left[1 + \frac{M^2 - m_1^2 - m_2^2}{\Lambda} \ln \frac{2m_1m_2}{M^2 - m_1^2 - m_2^2 + \Lambda}\right] \ln \frac{m_{\gamma}^2}{4\omega^2} \\ + \frac{M^2 - m_1^2 - m_2^2}{2\Lambda} \left[\operatorname{Li}_2\left(\frac{-2\Lambda}{M^2 + m_1^2 - m_2^2 - \Lambda}\right) - \operatorname{Li}_2\left(\frac{2\Lambda}{M^2 + m_1^2 - m_2^2 + \Lambda}\right) + (1 \leftrightarrow 2)\right] \\ - \frac{M^2 + m_1^2 - m_2^2}{\Lambda} \ln \frac{2Mm_1}{M^2 + m_1^2 - m_2^2 + \Lambda} - (1 \leftrightarrow 2)$$

CERN, May 2006

- Hard photon contribution

$$d\Gamma^{\text{Hard}} = |A^{\text{Born}}|^2 4\pi \alpha \left(q_1 \frac{k_1 \cdot \epsilon}{k_1 \cdot k_\gamma} - q_2 \frac{k_2 \cdot \epsilon}{k_2 \cdot k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

- $\Lambda = \lambda^{1/2}(M^2, m_1^2, m_2^2)$
- The infrared divergency, is regularized by m_γ , it cancels in the sum of virtul and soft contributions
- The virtual correction depends on ultraviolet scale μ_{UV}
- The total width is free of ω and of the final meson mass singularity (KLN theorem), we will choose the scale to make an overall correction of order of zero.
- for **Charged meson decay channels** we have:

- Virtual photon contribution

$$\begin{split} \delta^{virt}(m_{\gamma},\mu_{UV}) &= \left[1+\frac{M^2+m_1^2-m_2^2}{\Lambda}\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\right]\ln\frac{Mm_1}{m_{\gamma}^2} + \frac{3}{2}\ln\frac{\mu_{UV}^2}{Mm_1} \\ &+ \frac{M^2+m_1^2-m_2^2}{2\Lambda}\left[\operatorname{Li}_2\left(\frac{M^2-m_1^2-m_2^2+\Lambda}{2\Lambda}\right) - \operatorname{Li}_2\left(\frac{M^2-m_1^2-m_2^2-\Lambda}{-2\Lambda}\right) \right. \\ &+ \operatorname{Li}_2\left(\frac{M^2+m_2^2-m_1^2-\Lambda}{-2\Lambda}\right) - \operatorname{Li}_2\left(\frac{M^2+m_2^2-m_1^2+\Lambda}{2\Lambda}\right) \\ &+ 2\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\ln\frac{\Lambda}{Mm_2} - \ln\frac{2Mm_2}{M^2+m_2^2-m_1^2+\Lambda}\ln\frac{M^2}{m_1^2} \end{split}$$

$$+ \frac{\Lambda}{2m_2^2} \ln \frac{2Mm_1}{M^2 + m_1^2 - m_2^2 + \Lambda} - \frac{M^2 - m_1^2}{4m_2^2} \ln \frac{m_1^2}{M^2} + 1;$$

– Soft photon contribution

$$\begin{split} \delta^{soft}(m_{\gamma},\omega) &= \left[1 + \frac{M^2 + m_1^2 - m_2^2}{\Lambda} \ln \frac{2Mm_1}{M^2 + m_1^2 - m_2^2 + \Lambda}\right] \ln \frac{m_{\gamma}^2}{4\omega^2} \\ &+ \frac{M^2 + m_1^2 - m_2^2}{2\Lambda} \left[\operatorname{Li}_2\left(\frac{-2\Lambda}{M^2 + m_1^2 - m_2^2 - \Lambda}\right) - \operatorname{Li}_2\left(\frac{2\Lambda}{M^2 + m_1^2 - m_2^2 + \Lambda}\right)\right] \\ &- \frac{M^2 + m_1^2 - m_2^2}{2\Lambda} \ln \frac{2Mm_1}{M^2 + m_1^2 - m_2^2 + \Lambda} \end{split}$$

CERN, May 2006

25

Z. Was

- Hard photon contribution

$$d\Gamma^{\text{Hard}} = |A^{\text{Born}}|^2 4\pi \alpha \left(q_1 \frac{k_1 \cdot \epsilon}{k_1 \cdot k_\gamma} - q \frac{P \cdot \epsilon}{P \cdot k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

- These formulas are far easier to work on than for Z.
- They do not require control of the quantization axis of B's as their spin is zero.
- The complete matrix element can be thus installed into PHOTOS with standard organization of information flow.
- Once matrix element is clearly defined. It can be also replaced.
- Gate for shape-factors from fits to data is open!

\mathcal{N} umerical results –first order

- $\bullet\,\,{\rm tests}\,\,{\rm for}\,\,Z\to\mu^+\mu^-$
- $\bullet\,\,{\rm tests}\,\,{\rm for}\,\,B^-\to K^-\pi^0$
- $\bullet\,\,{\rm tests}\,\,{\rm for}\,\,B^0\to K^+\pi^-$

CERN, May 2006



Similarity coefficients: T1=0.302959 %, T2=0.092415 %

Price paid for universality: see table and plots of the next slide.



With NLO, (singl em.) 100Mevts compared!

Part of formactor $F_R = \frac{3}{4} \frac{\alpha}{\pi}$ affecting total rate is watched after. Note: it decreases by $1 + F_R$ (e.g.) the final weight; and as consequence number of events with photon. This point is important for algorithm extensions.

Found decay modes:

Decay channel	Branching Ratio \pm Rough Errors		Max. shape
	Generator #1	Generator #2	dif. param.
$Z^0 \rightarrow \gamma \mu^+ \mu^-$	$21.9118 \pm 0.0047\%$	$21.9369 \pm 0.0047\%$	0.00000
$Z^0 \rightarrow \mu^+ \mu^-$	$78.0882 \pm 0.0088\%$	$78.0631 \pm 0.0088\%$	0.00000

Similarity coefficients: T1=0.050265 %, T2=0.000000 %





CERN, May 2006



• After all Higgs spin is zero. There must be similar simplifications as for B decay matrix element

- I do not have yet analytical, but only numerical support for that statement.
- The methodology of comparisons we have used in case of Z decay, is good to localize the differences, but somehow does not quantify agreements.
- \bullet we will use slightly different method for B decays.



Comparison With PHOTOS

- We used the same methodology as in the case of "W decay" (Acta Phys. Pol. B34, (2003) 4561-4569; hep-ph/0303260)
- To visualize the usually small differences between SANC and PHOTOS, we plot the ratios of the predictions from the two programs for the certain class of (pseudo-)observables
- These observables are

- <u>Photon energy in the decaying particle rest frame</u> sensitive to the collinear-soft componet of the distributions
- <u>Energy of final state charged particle</u> as the previous one
- <u>Angle of photon with final-state charged particle</u> sensitive to the collinear componet of the distributions
- <u>Acollinearity angle of the final-state particles</u> sensitive to the nonsoft and non-collinear, but non-leading componet of the distributions

















Z. Was

Kernels; summary

- We have demonstrated that complete first order kernel for single photon emission can be installed into PHOTOS.
- We have provided numerical tests that it works for samples of up to 10^9 events.
- Thanks to explicit form of matrix element it opens the gate for fits to the data and introduction of shape factors.
- but ...
- We need multiple photon radiation as well !!!



MC-Tester as measure on space of events. • For Z-decays and first order it was relatively simple to define comparison All invariant masses, which could be constructed from 3 four-vectors were histogramed. • MC-TESTER: Comput. Phys. Commun. 157 (2004) 39 was used for our automated comparisons for multiple photon configurations. Analysis has to be infrared safe; photons of energies below threshold added to

the nearest charged photon.

- Also if there was more than two hard photons, the softer ones were added to the nearest charged muon.
- In this way we define as identical, states of different photon multiplicity, if they differ by presence/absence of soft photons only.

CERN, May 2006

46

CERN, May 2006



Similarity coefficients: T1=0.027109 %, T2=0.000482 %

Improvement by a factor of 100 for shape difference parameter!



CERN, May 2006

NLO in PHOTOS (exp) included, 100Mevts, 2-ph test

Found decay modes:

Decay channel	Branching Ratio \pm Rough Errors		Max. shape
	Generator #1	Generator #2	dif. param.
$Z^0 ightarrow \mu^- \mu^+ \gamma$	$14.8164 \pm 0.0038\%$	$14.7829 \pm 0.0038\%$	0.00005
$Z^0 ightarrow \mu^- \mu^+$	$83.9177 \pm 0.0092\%$	$83.9303 \pm 0.0092\%$	0.00000
$Z^0 ightarrow \mu^- \mu^+ \gamma \gamma$	$1.2659 \pm 0.0011\%$	$1.2868 \pm 0.0011\%$	0.00293

Similarity coefficients: T1=0.066630 %, T2=0.004108 %



CERN, May 2006



ordering pictures suggest that it should be worse!



CERN, May 2006

51

Z. Was

SUMMARY – Applications. Topics:

- Installation of complete first order kernel in case of Z and two-body B-decays.
- Numerical tests of these kernels at single photon radiation level.
- \bullet Numerical tests for multiple photon radiation for Z decay only.
- New results: numerical size of genuine NLO effects in case of B-meson decays.
- Older results, for Z decays and size of genuine NLO, partly skipped. They were presented already.
- We usd technical pseudo-observables only. Interesting ones need long introduction.



- Definition of tangent phase space. Starting point for iteration.
- Relation: fixed order ME, parton shower-like iteration and exact phase space.
- Crude tangent space: all sources piled together. Algorithm branches.
- Solution is stable (0.01 %) for huge redefinitions of tangent space volumes.

- Algorithm works technically well, even for up to 10 charged particles in final state.
- Algorithm does not depend on tunable technical parameters.
- Nonetheless shape-factors can be introduced.
- No phase space slicing or ordering necessary, instead of degrading, we reproduce nice, normally only NNLO effects.
- Most of genuine NNLO effects escaped our tests; they are of order $\sim 10^{-4,-5}$.
- It is refreshing to see that PHOTOS parton shower like solution is realization of functional polynomial on the ring (polynomial on the field for tangent space).
- Relations between fixed order versions of the algorithm lead to intriguing coefficients: just ratios of integers. Tangent space is free of physics!
- Is PHOTOS 'toy model' instructive for some aspects of PS in QCD?
- May be, but it is quite 'heavy toy' by itself already.
- Fortunately represents established pheno-tool for QED as well.