Phenomenology

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BUSSTEPP
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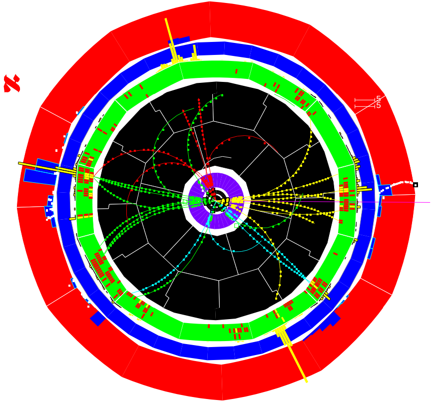
Phenomenology

Lecture 3

(QCD basics, jets)

Quarks $\to$ jets of hadrons



Aleph Higgs event:

- Claim: it corresponds to $ZH \to q\bar{q}b\bar{b}$.
- But actually just bunches ('jets') of hadrons.
- Can they be related? How?
  NB: not just '*are* they related?'

Need understanding of QCD
(and not just for this!)

QCD is not solved

Degrees of freedom of Lagrangian (quarks, gluons) $\neq$ physical particles (π, p, n, ...).

Lattice is not powerful enough to reach high energies; perturbative QCD only good for talking about *unphysical particles* (quarks, gluons).

So: phenomenology with QCD objects (jets, incoming protons) has to

work around these problems

- Choose the right observables (to let us ignore our ignorance).
- Learn from experiments what we cannot (yet) calculate.
- Know how to *quantify remaining ignorance...*

Lagrangian + colour

Quarks — 3 colours: $\psi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Quark part of Lagrangian:

$$\mathcal{L}_q = \bar{\psi}_a (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m)\psi_b$$

$SU(3)$ local gauge symmetry $\leftrightarrow$ 8 $(= 3^2 - 1)$ generators $t^1_{ab} \ldots t^8_{ab}$ corresponding to 8 gluons $\mathcal{A}^1_\mu \ldots \mathcal{A}^8_\mu$.

A representation is: $t^A = \frac{1}{2}\lambda^A$,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

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Lagrangian + colour (cont.)

Field tensor:

$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\nu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu \qquad [t^A, t^B] = i f_{ABC} t^C$

f_{ABC} are structure constants of $SU(3)$ (antisymmetric in all indices — $SU(2)$ equivalent was ϵ^{ABC}). Needed for gauge invariance of gluon part of Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4} F^{\mu\nu}_A F^{A\,\mu\nu}$$

Interaction vertices of Feynman rules:

Lagrangian + colour (cont.)

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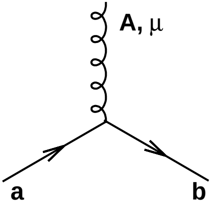
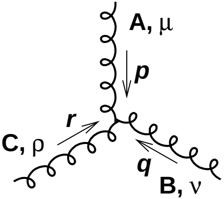
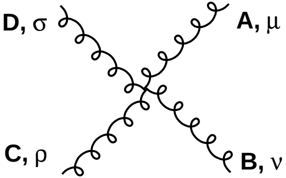


$$-i g_w t^A_{ba} \gamma^\mu$$



$$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$$



$$-i g_s^2 f^{XAC} f^{XBD}[g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$$

Quick guide to colour algebra

$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \qquad {\color{red} T_R = \tfrac{1}{2}}$$

$$\textstyle\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \qquad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

$$\textstyle\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}\,, \qquad C_A = N_c = 3$$

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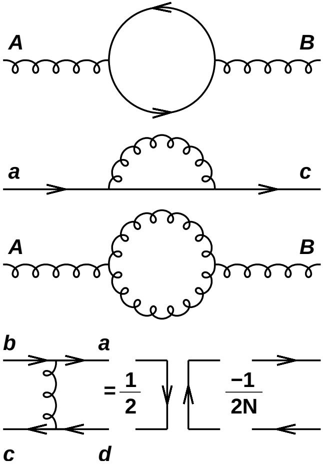

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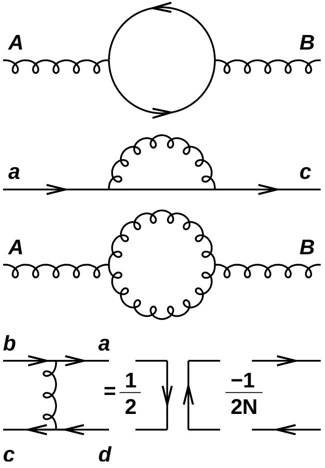

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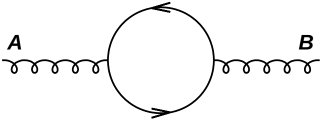

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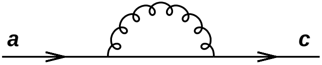

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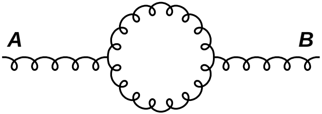

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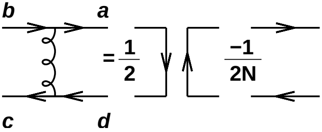

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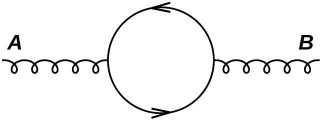

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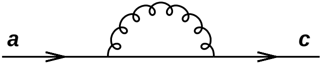

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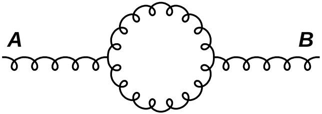

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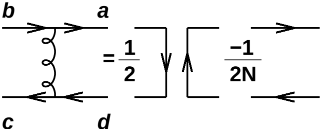

Running coupling

The strong coupling, α_s, *runs:*

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s)\,, \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)\,,$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}\,, \qquad b_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2} = \frac{153 - 19n_f}{24\pi^2}$$

Note sign: Asymptotic Freedom, due to gluon to self-interaction

- At high scales Q, coupling is weak
  ↪quarks and gluons are almost free, interactions are just a perturbation
- At low scales, coupling is strong
  ↪quarks and gluons interact strongly — they are confined into hadrons. Perturbation theory fails

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Solve $Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -b_0 \alpha_s^2 \Rightarrow \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - b\alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$

Λ (aka Λ_{QCD}) is the fundamental scale of QCD, at which coupling blows up.

- Λ sets the scale for hadron masses
  (NB: Λ not unambiguously defined wrt higher orders)
- Perturbative calculations valid for scales $Q \gg \Lambda$.

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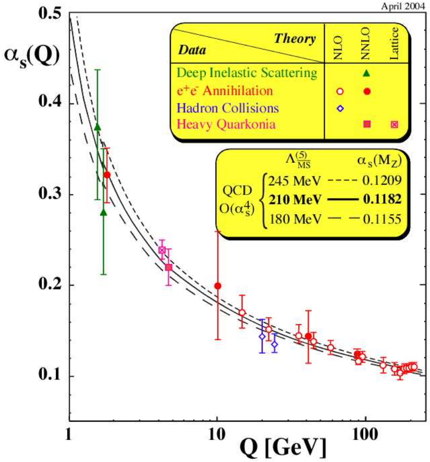

What is the right scale?

Say we have some observable V in e^+e^- collisions at centre of mass energy $Q = \sqrt{s}$. After *renormalisation* at scale μ

$$V = C_0 + C_1 \cdot \alpha_s(\mu^2) + \left(C_2 + C_1 b_0 \ln \frac{\mu^2}{Q^2}\right) \cdot \alpha_s^2(\mu^2) + \dots$$

- Coupling depends on μ^2; so do higher order coefficients.
- Sum of full series should be independent of μ.

But sum of truncated series *does* depend on μ. What do we take? Various scales in problem:

- centre of mass energy $Q \to$ result is perturbative
- masses of produced hadrons $\to$ result is non-perturbative

We'd *like* to say Q ('hard scale') is right one — but how do we know?

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Soft gluon amplitude

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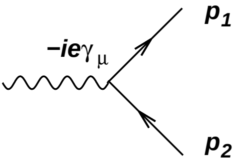
$$\mathcal{M}_{q\bar{q}} = -\bar{u}(p_1) i e_q \gamma_\mu v(p_2)$$



Emit a gluon:

$$\mathcal{M}_{q\bar{q}g} = \bar{u}(p_1) i g_s \not{\epsilon} t^A \frac{i}{\not{p}_1 + \not{k}} i e_q \gamma_\mu v(p_2) - \bar{u}(p_1) i e_q \gamma_\mu \frac{i}{\not{p}_2 + \not{k}} i g_s \not{\epsilon} t^A v(p_2)$$

Make gluon *soft* $\equiv k \ll p_{1,2}$; ignore terms suppressed by powers of k:

$$\mathcal{M}_{q\bar{q}g} \simeq \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k} \right)$$

$\not{p} v(p) = 0,$
$\not{p}\not{k} + \not{k}\not{p} = 2p.k$

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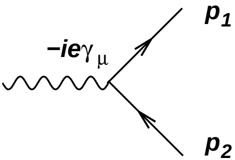

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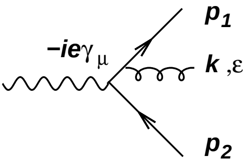

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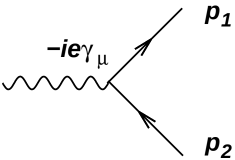
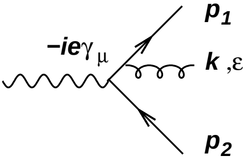


Emit a gluon:

$$\mathcal{M}_{q\bar{q}g} = \bar{u}(p_1) i g_s \not{\epsilon} t^A \frac{i}{\not{p}_1 + \not{k}} i e_q \gamma_\mu v(p_2) - \bar{u}(p_1) i e_q \gamma_\mu \frac{i}{\not{p}_2 + \not{k}} i g_s \not{\epsilon} t^A v(p_2)$$



Make gluon *soft* $\equiv k \ll p_{1,2}$; ignore terms suppressed by powers of k:

$$\mathcal{M}_{q\bar{q}g} \simeq \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k} \right)$$

$$\not{p} v(p) = 0,$$
$$\not{p}\not{k} + \not{k}\not{p} = 2p.k$$

Squared amplitude

$$|M^2_{q\bar{q}g}| \simeq \sum_{A,\text{pol}} \left| \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k} \right) \right|^2$$

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Include phase space:

$$d\Phi_{q\bar{q}g} |M^2_{q\bar{q}g}| \simeq (d\Phi_{q\bar{q}} |M^2_{q\bar{q}}|) \frac{d^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

Note property of factorisation into hard $q\bar{q}$ piece and soft-gluon emission piece, dS.

$$dS = \omega_k d\omega_k d\cos\theta \frac{d\phi}{2\pi} \cdot \frac{2\alpha_s C_F}{\pi} \frac{2p_1.p_2}{(2p_1.k)(2p_2.k)}$$

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Soft & collinear gluon emission

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So final expression for soft gluon emission is

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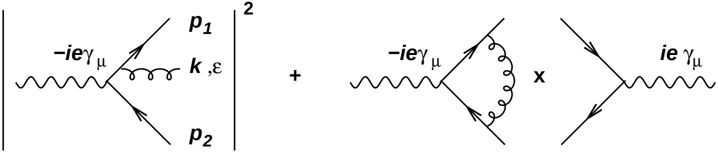
Total cross section: sum of all real and virtual diagrams



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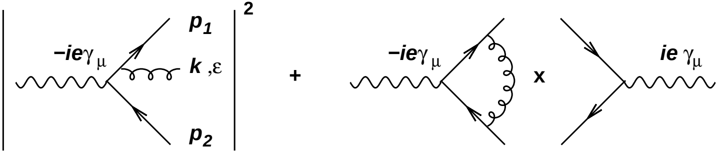

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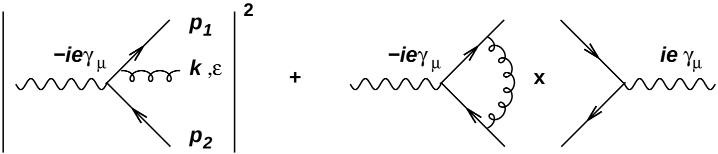

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total X-section (cont.)

Dependence of total cross section on only *hard* gluons is reflected in 'good behaviour' of perturbation series:

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + 1.045 \frac{\alpha_s(Q)}{\pi} + 0.94 \left(\frac{\alpha_s(Q)}{\pi} \right)^2 - 15 \left(\frac{\alpha_s(Q)}{\pi} \right)^3 + \cdots \right)$$

(Coefficients given for $Q = M_Z$)

Estimate uncertainties

Arguments say $\mu \sim Q$.

- $\mu = Q$?
- $\mu = Q/2$?
- $\mu = 2Q$?

No way to say — but at very high orders of perturbation theory it should not matter...

Impact illustrated in extraction of $\alpha_s(M_Z)$ from data on $\sigma_{e^+e^- \to \text{hadrons}}$.

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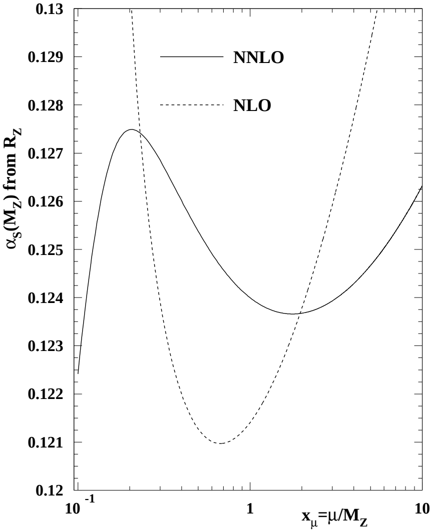

Naive jet X-section

In lecture 2 we associated each parton with a 'jet' ($HZ \to q\bar{q}b\bar{b}$).
So let's calculate X-section for 3 jets, as being that for 3 partons:

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Virtual piece absent: it only has 2 'jets'.

Result diverges (for $\omega \to 0, \theta \to 0$):

- perturbatively infinite cross section for producing an extra gluon
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➥ identifying jets as partons is a *bad idea*.

So what *do* we mean by a 'jet'?
Soft or collinear gluon should *not* be a separate jet.
Hard well-separated gluon should.

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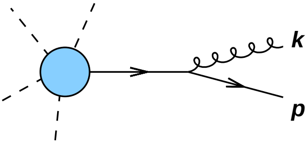
Infrared and Collinear Safety

To understand principles for defining a jet, first examine origin of divergence in general terms.

Take an n-parton amplitude and emit a soft collinear gluon k from parton p.



Combination of propagator and vertex give:

$$g_s t_p^A \frac{\epsilon . p}{k . p} \rightarrow^2 C_p \frac{g_s^2}{\omega_k^2 \theta^2}$$

There are soft and collinear divergences (real & virtual) for emission of a gluon off *any* coloured parton

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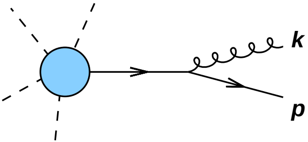

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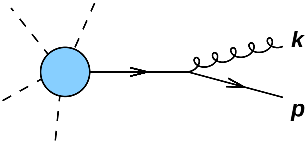

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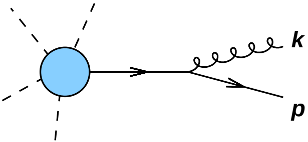

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Infrared and Collinear Safety (definition)

For an observable's distribution to be calculable in perturbation theory, the observable should be infra-red safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching

$$\vec{p}_i \longrightarrow \vec{p}_j + \vec{p}_k$$

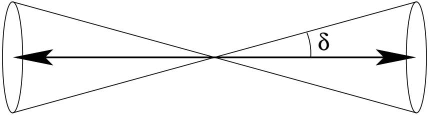
whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel [collinear] or one of them is small [infrared].

[QCD and Collider Physics (Ellis, Stirling & Webber)]

Sterman-Weinberg jets

The *original* (finite) jet definition

An event has 2 jets if at least a fraction $(1-\epsilon)$ of event energy is contained in two cones of half-angle δ.



$$\sigma_{2-jet} = \sigma_{q\bar{q}} \left(1 + \frac{2\alpha_s C_F}{\pi} \int \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi} \left(R\left(\frac{\omega}{Q}, \theta\right) \times \right.\right.$$
$$\left.\left. \times \left(1 - \Theta\left(\frac{\omega}{Q} - \epsilon\right)\Theta(\theta - \delta)\right) - V\left(\frac{\omega}{Q}, \theta\right)\right)\right)$$

- For small ω or small θ this is just like total cross section — full cancellation of divergences between real and virtual terms.
- For large ω and large θ a *finite piece* of real emission cross section is *cut out.*
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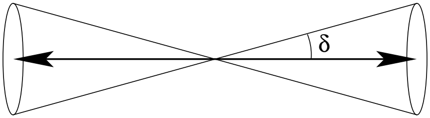

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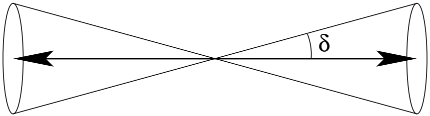

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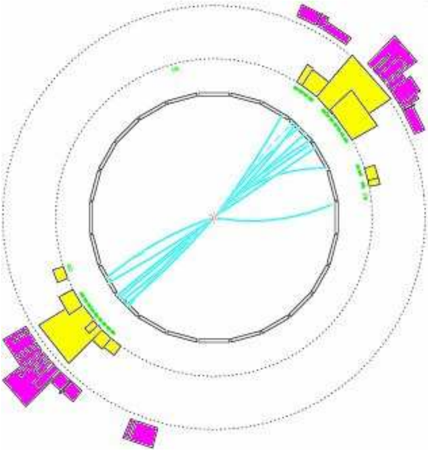


Near 'perfect' 2-jet event

2 well-collimated jets of particles.

All energy in two cones.

NB: picture of two quarks and a soft gluon does not reflect reality of event structure.

Origin of event structure?

- Multiple QCD radiation has *nested* soft and collinear divergences.

  Much of structure is calculable to all orders!

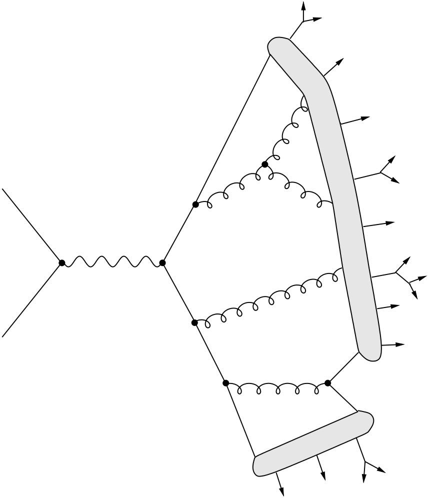
- Produce many soft and collinear gluons, $q\bar{q}$ pairs

- *Somehow* there is a transition from *partons* → *hadrons*

  Can only be modelled

- These elements are encoded in Monte Carlo simulation programs

  Extremely successful, ubiquitous
  e.g. Pythia, Herwig, Sherpa



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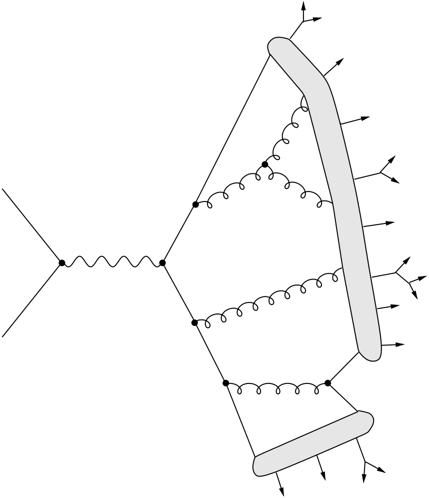

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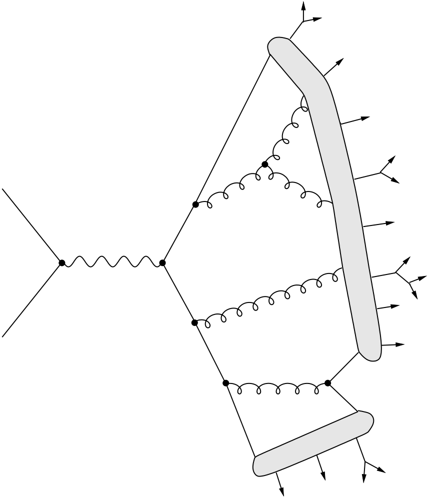

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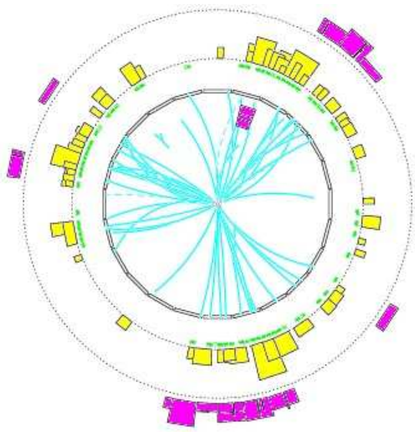

multi-jet event

How can we define jets for more complex events?

- Sterman-Weinberg ('cone') definition gets messy
- Jets may be broader than chosen cone
- Some of energy-momentum is outside jet cones ($\sum$ jet energy $\neq$ total energy)

Need a more sophisticated tool to relate real events to an *idealised* hard event.

Modern jet algorithms

Based on idea of successive clusterings and *resolution parameter* (y_{cut}):
Idea: try to *undo multiple QCD branching and 'hadronisation'*.

1. Calculate the *distance* y_{ij} (according to some measure) between all current pairs of particles/pseudo-jets i, j:

$$y_{ij} = min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$

'k_t' measure: closeness ⇔ structure of QCD divergences

2. If all $y_{ij} > y_{\text{cut}}$ *stop.*
3. Otherwise, select the i, j, with the smallest y_{ij} and *cluster* them to make a 'pseudojet'.
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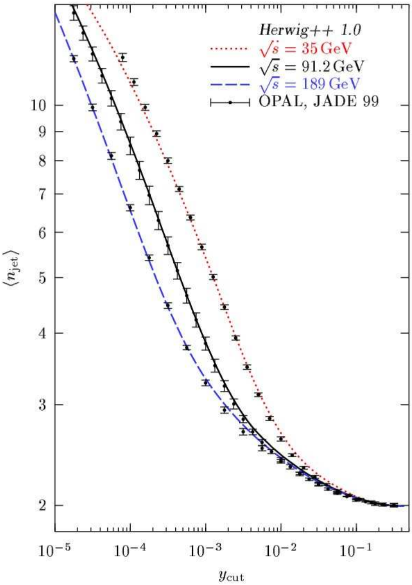
Number of jets v. resolution (e^+e^-)



Interim QCD summary

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  - confinement (low Q): quarks, gluon $\neq$ physical d.o.f.
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Next lecture

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