PHOTOS Monte Carlo – its phase-space and benchmarks

Z. Was

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Web pages: http://wasm.home.cern.ch/wasm/goodies.html

http://piters.home.cern.ch/piters/MC/PHOTOS-MCTESTER/

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Purpose of the talk

Because QED corrections affect interpretation of measured quantities: cut off induced corrections to the rates, to parity sensitive asymmetries, CKM ...

PHOTOS was used for many years in low precision regime for that purpose by practically all experiments.

Precision requirements increased; responsability on the project grows.

We have completed re-amalysis of program content in some of its aspects:

-1- matrix elements for $Z \to l\bar{l}$; QED.

-2- matrix elements for $B \to K\bar{\pi}$; scalar QED.

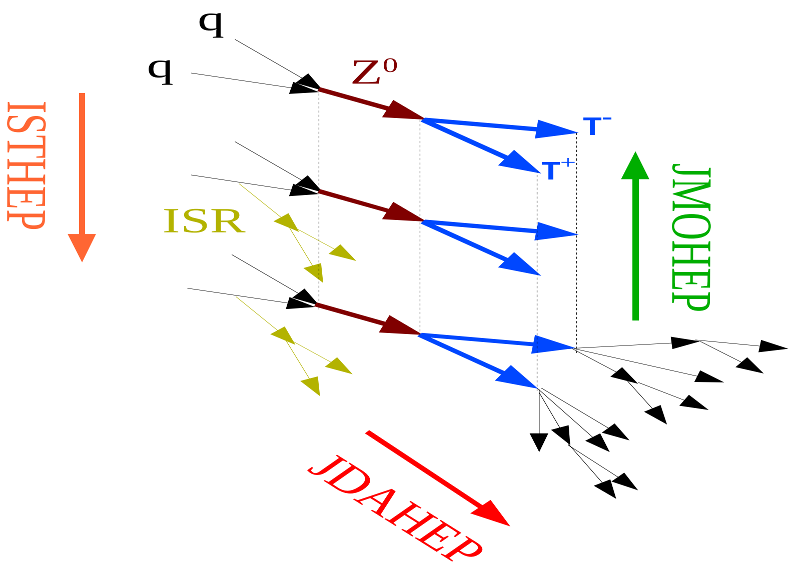
-3- phase space *of no approximations*, **also for multiple photon radiation!** On mass-shell iterative relations are attracting attention, technique used in PHOTOS may become useful outside QED?

PHOTOS: short presentation

Presentation

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P.Golonka) is used to simulate the effect of radiatiative corrections in decays, since 1989.
- Full events combining complicated tree structure of production and subsequent decays have to be fed in, e.g. help of F77 HEPEVT event record.
- This is often source of technical difficulties as standard is often overruled.
- At every event decay branching, PHOTOS intervene. With certain probability extra photon(s) may be added and kinematics of other particles adjusted.
- PHOTOS works on four-momenta; watch numerical stability.
- I will not talk about those practical aspects; they are well known.
- The C++ version of program exist since 1999

Problems With Event Record



1. Hard process
2. with shower
3. after hadronization
4. Event record overloaded with physics beyond design $\longrightarrow$ gramar problems.
5. Here we have basically LL phenomenology only.

This Is Physics Not F77!

Similar problems are in any use of full scale Monte Carlos, lots of complaints at MC4LHC workshop, `HEPEVTrepair` utility (C. Biscarat and ZW) being probed in D0.

Design of event structure WITH some grammar requirements AND WITHOUT neglecting possible physics is needed NOW to avoid large problems later.

Main References

- E. Barberio, B. van Eijk and Z. Was, Comput. Phys. Commun. **66**, 115 (1991).
- E. Barberio and Z. Was, Comput. Phys. Commun. **79**, 291 (1994).
- P. Golonka, B. Kersevan, T. Pierzchala, E. Richter-Was, Z. Was and M. Worek, arXiv:hep-ph/0312240, Comput. Phys. Commun.174 (2006) 818.
- P. Golonka, T. Pierzchala and Z. Was, Comput. Phys. Commun. **157** (2004) 39
- Z. Was, Eur. Phys. J. C **44** (2005) 489
- P. Golonka and Z. Was, Eur. Phys. J. C **45** (2006) 97 and arXiv:hep-ph/0508015, arXiv:hep-ph/0604232, EPJC in print.
- G. Nanava, Z. Was, hep-ph/0607019, rewritten version submitted to EPJC

Plan

- Phase-space.
- First order matrix element for $Z \to l\bar{l}$.
- First order matrix element for $B \to K(\pi)K(\pi)$.
- Property of matrix elements used: on mass shell iterative relation.
- Tests at single photon emission level.
- Iteration of matrix element for multiple photon radiation.
- Behaviour in LL and infrared limits.
- Tests at multiple photon level.
- Summary

Phase Space: (trivialities)

Let us recall the element of Lorentz-invariant phase space (*Lips*):

$$
\begin{aligned}
& dLips_{n+1}(P) = \\
& \frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3}\frac{d^3q}{2q^0(2\pi)^3}(2\pi)^4\delta^4\Big(P-\sum_1^n k_i - q\Big) \\
= \; & d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3}\frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3}(2\pi)^4\delta^4\Big(p-\sum_1^n k_i\Big) \\
= \; & d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3}dLips_n(p\to k_1 ... k_n).
\end{aligned}
$$

Integration variables, the four-vector p, compensated with $\delta^4\big(p-\sum_1^n k_i\big)$, and another integration variable M_1 compensated with $\delta\big(p^2-M_1^2\big)$ are introduced.

Phase Space: (cont.)

$$
\begin{aligned}
& dLips_{n+1}(P) = \\
= \quad & dM^2_{1\dots n}\left[d\cos\hat\theta d\hat\phi \frac{1}{8(2\pi)^3}\frac{\lambda^{\frac{1}{2}}(M^2, M^2_{1\dots n}, m^2_{n+1})}{M^2}\right] \times dLips_n(p \to k_1 \dots k_n) \\
= \quad & \left[k_\gamma dk_\gamma d\cos\theta d\phi \frac{1}{2(2\pi)^3}\right] \times dLips_n(p \to k_1 \dots k_n).
\end{aligned}
$$

The expression is used since many decades in phase-space parametrizations and Monte Carlos such as FOWL, TAUOLA SANC. There is a good reason in that; it exhibits those left by energy-momentum conservation, remnants of Lorentz group symmetry. The formula can be also nested.

The expression is only slightly more complicated if instead of photon massive particle is added, such a form is needed for proof of our main formula of next transparency..

In the following we will show that it can be used for matrix elements with soft and collinear singularites for many charged lines.

$\mathcal{M}$ain $\mathcal{F}$ormula of the talk

$$dLips_{n+1}(P \to k_1...k_n, k_{n+1}) = dLips_n^{+1\ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1\ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$
$$\{k_1, \ldots, k_{n+1}\} = \mathbf{T}\big(k_\gamma, \theta, \phi, \{\bar{k}_1, \ldots, \bar{k}_n\}\big). \quad (1)$$

1. One can verify that this formula lead to exact parametrization.
2. A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent properly.
3. Variables k_γ, θ, ϕ are at first *free of any geometrical interpretation*.
4. PHOTOS application: Take the configurations from n-body phase space.
5. Turn it back into some coordinate variables.
6. construct new kinematical configuration from all variables.
7. **Forget about coordinates used in the step.**

Phase Space: (main formula)

If we choose

$$G_n \ : \ M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \ \rightarrow \ \bar{k}_1 \dots \bar{k}_n \tag{2}$$

and

$$G_{n+1} \ : \ k_\gamma, \theta, \phi, M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \ \rightarrow \ k_1 \dots k_n, k_{n+1} \tag{3}$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \dots, \bar{k}_n)). \tag{4}$$

The ratio of the Jacobians (factors $\lambda^{1/2}$ etc.) form the factor W_n^{n+1}, which in our case is rather simple,

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1\dots n}, M^2_{2\dots n}/M^2_{1\dots n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2\dots n}/M^2)}, \tag{5}$$

- All details depend on definition of G_n.

1. Because we are using this type of parametrization based on Lorentz group, we can express it with the help of consecutive boosts and rotations
2. It is particularly convenient for Monte Calro when we need to build events!
3. For the definition of coordinate system in the P-rest frame the $\hat{x}$ and $\hat{y}$ axes of the laboratory frame boosted to the rest frame of P can be used. The orthogonal right-handed system can be constructed with their help in a standard way.
4. We choose polar angles θ_1 and ϕ_1 defining the orientation of the four momentum $\bar{k}_2$ in the rest frame of P. In that frame $\bar{k}_1$ and $\bar{k}_2$ are back to back[a], see fig. (1).
5. The previous two points would complete the definition of the two-body phase space, if both $\bar{k}_1$ and $\bar{k}_2$ had no measurable spin degrees of freedom visualizing themselves e.g. through correlations of the secondary decay products' momenta. Otherwise we need to know an additional angle ϕ_X to complete the set of Euler angles defining the relative orientation of the axes of the P rest-frame system with the coordinate system used in the rest-frame of $\bar{k}_2$ (and possibly also of $\bar{k}_1$), see fig. (2).

[a] In the case of phase space construction for multi-body decays $\bar{k}_2$ should read as a state representing the sum of all decay products of P but $\bar{k}_1$.

6. If both rest-frames of $\bar{k}_1$ and $\bar{k}_2$ are of interest, their coordinate systems are oriented with respect to P with the help of θ_1, ϕ_1, ϕ_X. We assume that the coordinate systems of $\bar{k}_1$ and $\bar{k}_2$ are connected by a boost along the $\bar{k}_2$ direction, and in fact share axes: $z' \uparrow\downarrow z''$, $x' \uparrow\uparrow x''$, $y' \uparrow\downarrow y''$.

7. For the three-body phase space: We take the photon energy k_γ in P rest frame. We calculate: photon, k_1 and k_2 energies, all in $k_1 + k_2$ frame.

8. We use the angles θ, ϕ, in the rest-frame of the $k_1 + k_2$ pair: angle θ is an angle between the photon and k_1 direction (i.e. $-z''$). Angle ϕ defines the photon azimuthal angle around z'', with respect to x'' axis (of the k_2 rest-frame), see fig. (3).

9. If all k_1, k_2 and $k_1 + k_2$ rest-frames exist, then the x-axes for the three frames are chosen to coincide. It is OK, all frmes connected by boosts along z'' see fig. (3).

10. To define orientation of k_2 in P rest-frame coordinate system, and to complete construction of the whole event, we will re-use Euler angles of $\bar{k}_2$: ϕ_X, θ_1 and ϕ_1 (see figs. 4 and 5), defined again of course in the rest frame of P.

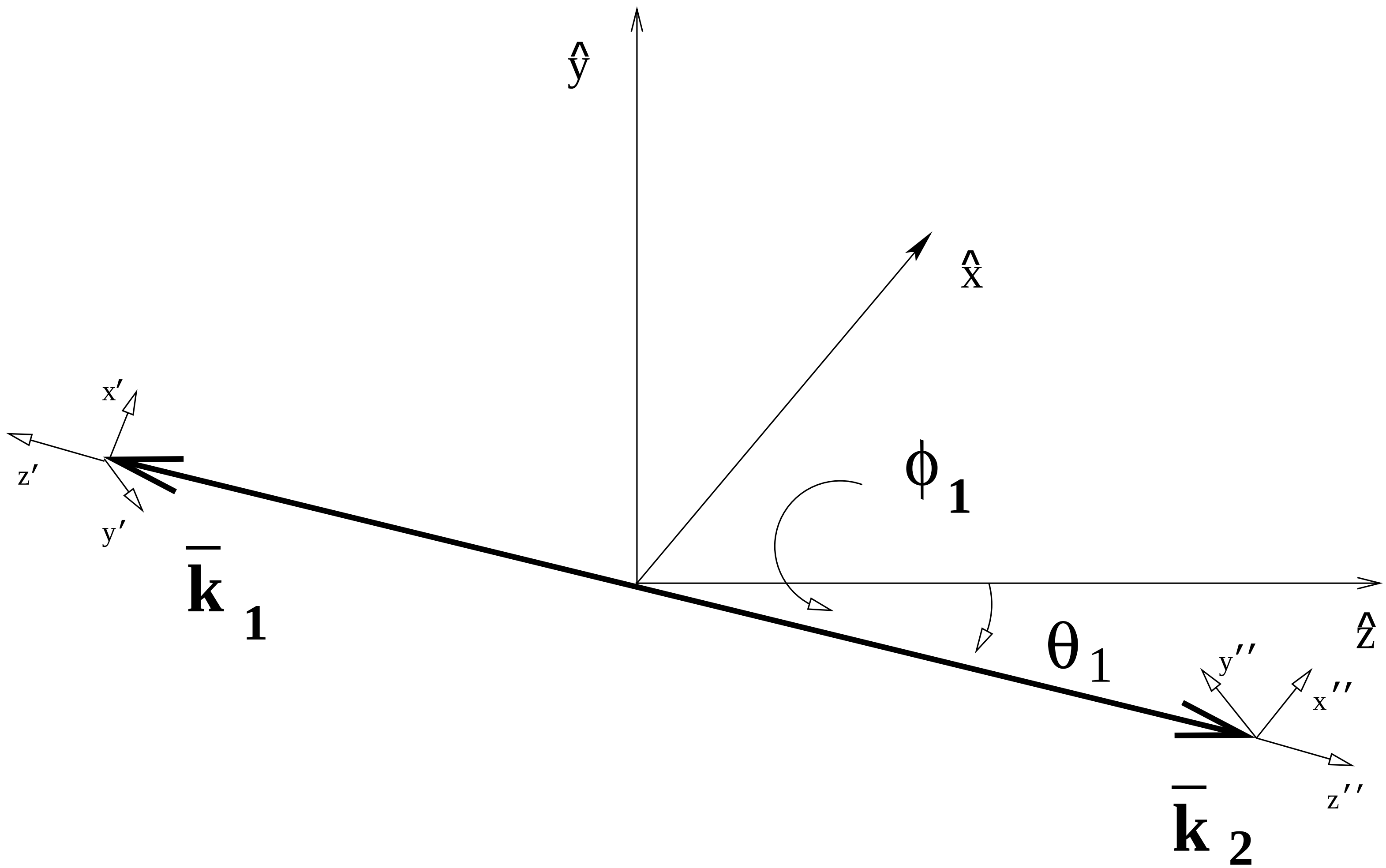


Figure 1: The angles θ_1, ϕ_1 defined in the rest-frame of P and used in parametrization of two-body phase-space.

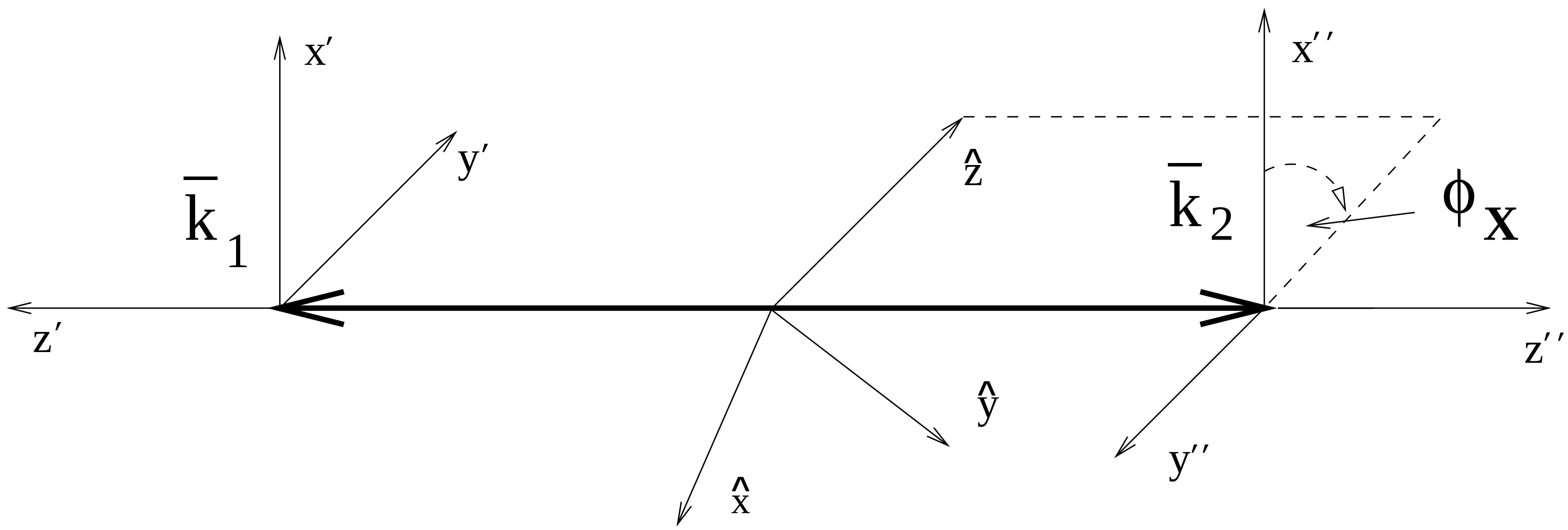


Figure 2: Angle ϕ_X is also defined in the rest-frame of P as an angle between (oriented) planes spanned on: (i) $\bar{k}_1$ and $\hat{z}$-axis of the P rest-frame system, and (ii) $\bar{k}_1$ and x''-axis of the $\bar{k}_2$ rest frame. It completes definition of the phase-space variables if internal orientation of $\bar{k}_1$ system is of interest. In fact, Euler angle ϕ_X is inherited from unspecified in details, parametrization of phase space used to describe possible future decay of $\bar{k}_2$ (or $\bar{k}_1$).

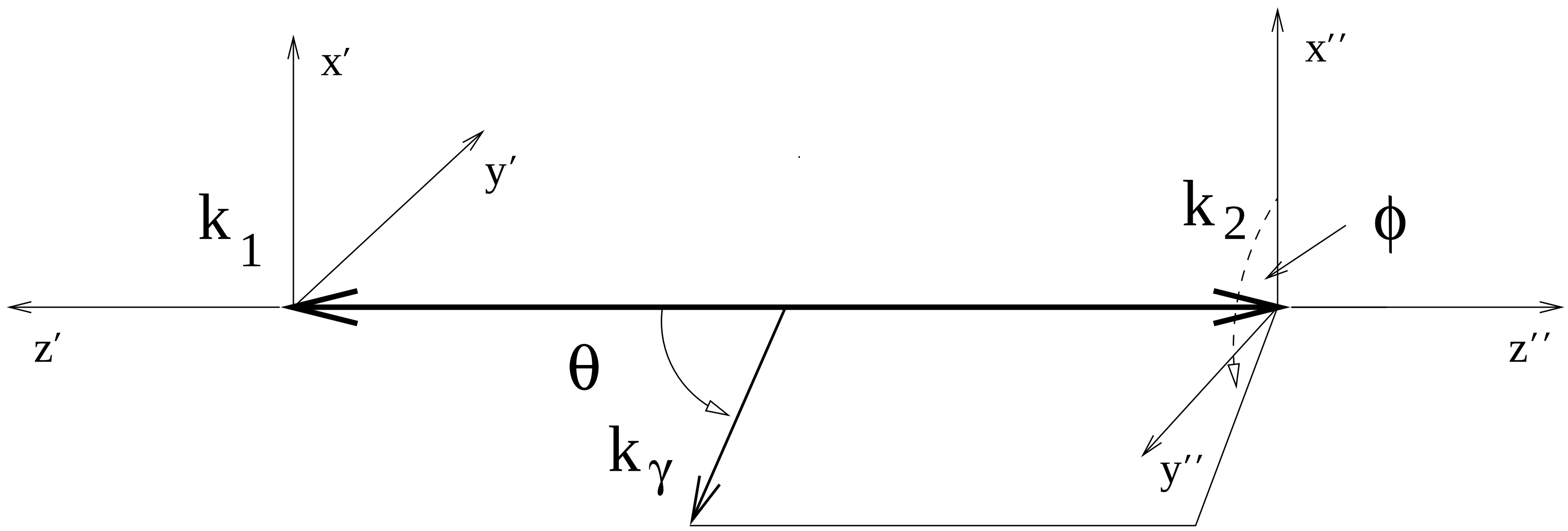


Figure 3: The angles θ, ϕ are used to construct the four-momentum of k_γ in the rest-frame of $k_1 + k_2$ pair (itself not yet oriented with respect to P rest-frame). To calculate energies of k_1, k_2 and photon, it is enough to know m_1, m_2, M and photon energy k_γ of the P rest-frame.

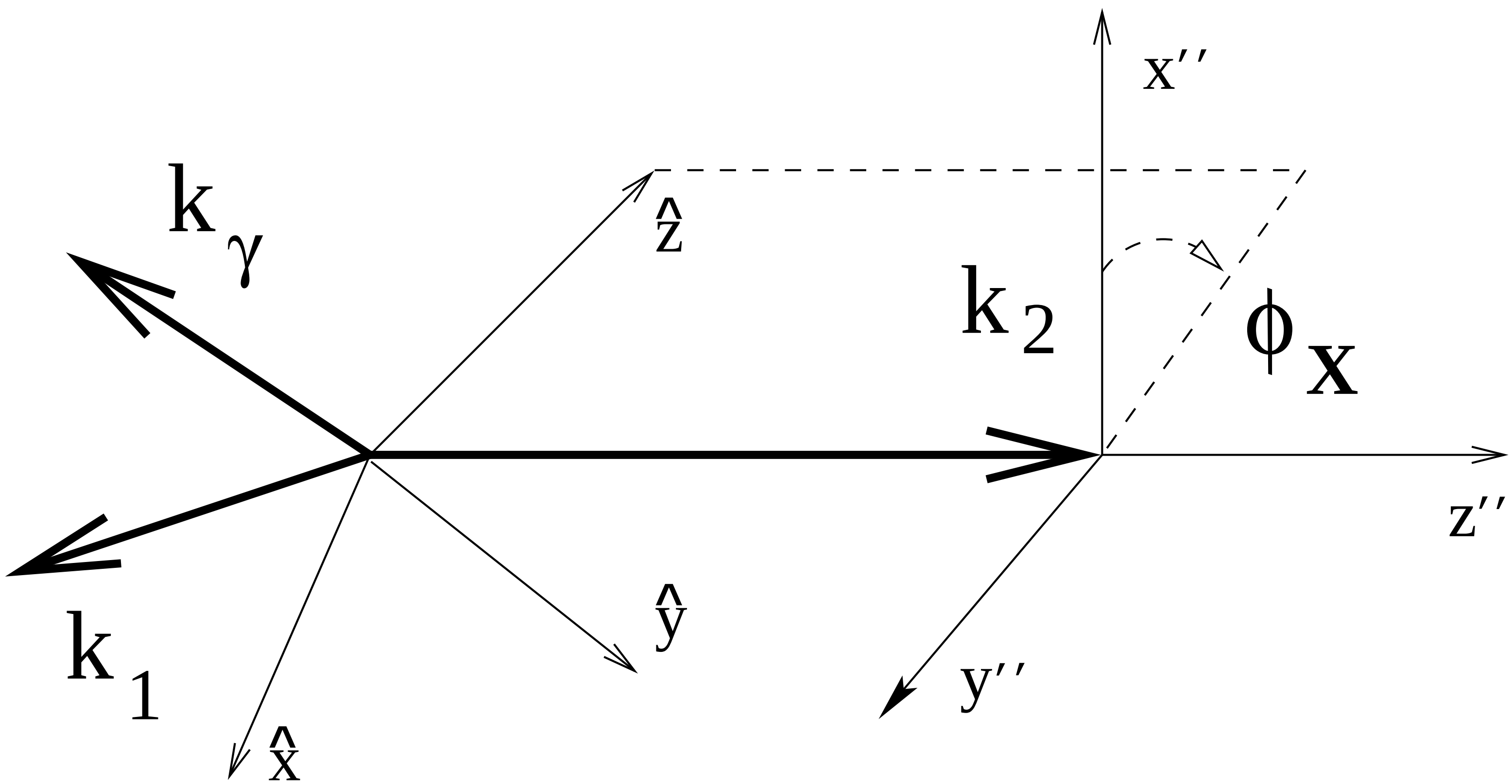


Figure 4: Use of angle ϕ_x in defining orientation of k_1, k_2 and photon in the rest-frame of P. At this step only the plane spanned on P frame axis $\hat{z}$ and k_2 is oriented with respect to $k_2 \times x''$ plane.

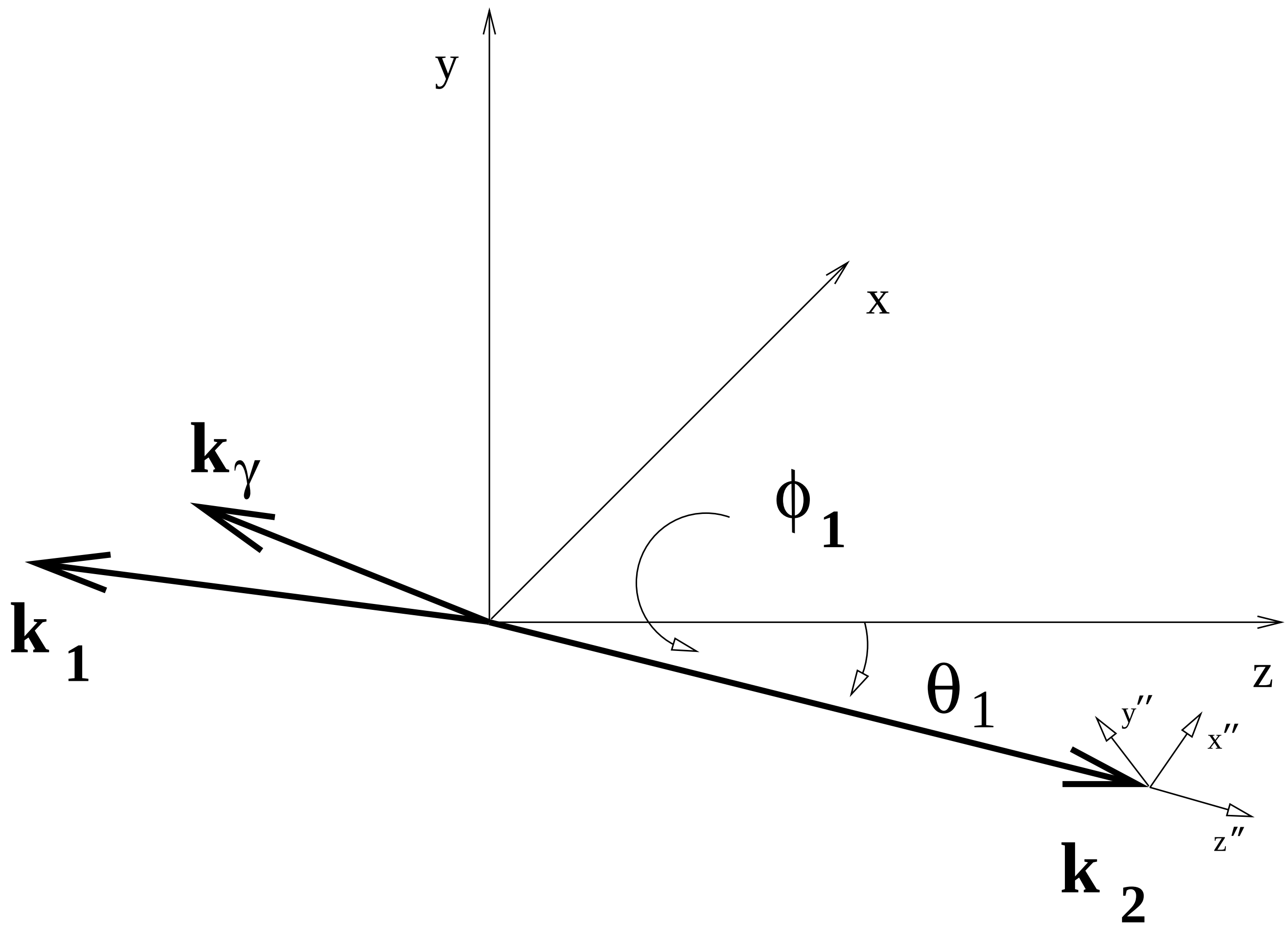


Figure 5: Final step in event construction. Angles θ_1, ϕ_1 are used. The final orientation of k_2 coincide with this of $\bar{k}_2$.

Phase Space: (multiple photon radiation)

By iteration, we can generalize formula (1) to the case of l photons and we write:

$$dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) = \frac{1}{l!} \prod_{i=1}^{l} \Big[dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W_{n+i-1}^{n+i} \Big]$$
$$\times dLips_n(P \to \bar{k}_1...\bar{k}_n), \tag{6}$$
$$\{k_1, \dots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\dots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \dots, \bar{k}_n\}\big) \dots\big).$$

Note that variables $k_{\gamma_m}, \theta_{\gamma_m}, \phi_{\gamma_m}$ are used at a time of the $m-$th step of iteration only, and are not needed elsewhere in construction of the physical phase space; the same is true for invariants and angles $M^2_{2\dots n}, \theta_1, \phi_1, \dots, \theta_{n-1}, \phi_{n-1} \ \to \ \bar{k}_1 \dots \bar{k}_n$ of (2,3), which are also redefined at each step of the iteration. Also intermediate steps require explicit construction of temporary $\bar{k}'_1 \dots \bar{k}'_n \dots \bar{k}'_{n+m}$

This construction gives **exact distribution** of weighted events over $n+l$ body phase space.

Crude Ddistribution

If we add factors $f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i})$ and F to

$$dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) = \exp(-F)\frac{1}{l!}\prod_{i=1}^{l}$$

$$\left[f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W^{n+i}_{n+i-1} \right] \times dLips_n(P \to \bar{k}_1...\bar{k}_n), \qquad (7)$$

$$\{k_1, \ldots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\ldots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \ldots, \bar{k}_n\}\big) \ldots\big),$$

$$F = \int_{k_{min}}^{k_{max}} dk_\gamma d\cos\theta_\gamma d\phi_\gamma f(k_\gamma, \theta_\gamma, \phi_\gamma).$$

- and if we will take only Green parts, we will get crude distribution over tangent space. The k_{min}, k_{max} must be sufficently small/large, but otherwise are arbitrary.

- True matrix element is still missing of course.

Phase Space: (what if more than one charged particle?)

1. I should discuss now matrix elements. Without, the following points look like irrelevant/arbitrary spurious complications, but nonetheless correct:

a) The parametrizations of the two-body and three-body phase-space (photon included) are used for the explicit kinematical construction denoted by formula (1). We can replace the roles played by k_1 and k_2. This simple operation leads to a new phase-space parametrization, which can be used in a second branch of the Monte Carlo generation.

b) The phase-space Jacobians (factor W_n^{n+1} of (1)) are identical for the two branches; this factor is also never larger than $k_\gamma \frac{1}{2(2\pi)^3}$.

c) Angle θ of the first branch coincides with $\pi - \theta$ of the second one.

d) In the soft ($k_\gamma \to 0$) and collinear ($\theta \to 0$ or π) limits, angles θ_1, ϕ_1, ϕ_X of the two branches converge to each other (in these limits they may differ by π or 2π).

e) Properties (c) and (d) are convenient for our construction of the weights given by formula (9), because they coincide with the similar properties of the exact matrix element.

f) Thanks to (b), the first version of (9) is exact. In fact, it is more suitable for multi-photon radiation, if first order matrix element is used only. This required comparisons with second order matrix elements. The choice of $\bar{k}_2$ (or k_2) direction to define θ_1, ϕ_1, rather than $\bar{k}_1$, was also motivated by the properties of the decay matrix elements.

2. Property (d) extends to multi-body decays, and to cases of more than two charged particles in the final state. The relation between angles θ_1, ϕ_1, ϕ_X of the distinct branches is more complex, but in the discussed limits still independent from θ and ϕ.

3. Extended property (d) and (e) enable the use of (9) for multi-photon radiation; this also holds in the case when more than two charged particles are present in the final state.

4. That is why, in the case of two-body decays (plus bremsstrahlung photons), such type of phase-space treatment is sufficient for the NLO precision.

5. For the NNLO precision, in matching of the two mappings for the collinear singularities[a] another factor of the type $\lambda^{1/2}(\ldots)/\lambda^{1/2}(\ldots)$ would have to be included in W_n^{n+1} of formulas (1,9). In fact in such a case the exact multi-photon phase space parametrization would be preserved.

[a] Such matching is necessary for the two branches of the generation, used to presample collinear singularities along the directions of k_1 and k_2, to be used simultaneously in construction of each event.

6. For each additional charged decay product present in the final state, still another factor of the type $\lambda^{1/2}(...)/\lambda^{1/2}(...)$ is needed in W_n^{n+1} to assure multichannel generation with the exact treatment of the phase space.

7. Even without refinements (of the previous two points) our phase space parametrization is sufficient for NLO and NLL precision for the two-body (two-charges) decays, accompanied with arbitrary number of photons. In a general case, when more than two charged particles are present in final state, such phase space parametrization remains sufficient for LL only, also then the full multi-photon phase space is covered.

8. In our choice of phase space parametrization (point 1), we have dropped some details, the choice of $\hat{x},\ \hat{y},\ \hat{z}$ axes of the P rest-frame were not specified. Indeed, for the decay of a scalar object, such as that discussed in the present paper, every choice is equivalent. In general, it is not the case. Already in case of the Z boson decay, the choice of the $\hat{z}$ axis parallel to the direction of the incoming beam of the same charge as k_2 is advantageous, where the process $e^+e^- \to Z \to l^+l^- n(\gamma)$ was studied. In this case the direction of the incoming beam coincides with the spin state of Z, and the choice simplify expression for matrix element.

$\mathcal{F}$irst order first

- Historically PHOTOS was developped as a first order Monte Carlo for single photon emission in decays
- For that purpose first order exact matrix element of $Z \to \mu^+\mu^-$ decay was used.
- It was downgraded so it could be used in decay of any particle or resonance.
- In cases the benchmarks became available program worked good, also it helped to improve overall agreement between data and Monte Carlos of such experiments as Belle, Cleo BaBar

- The fully differential distribution from MUSTRAAL (used also in KORALZ for single photon mode) reads:

$$X_f = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \left\{ \frac{1}{(k'_+ k'_-)} \left[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \right] \right\}$$

- Here:

$$\begin{aligned} s &= 2p_+ \cdot p_-, \quad & s' &= 2q_+ \cdot q_-, \\ t &= 2p_+ \cdot q_+, \quad & t' &= 2p_+ \cdot q_-, \\ u &= 2p_+ \cdot q_-, \quad & u' &= 2_- \cdot q_+, \\ k'_\pm &= q_\pm \cdot k, \quad & x_k &= 2E_\gamma/\sqrt{s} \end{aligned}$$

- The Δ term is responsable for final state mass dependent terms, p_+, p_-, q_+, q_-, k denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

- after trivial manipulation it can be written as:

$$X_f = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \Bigg\{ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_-} \left[\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right]$$

$$+ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_+} \left[\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right] \Bigg\}$$

- In PHOTOS the following expression is used in universal application (AP adj.):

$$X_f^{PHOTOS} = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \Bigg\{$$

$$\frac{1}{k'_+ + k'_-} \frac{1}{k'_-} \quad \left[(1 + (1-x_k)^2) \frac{\mathrm{d}\sigma_B}{d\Omega} \Big(s, \frac{s(1-\cos\Theta_+)}{2}, \frac{s(1+\cos\Theta_+)}{2} \Big) \right] \frac{(1+\beta\cos\Theta_\gamma)}{2}$$

$$+ \frac{1}{k'_+ + k'_-} \frac{1}{k'_+} \quad \left[(1 + (1-x_k)^2) \frac{\mathrm{d}\sigma_B}{d\Omega} \Big(s, \frac{s(1-\cos\Theta_-)}{2}, \frac{s(1+\cos\Theta_-)}{2} \Big) \right] \frac{(1-\beta\cos\Theta_\gamma)}{2} \Bigg\}$$

$$\text{where}: \Theta_+ = \angle(p_+, q_+), \ \Theta_- = \angle(p_-, q_-)$$

$\Theta_\gamma = \angle(\gamma, \mu^-)$ are defined in (μ^+, μ^-)-pair rest frame

- also factor $\Gamma^{total}/\Gamma^{Born} = 1 + 3/4\alpha/\pi$ defines first order weight.

The differences are important

- The two expressions define weight to make out of PHOTOS complete first order.
- The PHOTOS expression separates (i) Final state bremsstrahlung (ii) electroweak parameters of the Born Cross section (iii) Initial state bremsstrahlung that is orientation of the spin quantization axix for Z.
- That would be heavy burden for managing PHOTOS interfaces. I know, because we encounter such difficulties for universal interface for TAUOLA.
- It is possible but extremenly inconvenient. Parts of generation managed by distinct authors.
- Of course all this has to be understood in context of Leading Pole approximaition. For example initial-final state interference breaks the simplification. Limitations need to be controlled: Phys. Lett. B219:103,1989.

Scalar QED for matrix elements in B decays

- Scalar QED is not an ultimate theory in the case of decays like $B^- \to \pi^0 K^-$ or $B^0 \to \pi^+ K^-$
- Nonetheless matrix elements can be calculated and provie good input for tests.
- Massive final states, $m_\pi/m_B \neq m_K/m_B \simeq 0.1$.
- Scalar particles.
- In fact much simpler matrix element than in case of Z decay.
- The one-loop QED correction to the decay width can be represented as the sum of the Born contribution with the contributions due to virtual loop diagrams and soft and hard photon emissions.

$$d\Gamma^{\text{Total}} = d\Gamma^{\text{Born}} \left\{ 1 + \frac{\alpha}{\pi} \left[\delta^{\text{Soft}}(m_\gamma, \omega) + \delta^{\text{Virt}}(m_\gamma, \mu_{UV}) \right] \right\} + d\Gamma^{\text{Hard}}(\omega)$$

- where for **Neutral meson decay channels** we have:
  – Virtual photon contribution

$$\delta^{\mathrm{Virt}}(m_\gamma,\mu_{UV}) = \left[1+\frac{M^2-m_1^2-m_2^2}{\Lambda}\ln\frac{2m_1m_2}{M^2-m_1^2-m_2^2+\Lambda}\right]\ln\frac{M^2}{{m_\gamma}^2}+\frac{3}{2}\ln\frac{\mu_{UV}^2}{M^2}$$
$$+\frac{M^2-m_1^2-m_2^2}{2\Lambda}\left[\mathrm{Li}_2\left(\frac{M^2+m_1^2-m_2^2+\Lambda}{2\Lambda}\right)-\mathrm{Li}_2\left(\frac{-M^2+m_2^2-m_1^2+\Lambda}{2\Lambda}\right)\right.$$
$$\left.+2\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\ln\frac{m_1\Lambda}{M^3}+(1\leftrightarrow 2)+\pi^2\right]$$
$$-\frac{\Lambda}{2M^2}\ln\frac{2m_1m_2}{M^2-m_1^2-m_2^2+\Lambda}+\frac{m_2^2-m_1^2}{4M^2}\ln\frac{m_2^2}{m_1^2}-\frac{1}{2}\ln\frac{m_1m_2}{M^2}+1$$

  – Soft photon contribution

$$\delta^{\mathrm{Soft}}(m_\gamma,\omega) = \left[1+\frac{M^2-m_1^2-m_2^2}{\Lambda}\ln\frac{2m_1m_2}{M^2-m_1^2-m_2^2+\Lambda}\right]\ln\frac{{m_\gamma}^2}{4\omega^2}$$
$$+\frac{M^2-m_1^2-m_2^2}{2\Lambda}\left[\mathrm{Li}_2\left(\frac{-2\Lambda}{M^2+m_1^2-m_2^2-\Lambda}\right)-\mathrm{Li}_2\left(\frac{2\Lambda}{M^2+m_1^2-m_2^2+\Lambda}\right)+(1\leftrightarrow 2)\right]$$
$$-\frac{M^2+m_1^2-m_2^2}{\Lambda}\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}-(1\leftrightarrow 2)$$

– Hard photon contribution

$$d\Gamma^{\,\mathrm{Hard}} = |A^{\mathrm{Born}}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_2 \frac{k_2.\epsilon}{k_2.k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

- $\Lambda = \lambda^{1/2}(M^2, m_1^2, m_2^2)$
- The infrared divergency, is regularized by m_γ, it cancels in the sum of virtul and soft contributions
- The virtual correction depends on ultraviolet scale μ_{UV}
- The total width is free of ω and of the final meson mass singularity (KLN theorem), we will choose the scale to make an overall correction of order of zero.
- for **Charged meson decay channels** we have:

– Virtual photon contribution

$$\delta^{virt}(m_\gamma,\mu_{UV}) = \left[1+\frac{M^2+m_1^2-m_2^2}{\Lambda}\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\right]\ln\frac{Mm_1}{m_\gamma{}^2}+\frac{3}{2}\ln\frac{\mu_{UV}^2}{Mm_1}$$
$$+\frac{M^2+m_1^2-m_2^2}{2\Lambda}\left[\mathrm{Li}_2\left(\frac{M^2-m_1^2-m_2^2+\Lambda}{2\Lambda}\right)-\mathrm{Li}_2\left(\frac{M^2-m_1^2-m_2^2-\Lambda}{-2\Lambda}\right)\right.$$
$$+\mathrm{Li}_2\left(\frac{M^2+m_2^2-m_1^2-\Lambda}{-2\Lambda}\right)-\mathrm{Li}_2\left(\frac{M^2+m_2^2-m_1^2+\Lambda}{2\Lambda}\right)$$
$$\left.+2\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\ln\frac{\Lambda}{Mm_2}-\ln\frac{2Mm_2}{M^2+m_2^2-m_1^2+\Lambda}\ln\frac{M^2}{m_1^2}\right]$$
$$+\frac{\Lambda}{2m_2^2}\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}-\frac{M^2-m_1^2}{4m_2^2}\ln\frac{m_1^2}{M^2}+1;$$

– Soft photon contribution

$$\delta^{soft}(m_\gamma,\omega) = \left[1+\frac{M^2+m_1^2-m_2^2}{\Lambda}\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\right]\ln\frac{m_\gamma{}^2}{4\omega^2}$$
$$+\frac{M^2+m_1^2-m_2^2}{2\Lambda}\left[\mathrm{Li}_2\left(\frac{-2\Lambda}{M^2+m_1^2-m_2^2-\Lambda}\right)-\mathrm{Li}_2\left(\frac{2\Lambda}{M^2+m_1^2-m_2^2+\Lambda}\right)\right]$$
$$-\frac{M^2+m_1^2-m_2^2}{2\Lambda}\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}$$

– Hard photon contribution

$$d\Gamma^{\rm Hard} \quad = \quad |A^{\rm Born}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q \frac{P.\epsilon}{P.k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

- Once matrix element is clearly defined. It can be used instead of universal one..
- **Gate for shape-factors from fits to data is open!**
- **It is essential that in all cases matrix element can be interpreted as transformation from Born to bremsstrahlung amplitude**
- encouraging observation, but first order and just two processes only.
- **Transformation can be applied to configurations with some photons already present.**
- Pure mechanical observation must be verified!

The weight for complete ME in Z decay

$$wt_i \quad = \quad \left.\frac{X_f}{X_f^{PHOTOS}}\right|_i \; \frac{\Gamma^{\rm Born}}{\Gamma^{\rm Total}} \qquad (8)$$

- The $i = +, -$ denote parts: terms proportional to $\frac{1}{k'_+}$ or $\frac{1}{k'_-}$ should be taken separately.

- Completely independent branches of generation for emissions from μ^+ and μ^-.

- **Virtual corrections** are different in tangent and real space, long formulas but enter with the factor $\frac{\Gamma^{\rm Born}}{\Gamma^{\rm Total}}$ only. Rejection must be performed $\longrightarrow$ sum rules.

- Sum rule for FSR is easy, but not essential in construction.

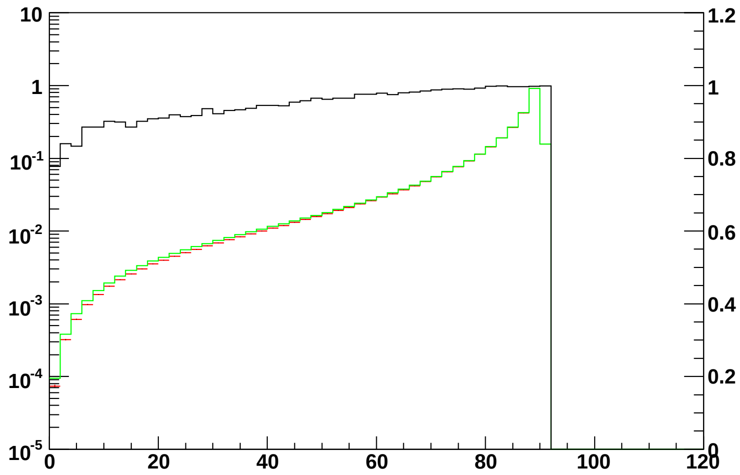
The weight for complete ME in B decay

$$
\begin{aligned}
wt &= \sum_{i=1,2} \frac{|\mathcal{M}|^2_{exact}}{|\mathcal{M}|^2_{PHOTOS}}\Big|_i \; \frac{\Gamma^{\mathrm{Born}}}{\Gamma^{\mathrm{Total}}} \; WT^i_{INT} \\
WT^i_{INT} &= \frac{\left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_2 \frac{k_2.\epsilon}{k_2.k_\gamma}\right)^2}{\left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_1 \frac{P.\epsilon}{P.k_\gamma}\right)^2 + \left(q_2 \frac{k_2.\epsilon}{k_2.k_\gamma} - q_2 \frac{P.\epsilon}{P.k_\gamma}\right)^2} \\
WT^i_{INT-option} &= J_i \frac{\left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_2 \frac{k_2.\epsilon}{k_2.k_\gamma}\right)^2}{\left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_1 \frac{P.\epsilon}{P.k_\gamma}\right)^2 J_1 + \left(q_2 \frac{k_2.\epsilon}{k_2.k_\gamma} - q_2 \frac{P.\epsilon}{P.k_\gamma}\right)^2 J_2} \\
J_1 &= \frac{1}{WT_1(P,k_1,k_2,k_\gamma) WT_2(P,k_1,k_2,k_\gamma)} \sim W_n^{n+1} \frac{1-\beta\cos\theta_1}{k_\gamma} \\
J_2 &= \frac{1}{WT_1(P,k_2,k_1,k_\gamma) WT_2(P,k_2,k_1,k_\gamma)} \sim W_n^{n+1} \frac{1+\beta\cos\theta_1}{k_\gamma}
\end{aligned}
\tag{9}
$$

- Options diifer only if multiphoton radiation is on !

First order first

- How it works in practice ?
- To answer, we have sliced PHOTOS (first order) into parts resonsable for Phase-space and matrix element
- We will show results when universal or exact matrix elements for some channel is used
- We can do that for $Z \to l\bar{l}$ (matrix element of QED was used) and $B^0 \to \pi^+ K^-$ and $B^+ \to \pi^0 K^+$ (scalar QED) only.
- This channels are also of importance for phenomenology.
- This is not a proof that universal solution will work as good in every decay.



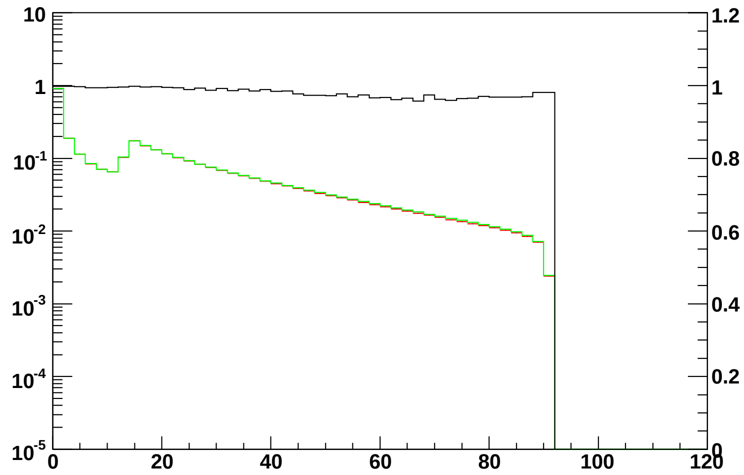


Figure 1: Comparison of standard PHOTOS and KORALZ for single photon emission. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00534. In the right frame the invariant mass of $\mu^-\gamma$; SDP=0.00296. The histograms produced by the two programs (logarithmic scale) and their ratio (linear scale, black line) are plotted in both frames. The fraction of events with hard photon was 17.4863 $\pm$ 0.0042% for KORALZ and 17.6378 $\pm$ 0.0042% for PHOTOS.

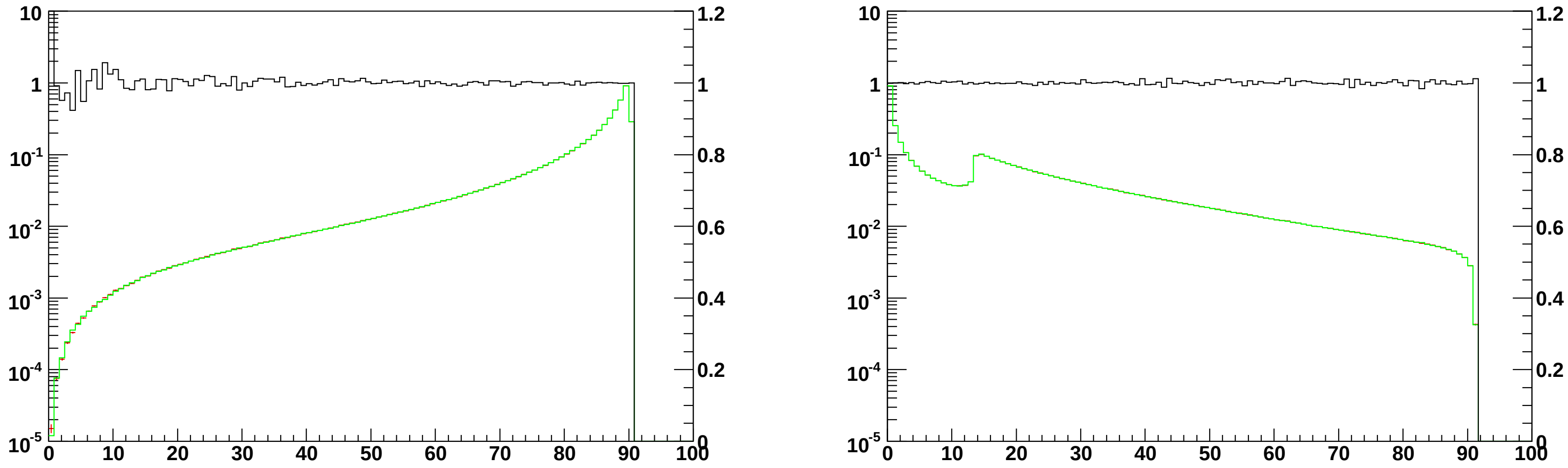
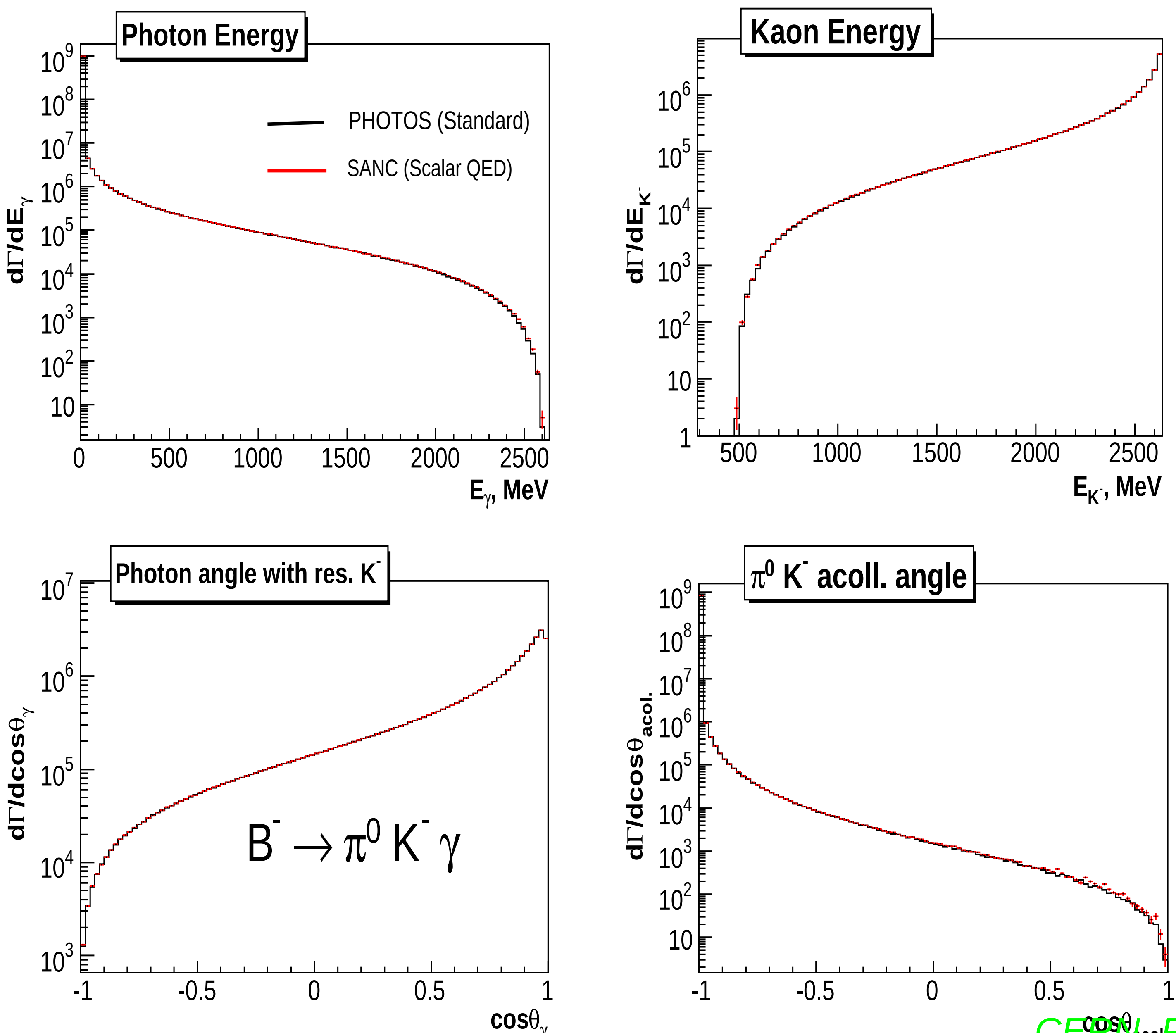
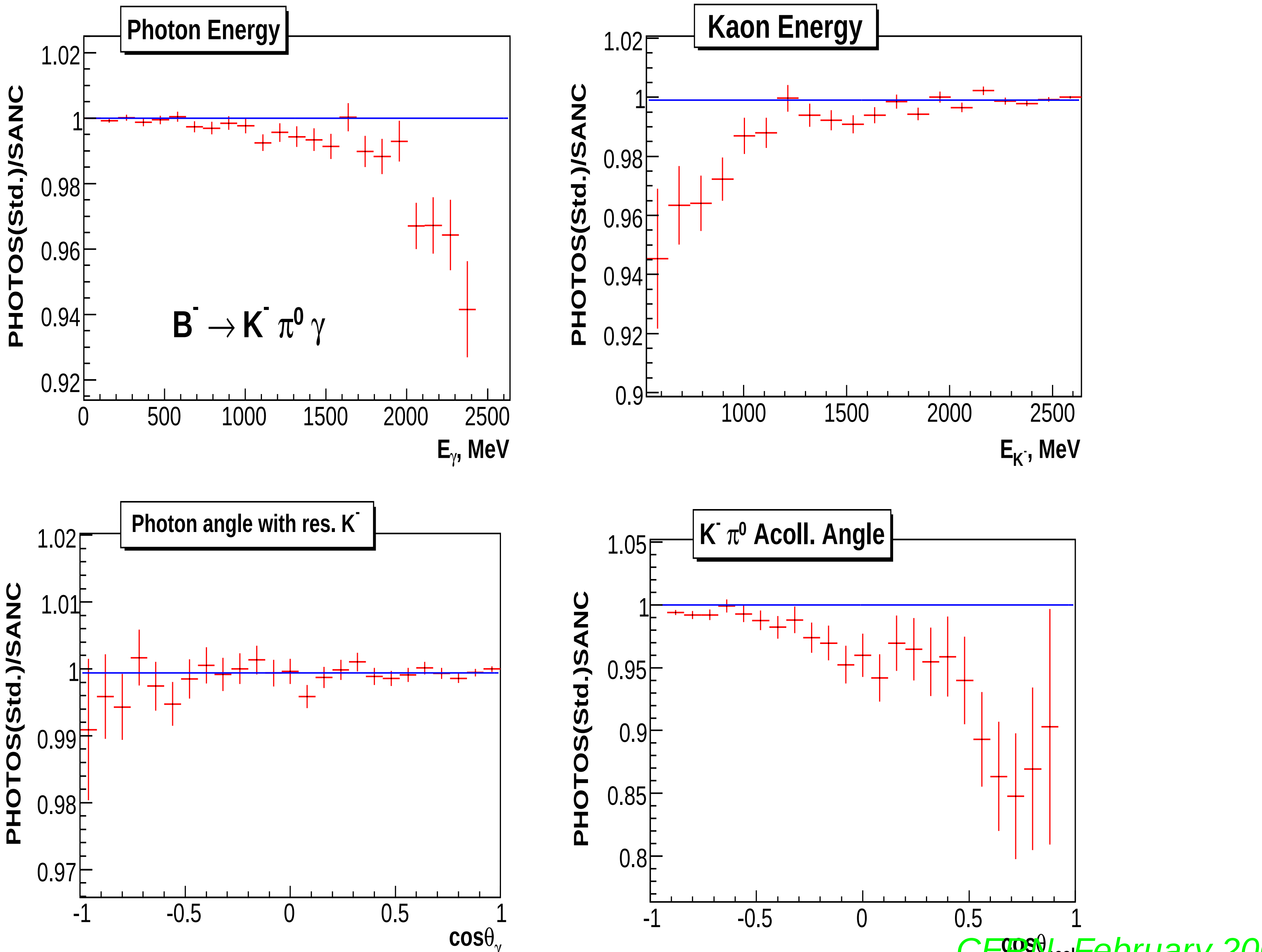


Figure 2: Comparisons of improved PHOTOS and KORALZ for single photon emission. In the left frame the invariant mass of the $\mu^+\mu^-$ pair. In the right frame the invariant mass of $\mu^-\gamma$ pair is shown. In both cases differences between PHOTOS and KORALZ are below statistical error. The fraction of events with hard photon was $17.4890 \pm 0.0042\%$ for KORALZ and $17.4926 \pm 0.0042\%$ for PHOTOS.

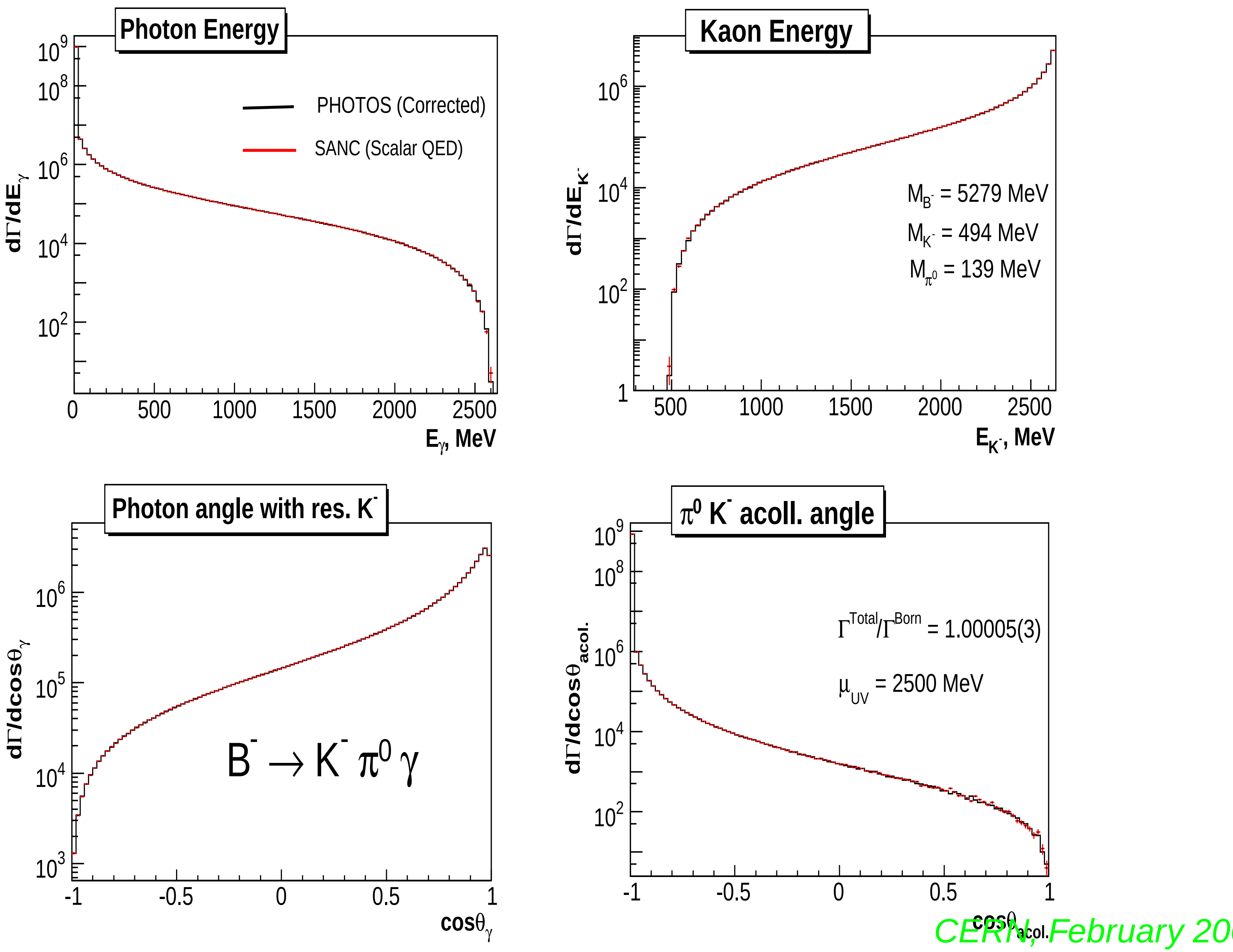
$B^- \to \pi^0 K^-$: *standard PHOTOS* looks good, but ...



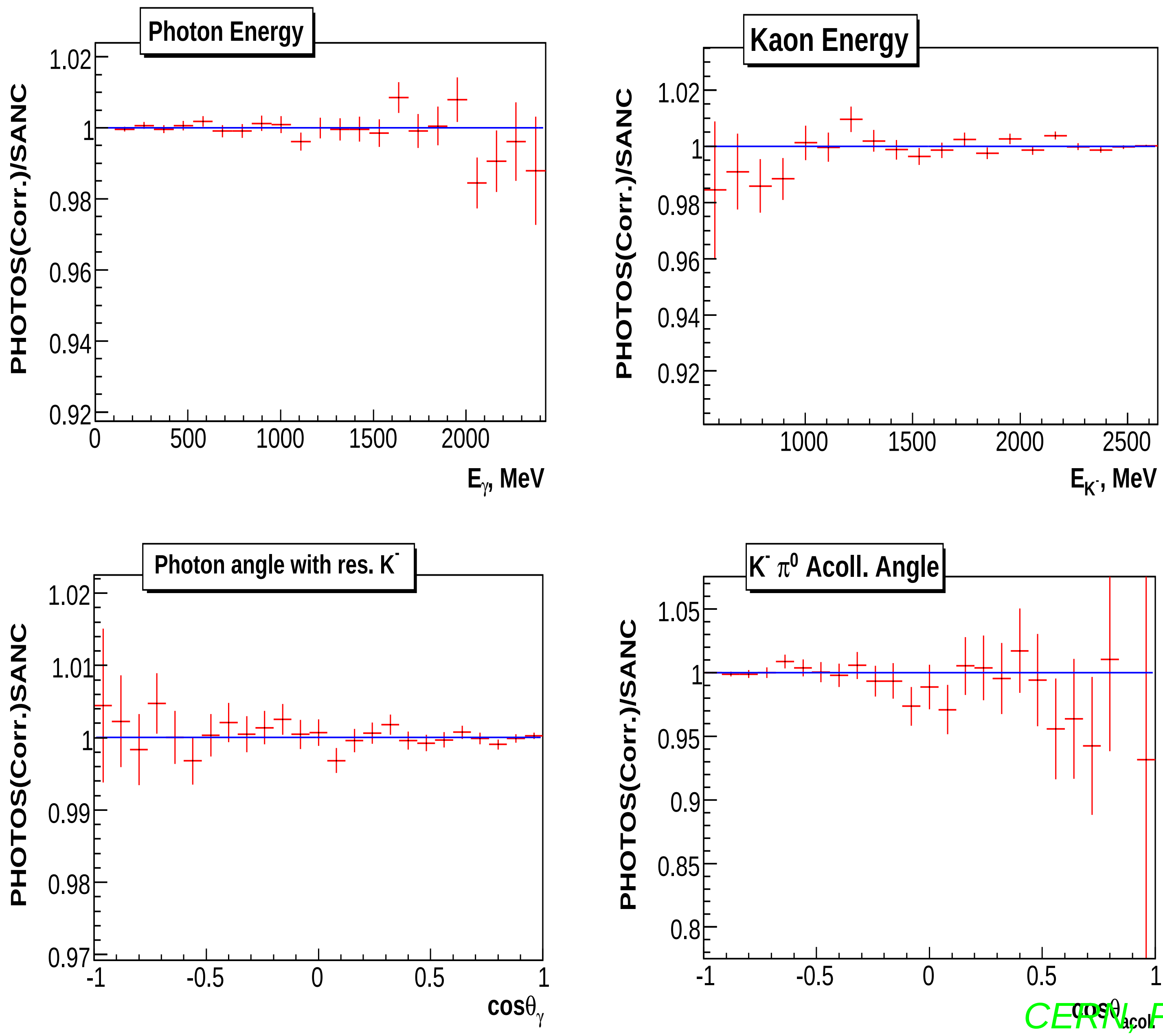
$B^- \to \pi^0 K^-$: standard PHOTOS not perfect



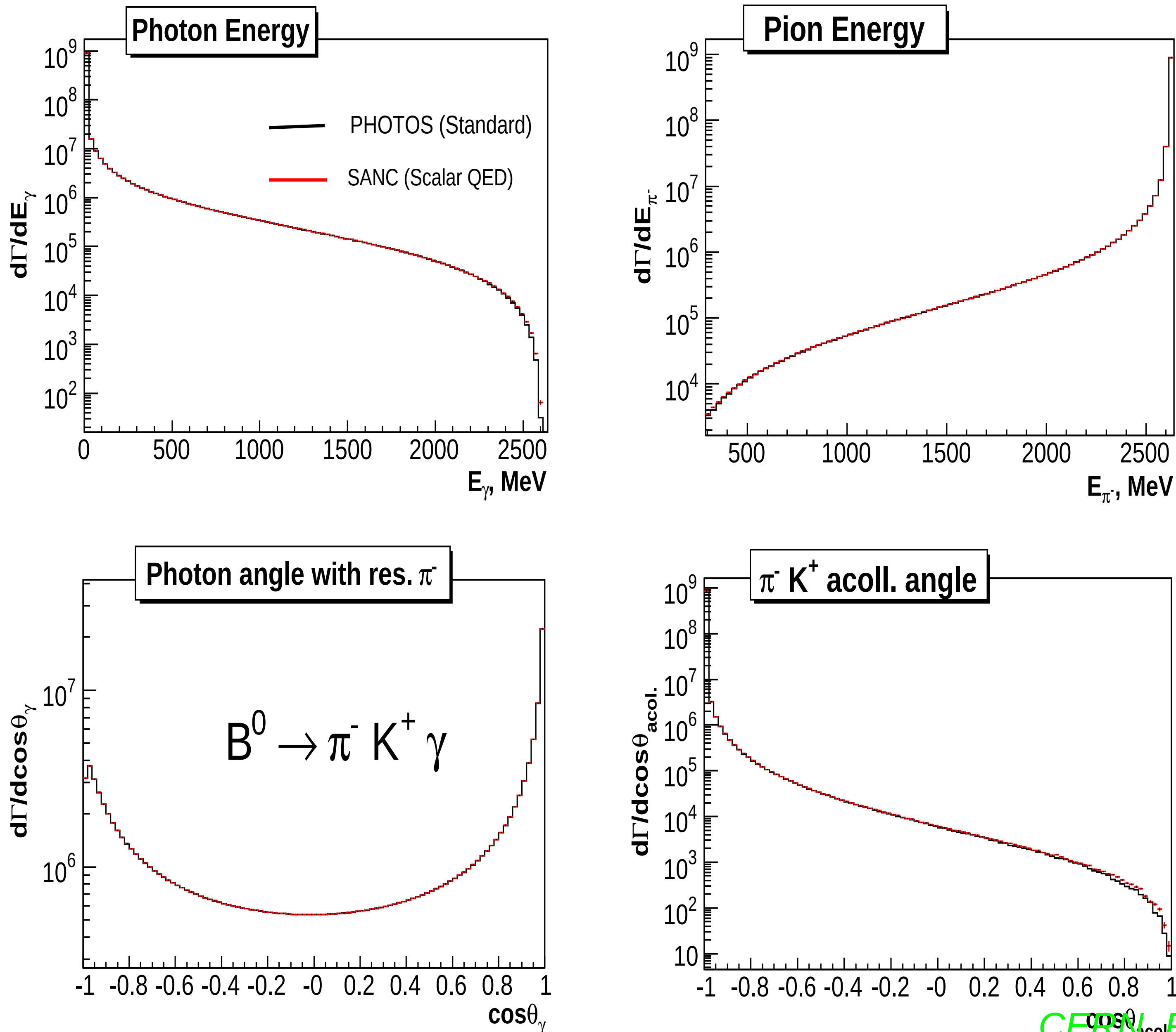
$B^- \to \pi^0 K^-$: NLO improved PHOTOS Looks good



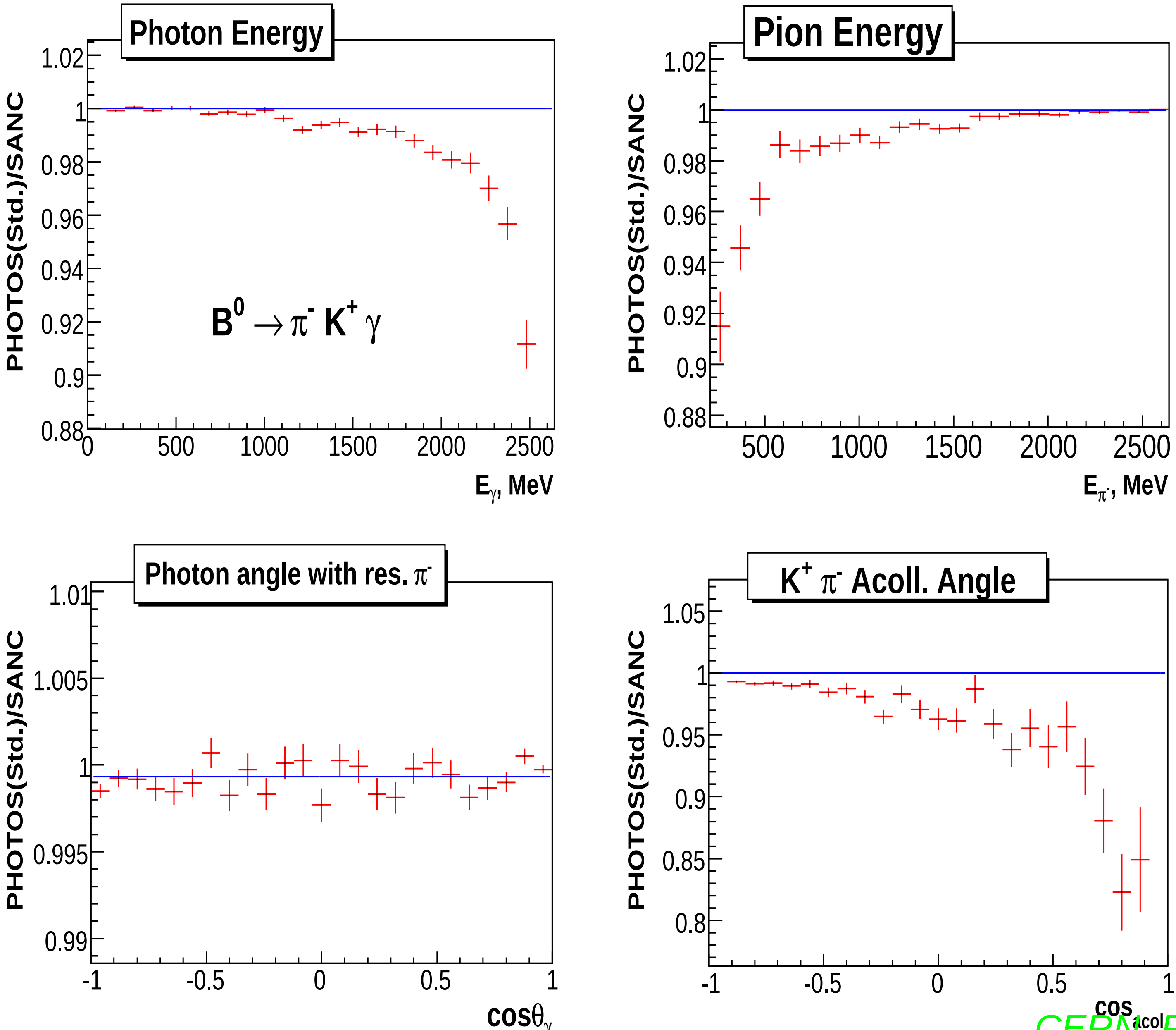
$B^- \to \pi^0 K^-$: NLO improved PHOTOS ... and is good.



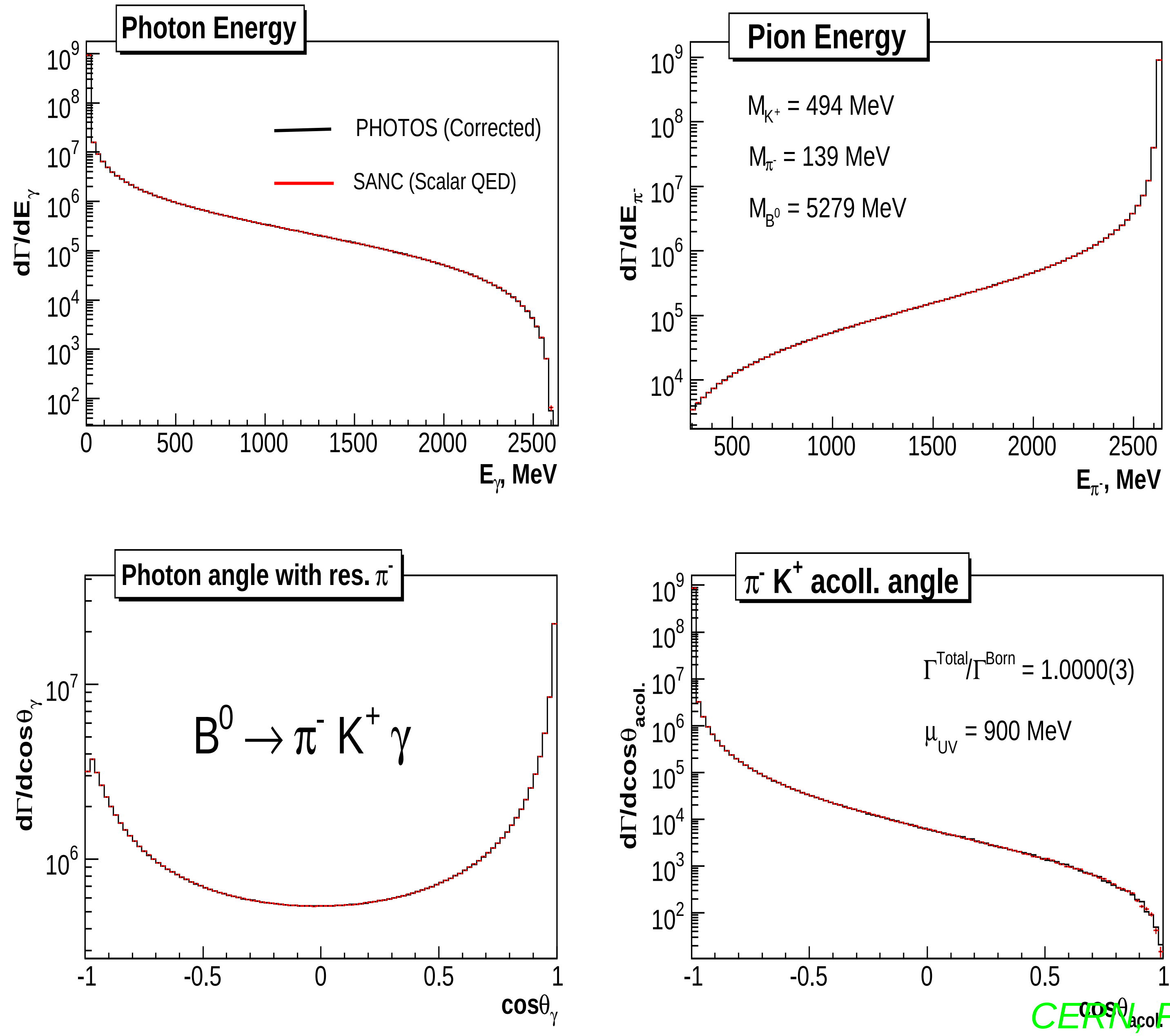
$B^0 \to \pi^- K^+$: standard PHOTOS Looks good ...



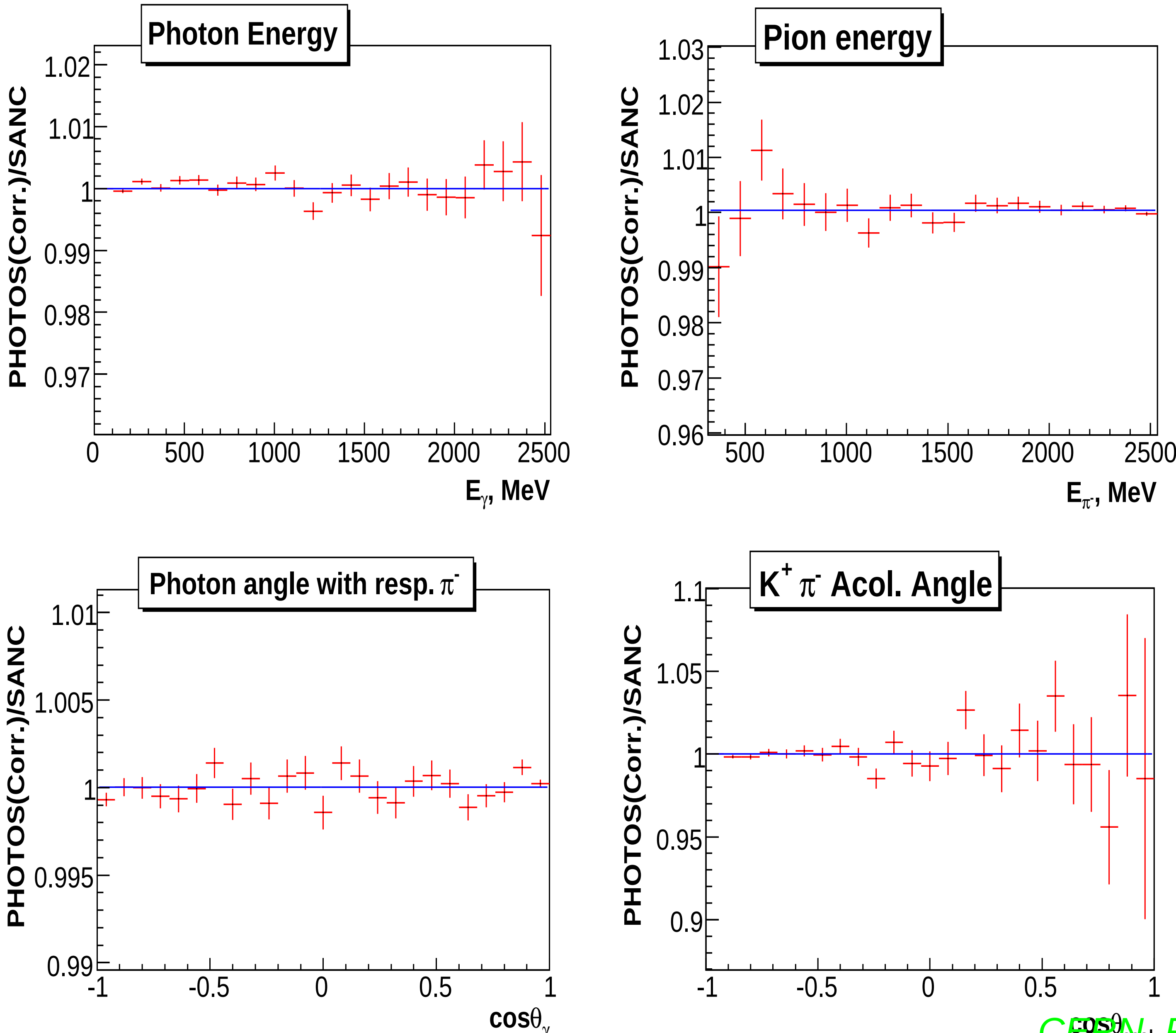
$B^0 \to \pi^- K^+$: *standard PHOTOS* ... but not perfect.



$B^0 \to \pi^- K^+$: NLO improved PHOTOS Looks good ...

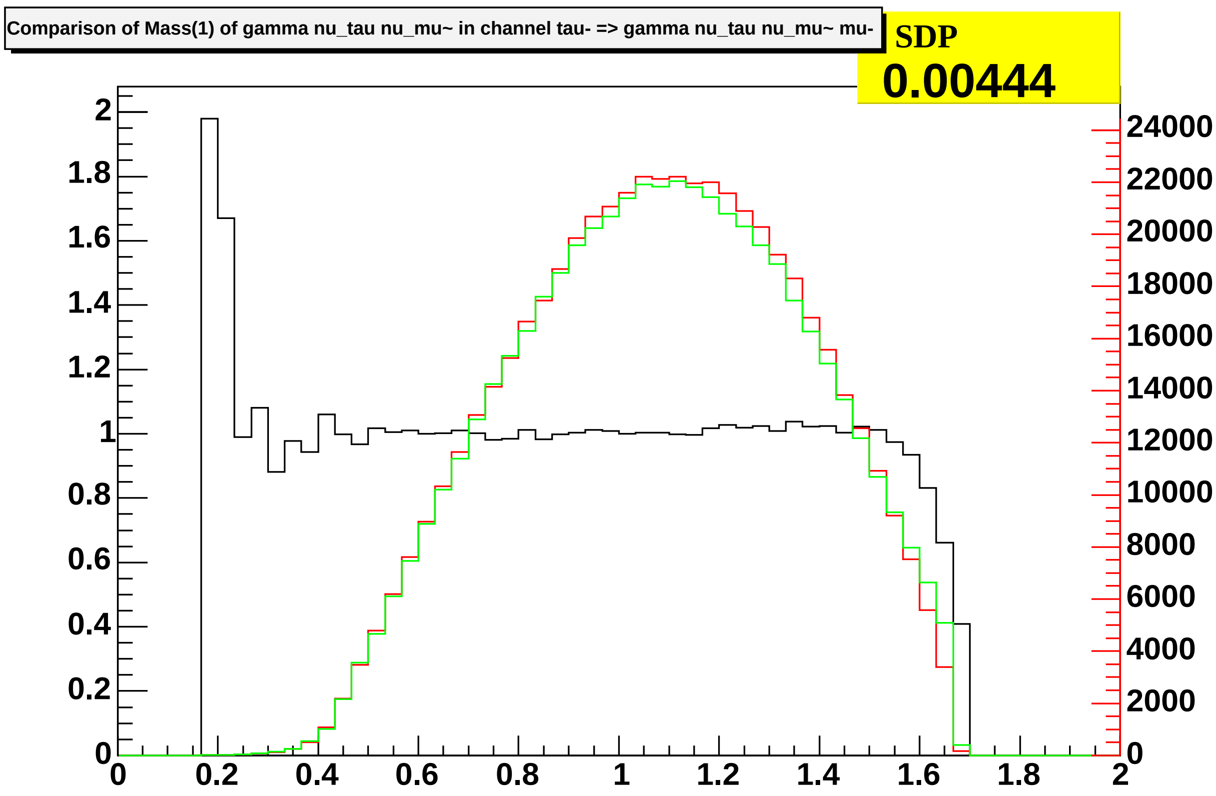


$B^0 \to \pi^- K^+$; *NLO improved PHOTOS* ... also perfect !



$\tau \to l\nu\bar{\nu}(\gamma)$ PHOTOS vs TAUOLA

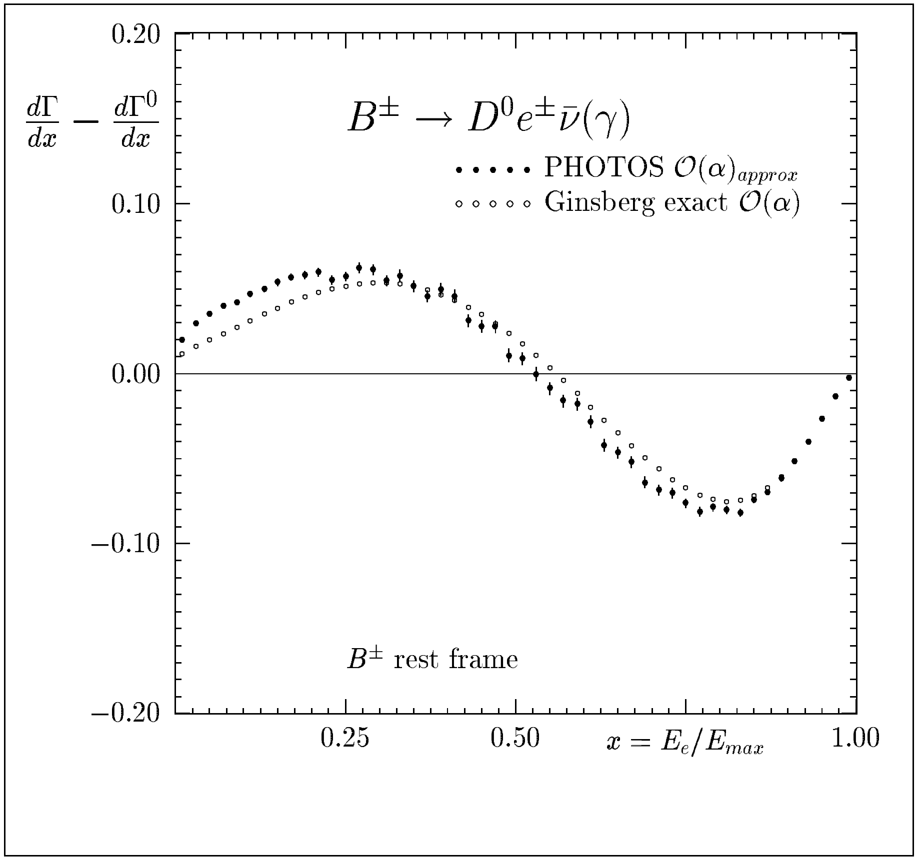
Plot of worst agreement for the channel. Distribution of $\gamma\nu_\tau\nu_\mu$ system mass is shown .



Also the fraction of events with photon above threshold agrees better than permille level.

In TAUOLA complete matrix element, comparison test PHOTOS approximations and design.

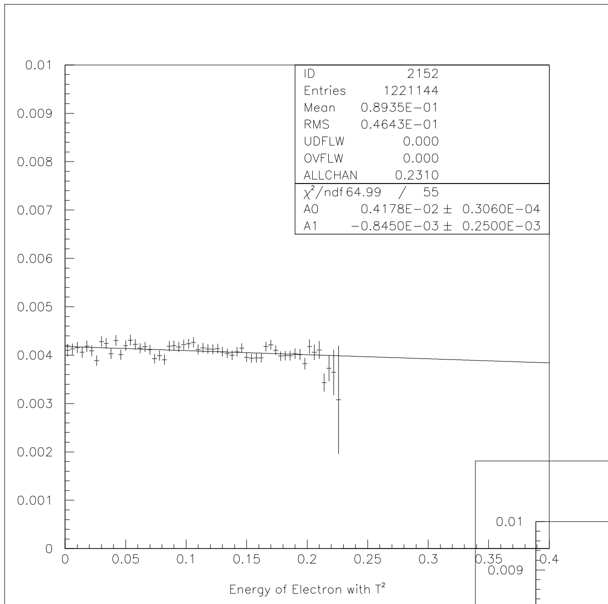
Phys. Lett, B 303 (1993) 163-169

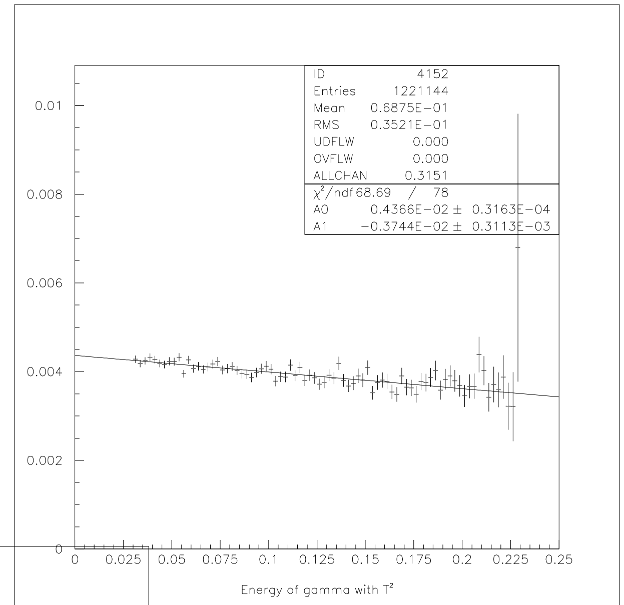


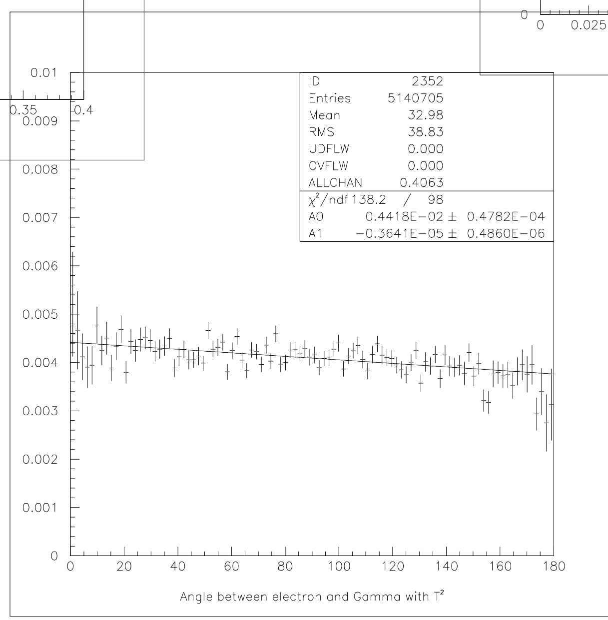
Radiative correction to the decay rate ($d\Gamma/dx - d\Gamma^0/dx$) for $B^\pm \to D^0 e^\pm \bar{\nu}(\gamma)$ in the $B^\pm$ rest frame. Open circles are from the exact analytical formula [2], points with the marked statistical errors from PHOTOS applied to JETSET 7.3. A total of 10^7 events have been generated. The results are given in units of $(G_\mu^2 m_B^5/32\pi^3)N_\eta|V_{cb}|^2|f_+^D|^2$, where $N_\eta = \eta^5 \int_0^1 x^2(1-x)^2/(1-\eta x)dx$ and $\eta = 1 - m_D^2/m_B^2$.

- *“QED bremsstrahlung in semileptonic B and leptonic τ decays”* by E. Richter-Was.
- agreement up to 1%
- disagreement in the low-x region due to missing sub-leading terms
- study performed in 1993.

$K \to \pi e \nu(\gamma)$ PHOTOS w/Interf vs Gasser







This was OK in 2005 but it is not systematic work.

Events with and without photon:

$R = \frac{\Gamma_{K_{e3\gamma}}}{\Gamma_{K_{e3}}}$	PHOTOS %	GASSER %
$5 < E_\gamma < 15\,MeV$	2.38	2.42
$15 < E_\gamma < 45\,MeV$	2.03	2.07
$\Theta_{e,\gamma} > 20$	0.876	0.96

courtesy of NA48 and Prof. L.Litov

This results can be obtained starting from PHOTOS version 2.13.

Multiphoton radiation

1. So far we were talking only about constructing configuration with single extra photon.
2. It is important, because that is the option used by experiments.
3. The part on phase space is basically explained already.
4. Even though one has to work on more details simultaneously, than before.
5. Construction of matrix elements rely on iteration.
6. We will concentrate on the $Z \to \mu^+\mu^-$ case
   (a) because we have matrix element at hand
   (b) and benchmarks with KKMC as well.
7. For other channels we will use general results and analogy only.

Phase Space: (multiple photon radiation)

$$\sum_{l=0}^{n_{max}} dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l})\rho_{cr} = \exp(-F)$$

$$\sum_{l=0}^{n_{max}} \frac{1}{l!} \prod_{i=1}^{l} \left[f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W_{n+i-1}^{n+i} \right] \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$

$$\{k_1, \ldots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\ldots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \ldots, \bar{k}_n\}\big) \ldots\big) \quad (10)$$

- We sum (7) over l, and to the case of arbitrary number of photons, limited by n_{max} or not.
- That is used in PHOTOS: maximal mulitiplicity can be set 1, 2, 3, 4 or can be arbitray large.
- WARNING: Without matrix elements the formula make no sense!
- We must have ME all over tangent AND physical spaces; virtual corrections are essential.
- We define tangent space distr. (blue colour) and get Poissonian distribution in l.
- With that ME we generate tangent multiplicity and all $k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}$ independently.

Matrix Element: (multiple photon radiation)

$$\sum_{l=0}^{n_{max}} dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l})\rho_{cr} = \exp(-F)$$

$$\sum_{l=0}^{n_{max}} \frac{1}{l!} \prod_{i=1}^{l} \left[f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W_{n+i-1}^{n+i} \hat{wt} \right] \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$

$$\{k_1, \ldots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\ldots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \ldots, \bar{k}_n\}\big) \ldots\big)$$

- At the time of introducing energy-momentum constraints (red parts of formula), we can also replace the tangent matrix elements with the ones we want to have at the end.
- For final state multiple bremsstrahlung, this procedure is relatively simple, because of small QED corrections to total rate no problems with four-momentum P (present for ISR).
- Iterative rejections due to differences in real and tangent space matrix elements and W_{n+i-1}^{n+i} simply remove candidates for consecutive photons.
- **HOW TO CONSTRUCT $\hat{wt}$, MULTIPLE PHOTON MATRIX ELEMENT?**

● **HOW TO CONSTRUCT $\hat{wt}$, MULTIPLE PHOTON MATRIX ELEMENT?**

● First, it is possible to naively "iterate" relation of the single photon matrix element with the one of Born level.

● It gives coverage of all multiphoton (at first two-photon) phase space. Virtual corrections can be introduced with sum rules.

● One can (1991,1994) see that such procedure works in soft photon phase space regions.

● Also, whatever the number of charged particles in final state energy spectrum of each these charged decay products get proper LL corrections (1994). Solution of QED evolution equation, possibly truncated to some order builds up automatically:

$$\begin{aligned} f^{\infty}(x) &= \delta(x) + P(x) + \frac{1}{2!}\Big\{P \otimes P\Big\}(x) + + \frac{1}{3!}\Big\{P \otimes P \otimes P\Big\}(x) + ..., \\ \Big\{P \otimes P\Big\}(x) &= \int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 + x_2 - x_1 x_2 - x) P(x_1) P(x_2). \end{aligned} \tag{11}$$

Here $P(x)$ denotes (basically) an Altarelli-Parisi kernel. Depends on spin of the charged particle and is proportional to $\frac{\alpha}{\pi} \log \frac{E_{ch}}{m_{ch}}$.

Matrix Element: (multiple photon radiation)

1. PHOTOS guarranties cover full phase space for bremsstrahlung photons.
2. maximum photon multiplicity of 1, 2 3, 4 or unlimited can be chosen.
3. Distribution in soft region of phase space is exact.
4. QED LL corrections for charged decay products energy spectra are OK.
5. For decays when complete first order matrix elements was available it was installed.
6. Only for $Z \to \mu^+\mu^-$ second order matrix element was used.
7. For other channels our choice of iteration details may not be the best one.
8. Recent progress in domain of on mass-shell iterative relations is encouraging. PHOTOS solution may find new applications?
9. Let us review some test of PHOTOS with KKMC (CEEX $\mathcal{O}(\alpha^2)$.

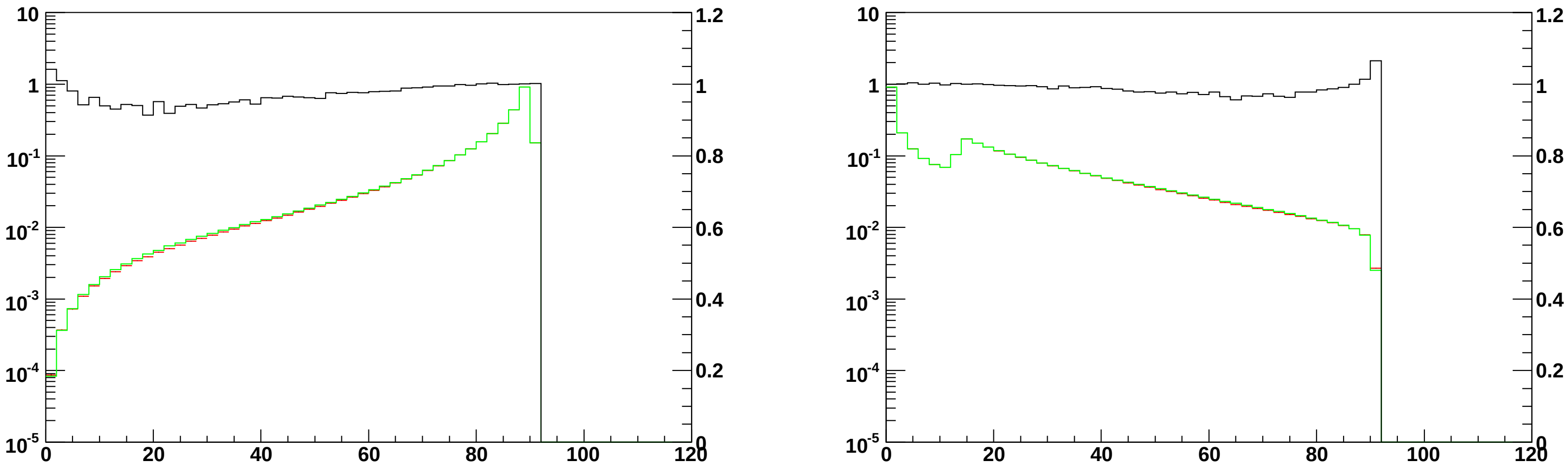


Figure 3: Comparison of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00409. In right frame the invariant mass of the $\mu^-\gamma$ pair; SDP=0.0025. The pattern of differences between PHOTOS and KKMC is similar to the one of Fig 1. The fraction of events with hard photon was 16.0824 $\pm$ 0.0040% for KKMC and 16.1628 $\pm$ 0.0040% for PHOTOS.

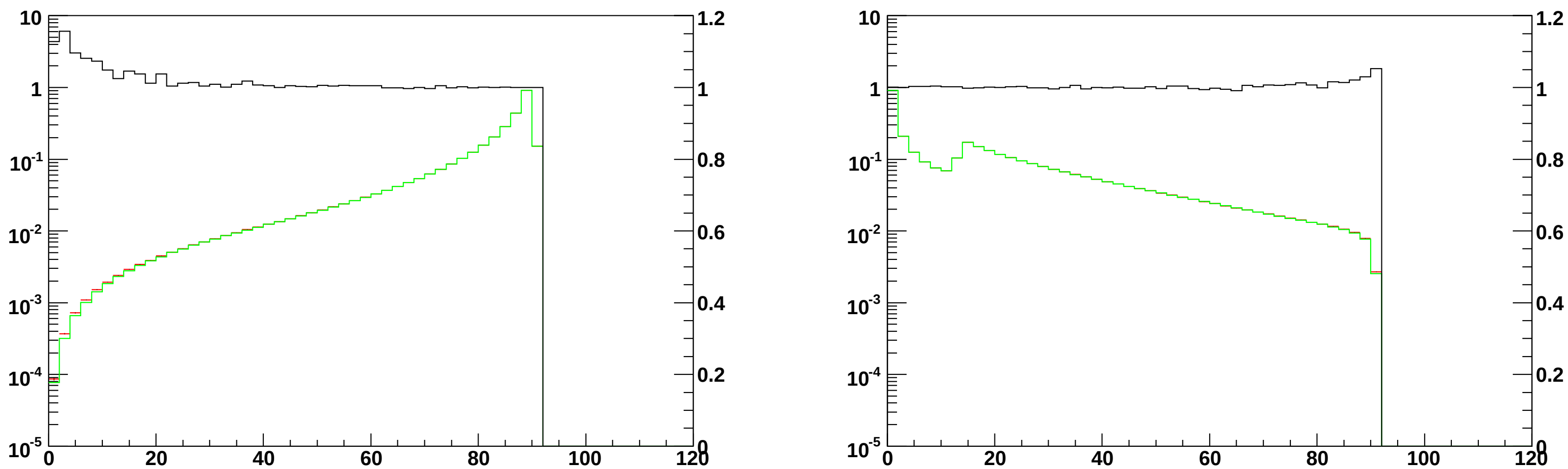


Figure 4: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.0000249. In the right frame the invariant mass of the $\mu^-\gamma$ pair; SDP=0.0000203. The fraction of events with hard photon was $16.0824 \pm 0.004\%$ for KKMC and $16.0688 \pm 0.004\%$ for PHOTOS.

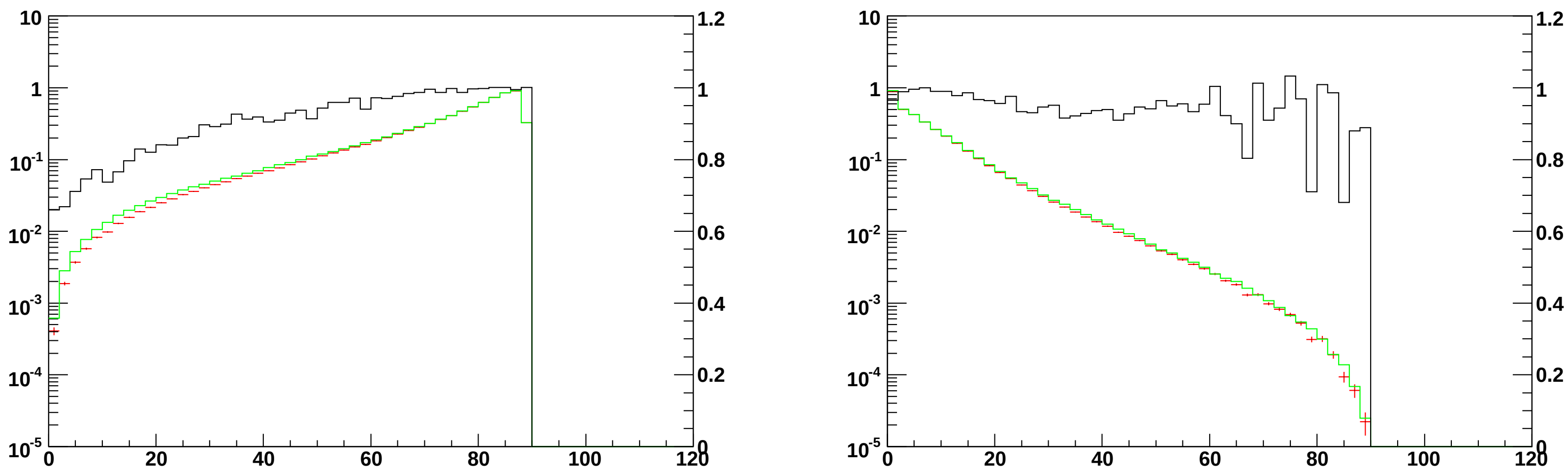
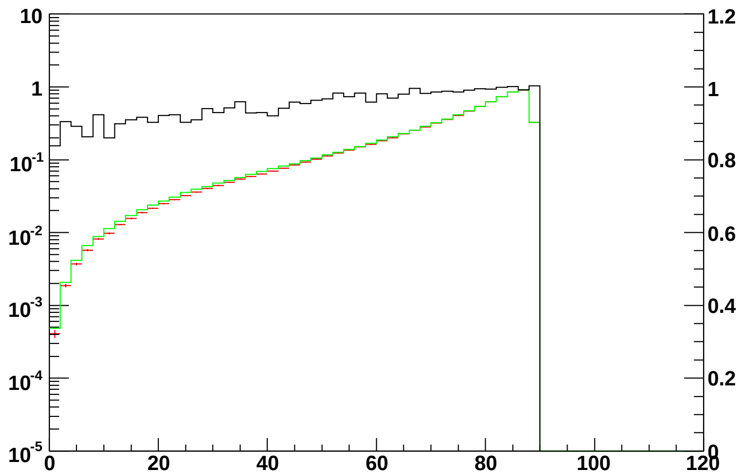


Figure 5: Comparisons of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ *pair; SDP= 0.00918. In the right frame the invariant mass of the* $\gamma\gamma$ *pair; SDP=0.00268. The fraction of events with two hard photons was* $1.2659 \pm 0.0011\%$ *for KKMC and* $1.2952 \pm 0.0011\%$ *for PHOTOS.*



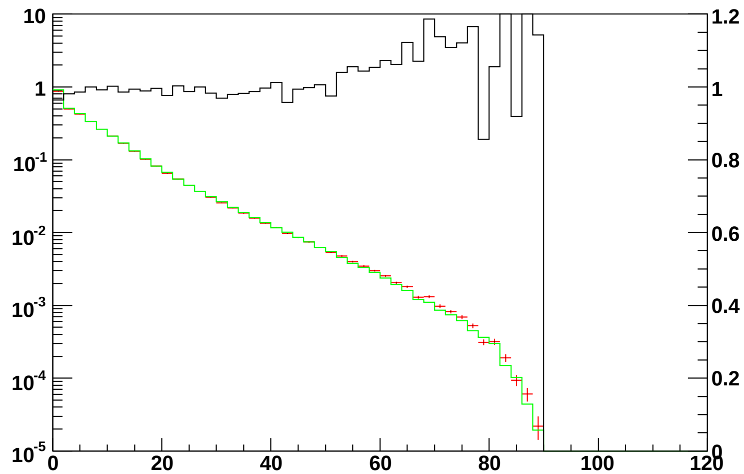
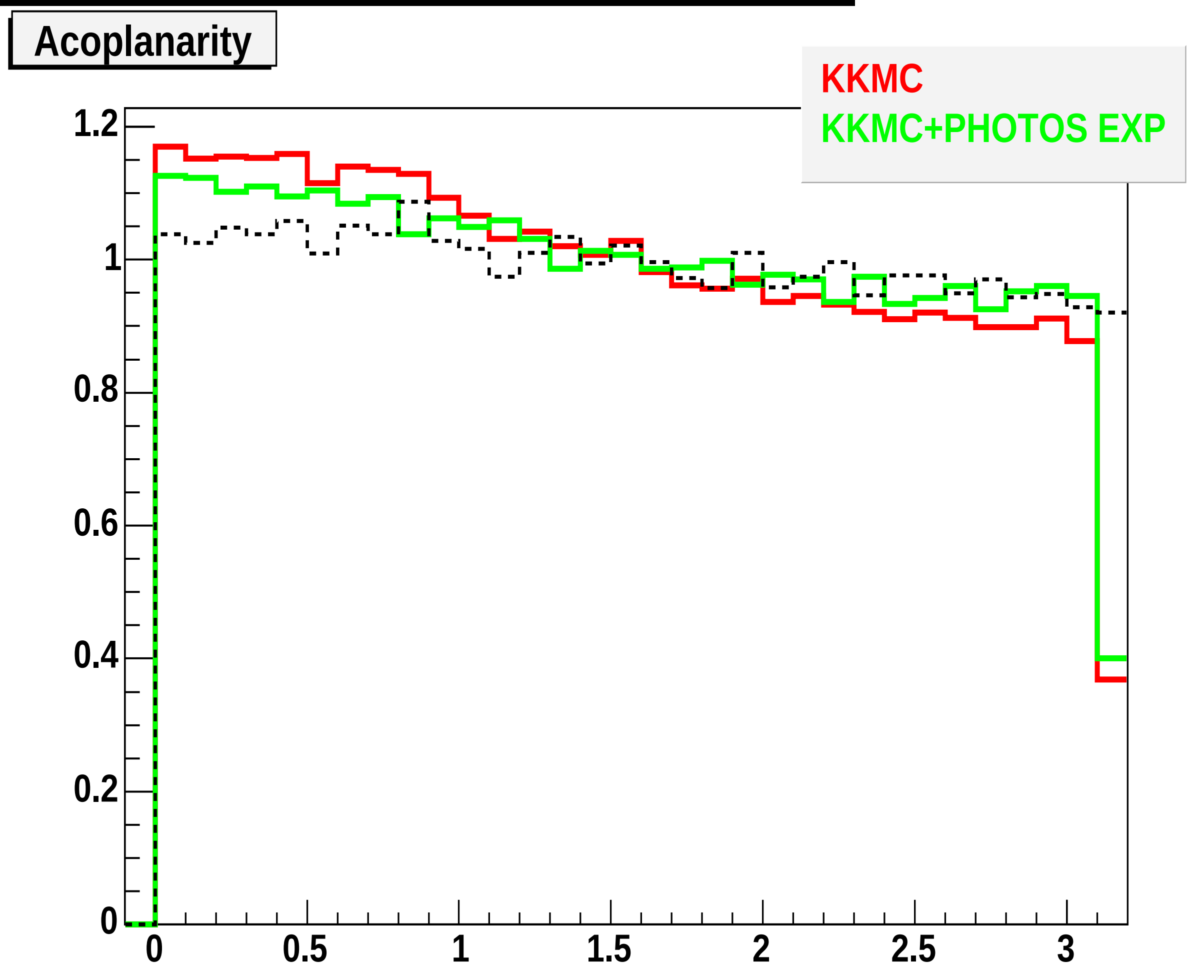


Figure 6: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP= 0.00142. In the right frame the invariant mass of the $\gamma\gamma$; SDP=0.00293. The fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KKMC and 1.2868 $\pm$ 0.0011% for PHOTOS.

Acoplanarity distribution – Looks good



Two plane spanned on μ^+ and respectively two hardest photons localized in the same hemisphere as μ^+. Why PHOTOS works so good?

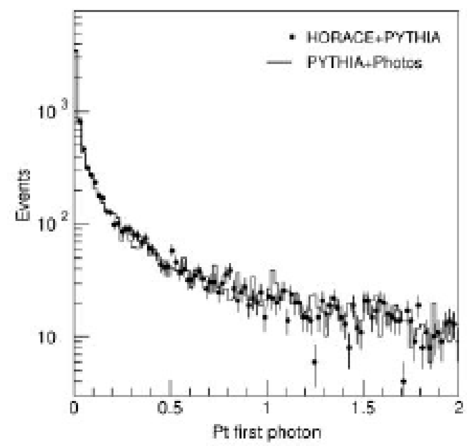
A successful validation example..

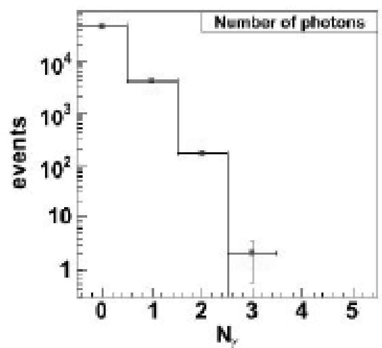
- Comparison between PHOTOS (supposed to be an approximate algorithm in principle) and HORACE (exact QED DGLAP solution):
  - Turns out that PHOTOS is doing an excellent job!

HORACE vs Photos (3)

- Photon multiplicity and transverse momentum spectrum done with standalone generators (outside Athena)

  perfect agreement for all p_T range





with cut $p_T(\gamma)$ > 500 MeV perfect agreement also in Athena iterfaced version to third hard photon

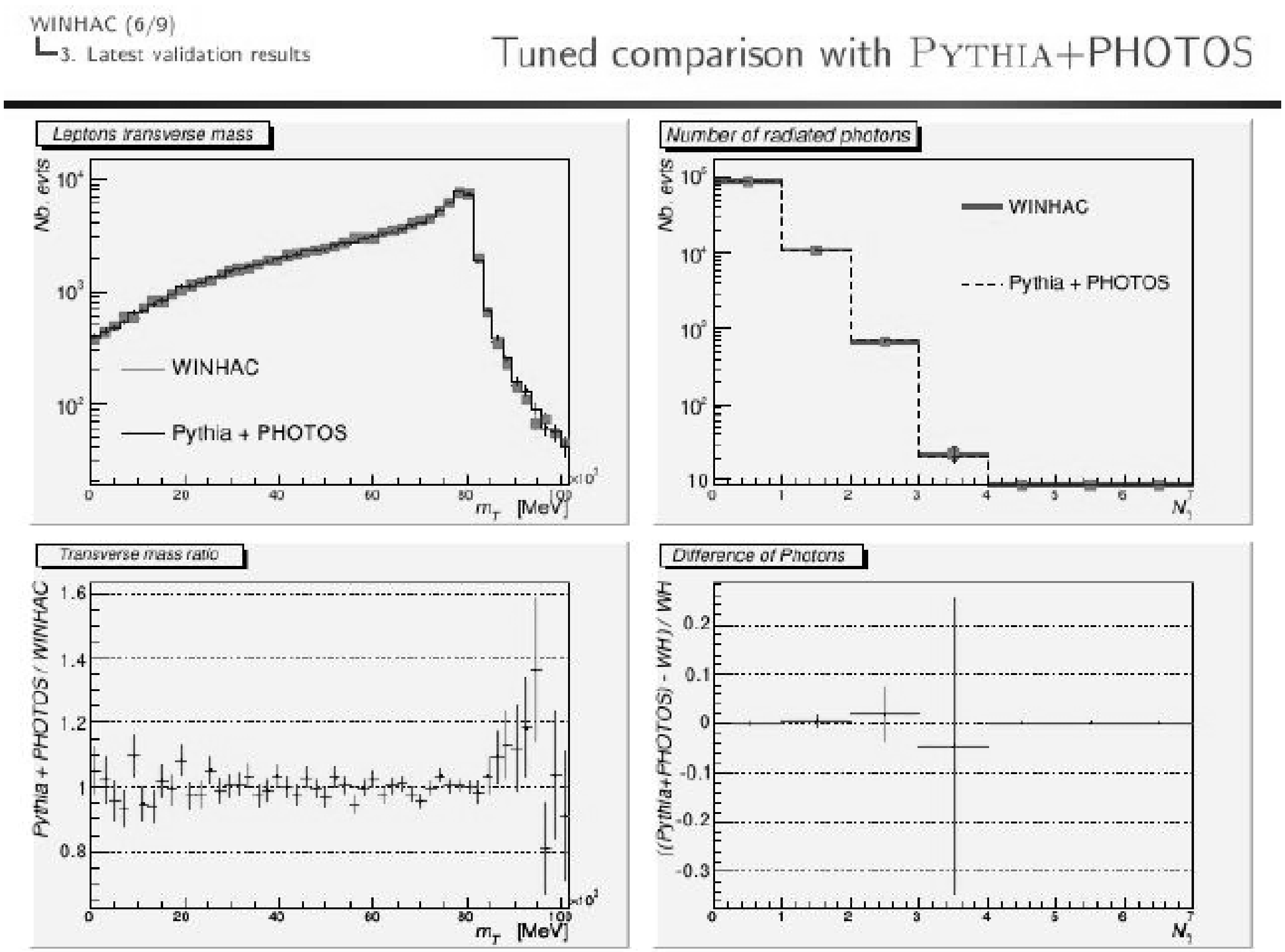
Pythia + HORACE

Pythia + Photos

29/05/06 M. Bellomo - ATLAS Monte Carlo 15

This is for Z production at LHC.

And another one.. Our Winhac effort



This is for W production at LHC.

MC Generators for LHC at ATLAS

ATLAS Overview Week (February 2007)

Borut Kersevan
Jozef Stefan Inst.
Univ. of Ljubljana

ATLAS experience:

- Generators used
- Validation procedures
- Interesting examples

Not systematic work on algorithm, but program validation for ATLAS. From today talk CERN main auditorium 11 am.

Summary

- Results look good !!

- Phase space, crude distribution: exact and explained. Separated from ME.

- In general case ME $\mathcal{O}(\alpha)$ is with approximations but in some already now it is exact. In every case its analytic form is explicitly given.

- For multiple photon radiation, still many unexplored options exist.

- In construction we rely on properties of factorization, my personal experience are summarized in paper on $e^+e^- \to \nu_e\bar{\nu}_e\gamma\gamma$, EPJC C44 (2005) 489.

- I need to know far more than now on mass shell iterative solutions for spin amplitudes to continue.

- For QED numerical results discourage this effort. They are good enough without.

- It is not bad news for program users !!!

- Shall we find some new area of applications for the method?

Good question, no answer today, but:

- **PHOTOS ready for "first data at LHC and for use with many main generators. It is observable builder fiendly.**
- **adequate for W-mass measurement to precision at 0.1 % precision level.**
- **adequate for energy scale calibration with $Z \to e^+e^-$ to 0.1%**
- **adequate for τ decays, for $Z \to \tau\tau$, $H \to \tau\ \tau$.**
- **no problem with radiative corrections in decays for SUSY discovery**
- **heavily used for B physcics.**

For the first data, most of the MC generstors will remain in FORTRAN! So is PHOTOS (and TAUOLA). Once HepMC event record stabilise in LHC we jump to C++ (version exist since 1999).

- Thank you.