Exercises for the lecture "Théorie des jets" at the École de Gif, 2007.

1. If you have never done so before, show that $\Delta y_{ij} = y_i - y_j$, with $y_i = \ln \frac{E_i + p_{zi}}{E_i - p_{zi}}$, is invariant under boosts in the z direction. It should be obvious that $\Delta\phi_{ij}$ and p_{ti}, p_{tj} are also boost invariant.

2. Show that the boost-dependent k_t algorithm measures

$$d_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})\,, \qquad d_{iB} = 2E_i^2(1 - \cos\theta_{iB})\,, \tag{1}$$

are identical to the longitudinally boost-invariant measures

$$d_{ij} = 2\min(k_{ti}^2, k_{tj}^2)\Delta R_{ij}^2\,, \qquad d_{iB} = k_{ti}^2\,, \tag{2}$$

when θ_{ij} and θ_{iB} are small respectively.

3. Take the event consisting of the following particles,

$$p_1 = (90, 0, 0; 90)\ \text{GeV} \tag{3a}$$
$$p_2 = (-90.5, 0, 0; 90.5)\ \text{GeV} \tag{3b}$$
$$p_3 = (1.2, 0.5, 0.0; 1.3)\ \text{GeV} \tag{3c}$$
$$p_4 = (-0.7, -0.5, 0.0; 0.9)\ \text{GeV} \tag{3d}$$

and draw it. Given an exclusive clustering that reduces the event to two jets, how would you expect the particles to be clustered?

Consider the e^+e^- JADE algorithm, a sequential recombination algorithm in which the dimensionless distance measure is related to the invariant pass of the pair under consideration,

$$y_{ij}^{\text{Jade}} = \frac{2E_iE_j(1 - \cos\theta_{ij})}{Q^2} \tag{4}$$

where Q is the total momentum in the event. If the closest pair is repeatedly clustered until only 2 pseudojets are left, what particles will appear in which jets? Is this sensible?

Now consider the e^+e^- k_t algorithm with distance measure,

$$y_{ij}^{k_t} = \frac{2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{Q^2} \tag{5}$$

What result do you obtain for the clustering? Is this more or less sensible than the result for the JADE algorithm?

4. Suppose you have the partonic process $q\bar{q} \to Z' \to q\bar{q}$, where the Z' has mass M, is produced at rest in the lab, and decays perpendicularly to the beam direction.

For an inclusive jet algorithm with small radius parameter, $R \ll 1$, calculate to order α_s the probability of the jet energy differeng from $M/2$ by more than an amount $E_0 \ll M$, using the approximations outlined below:

- Simplify the probability of radiation of a soft gluon, $E \ll M$ at an angle $\theta \ll 1$ from one of the outgoing quarks as,

$$\frac{dP}{dEd\theta} = \frac{2\alpha_s C_F}{\pi}\frac{1}{E\theta}\,. \tag{6}$$

- Ignore radiation from the incoming quarks. Why is this legitimate?
- Use a double logarithmic approximation, in which you ignore terms that don't contain at least two large logarithms per power of α_s.

If you wish to maintain a fixed probability of losing energy greater than E_0, should R be increased or decreased as M is taken larger?