QCD at hadron colliders
Lecture 3: Parton Distribution Functions

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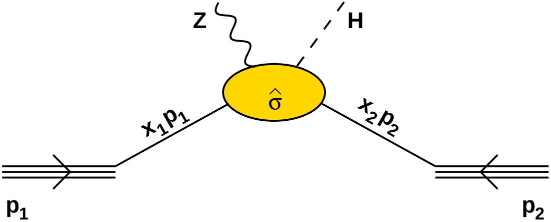
Factorization & parton distributions

Cross section for some hard process in hadron-hadron collisions



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- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section $\leftrightarrow$ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - how can we determine the PDFs? NB: non-perturbative
  - does picture really stand up to QCD corrections?

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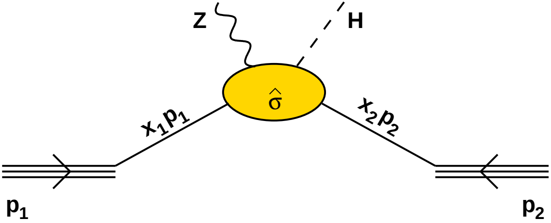

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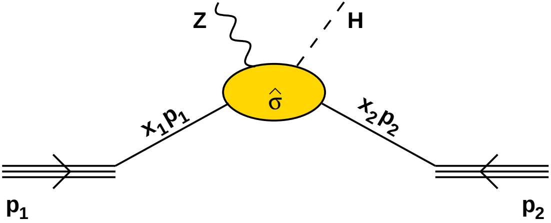

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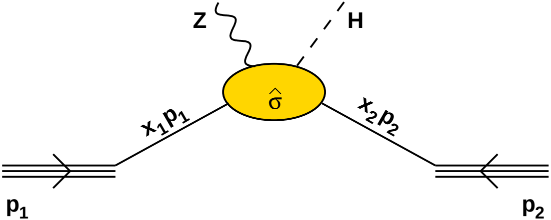

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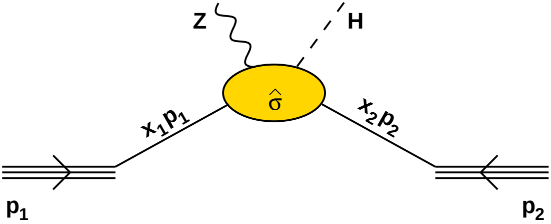

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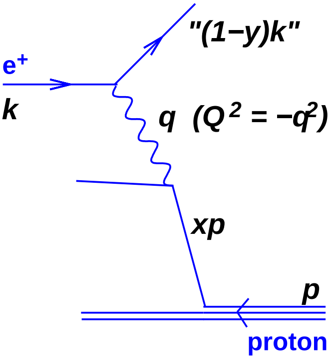
Deep Inelastic Scattering: kinematics

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



Kinematic relations:

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality ↔ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
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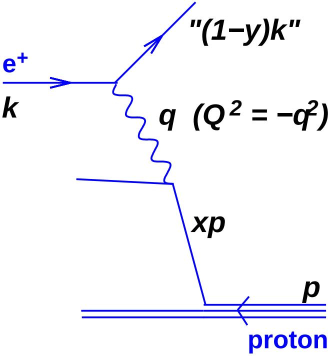

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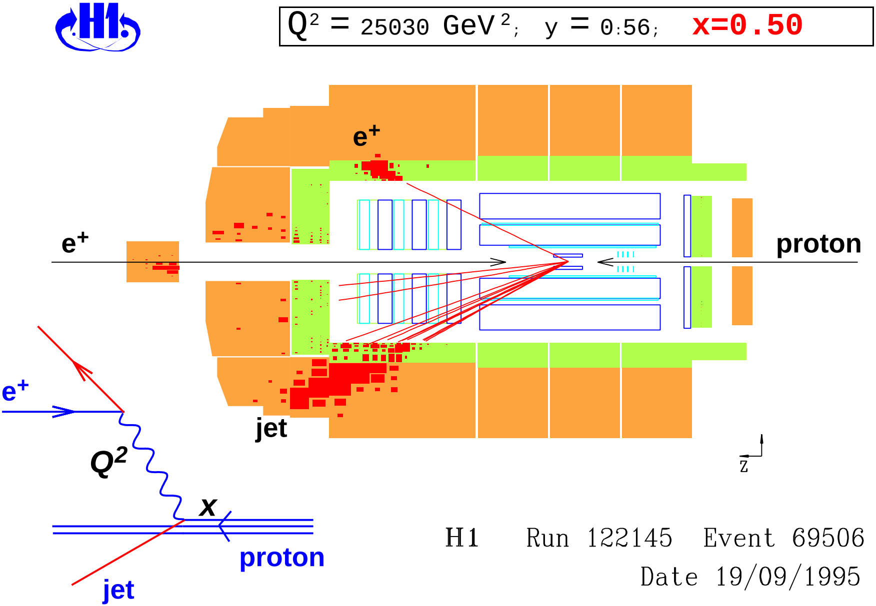
Deep Inelastic scattering (DIS): example



E.g.: extracting u & d distributions

Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

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F_2 gives us *combination* of u and d.
How can we extract them separately?

Extracting full flavour structure?

- Using neutrons and **isospin**

$$F_2^n = \frac{4}{9} u_n(x) + \frac{1}{9} d_n(x)$$

- Using charged-current ($W^\pm$) scattering
  [neutrinos instead of electrons in initial or final-state]
  - ν interacts only with $d, \bar{u}$
  - angular structure of interaction differs between d and $\bar{u}$

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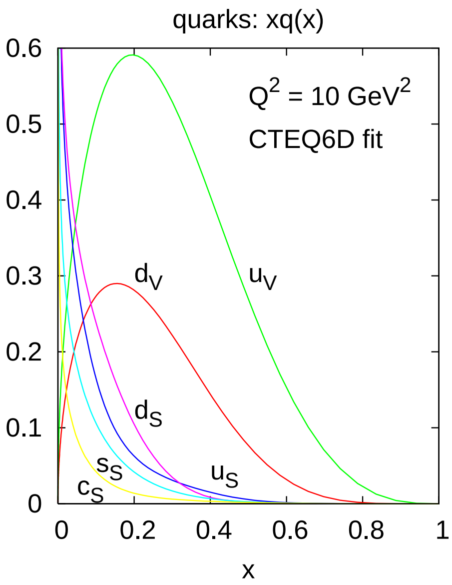
All quarks



These & other methods $\to$ whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*
  $x \to 1 : xq_V(x) \sim (1-x)^3$
  quark counting rules
  $x \to 0 : xq_V(x) \sim x^{0.5}$
  Regge theory
- sea quarks ($u_S = 2\bar{u}, \dots$) fairly *soft* (low-momentum)
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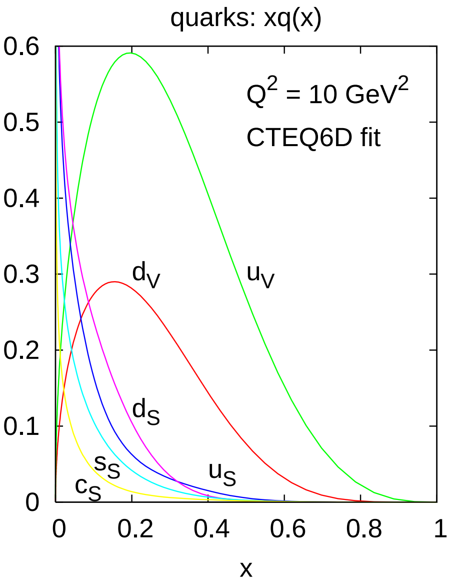

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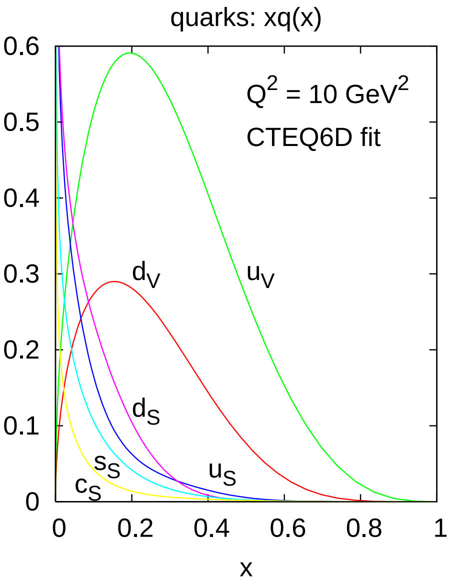

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Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^\pm$.

NB: need to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting...

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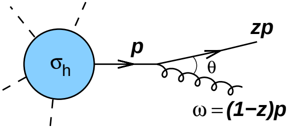
Recall final-state splitting

Tuesday's lecture: calculated $q \to qg$ $(\theta \ll 1,\ E \ll p)$ for final state of arbitrary hard process (σ_h):

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta^2}{\theta^2}$$



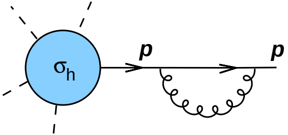
Rewrite with different kinematic variables

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

$$E = (1-z)p$$
$$k_t = E \sin\theta \simeq E\theta$$

If we avoid distinguishing $q+g$ final state from q (infrared-collinear safety), then divergent real and virtual corrections *cancel*

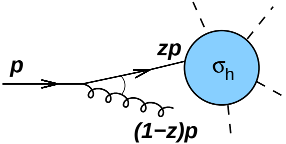
$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



Initial-state splitting

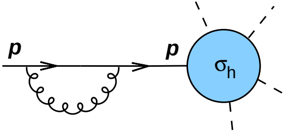
For initial state splitting, hard process occurs *after splitting*, and momentum entering hard process is modified: $p \to zp$.

$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



For virtual terms, momentum entering hard process is unchanged

$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



Total cross section gets contribution with *two different hard X-sections*

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\int \frac{dk_t^2}{k_t^2}\frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]$$

NB: We assume σ_h involves momentum transfers $\sim Q \gg k_t$, so ignore extra transverse momentum in σ_h

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

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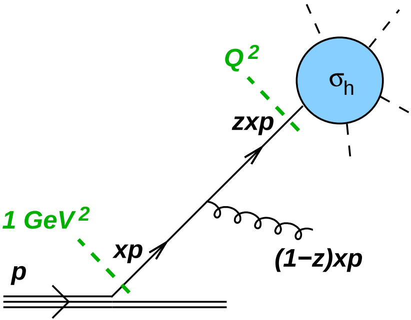
Collinear cutoff



By what right did we go to $k_t = 0$?

We assumed pert. QCD to be valid for all scales, but *below* 1 GeV *it becomes non-perturbative*.

Cut out this divergent region, & instead put non-perturbative quark distribution in proton.

$$\sigma_0 = \int dx\, \sigma_h(xp)\, q(x, 1\text{ GeV}^2)$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{1\text{ GeV}^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\, dz}{1-z} \left[\sigma_h(zxp) - \sigma_h(xp)\right] q(x, 1\text{ GeV}^2)}_{\text{finite}}$$

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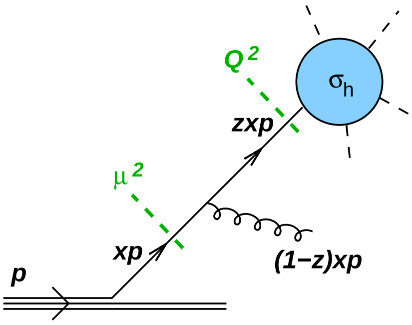

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- Collinear divergence for incoming partons *not cancelled* by virtuals.

  Real and virtual have different longitudinal momenta

- Situation analogous to renormalization: need to *regularize* (but in IR instead of UV).

  Technically, often done with dimensional regularization

- Physical sense of regularization is to separate *(factorize)* proton non-perturbative dynamics from perturbative hard cross section.

  Choice of factorization scale, μ^2, is arbitrary between 1 GeV^2 and Q^2

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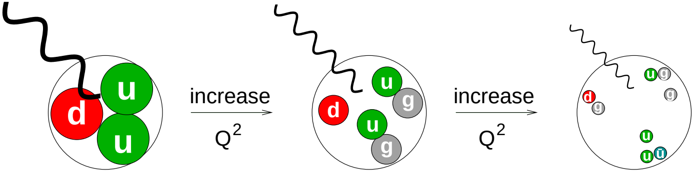
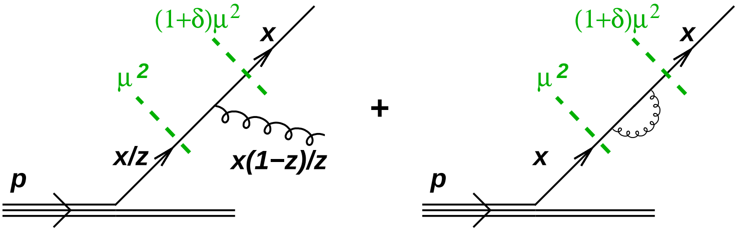
  Dokshizer Gribov Lipatov Altarelli Parisi equations (DGLAP)



DGLAP equation ($q \leftarrow q$)

Change convention: (a) now *fix outgoing* longitudinal momentum x; (b) *take derivative* wrt factorization scale μ^2



$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\, \frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F \frac{1+z^2}{1-z}$

Until now we approximated it in soft ($z \to 1$) limit, $p_{qq} \simeq \frac{2C_F}{1-z}$

DGLAP rewritten

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi} \underbrace{\int_x^1 dz\, P_{qq}(z)\, \frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F \left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_0^1 dz\, [g(z)]_+ f(z) = \int_0^1 dz\, g(z)\, f(z) - \int_0^1 dz\, g(z)\, f(1)$$

$z = 1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z = 1$

DGLAP flavour structure

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

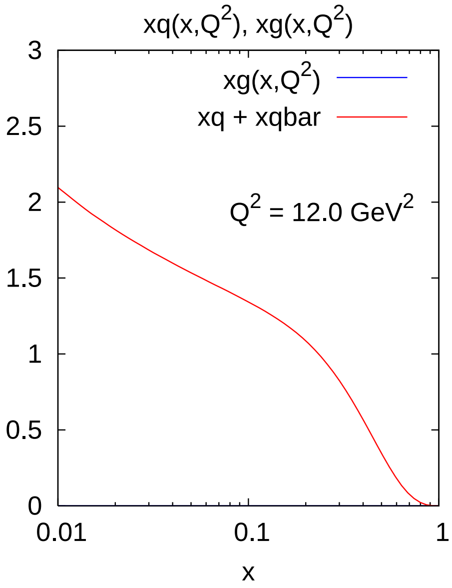
$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

Effect of DGLAP (initial quarks)



Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

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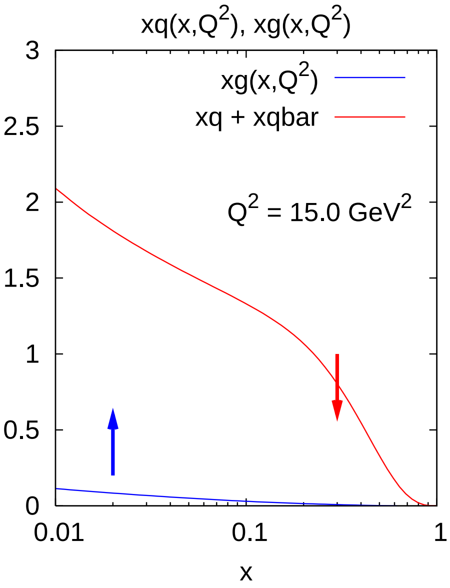

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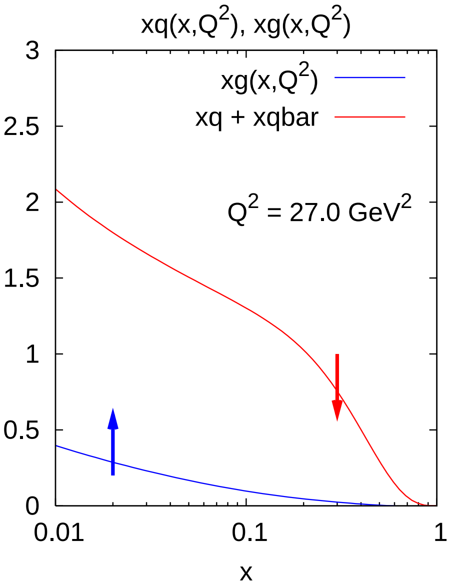

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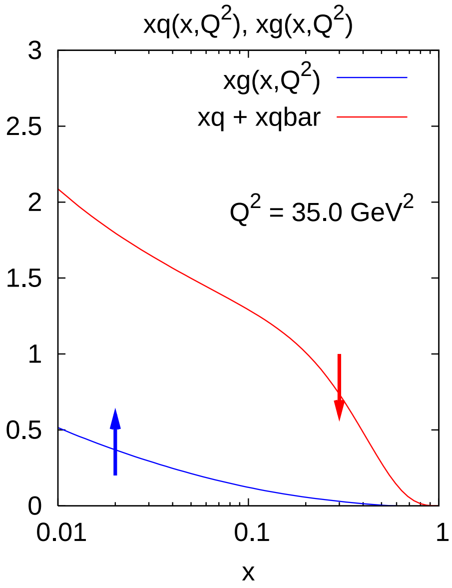

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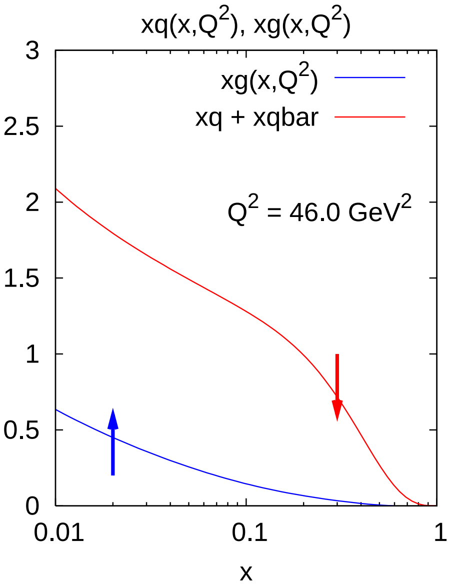

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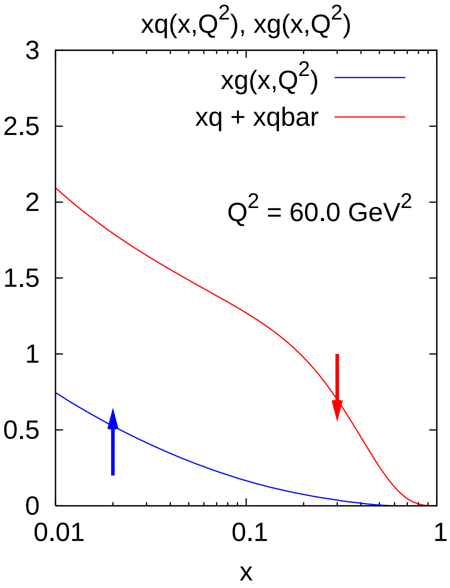

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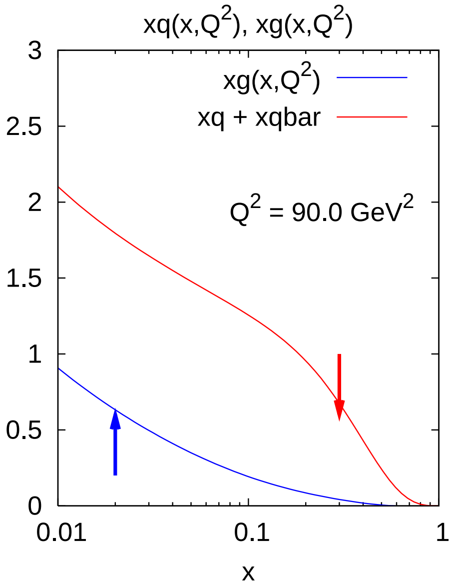

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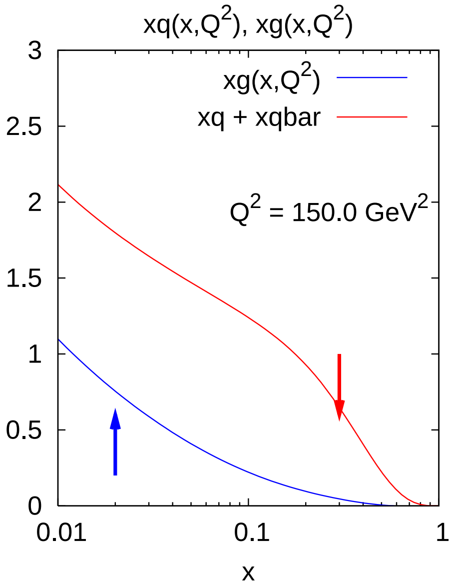

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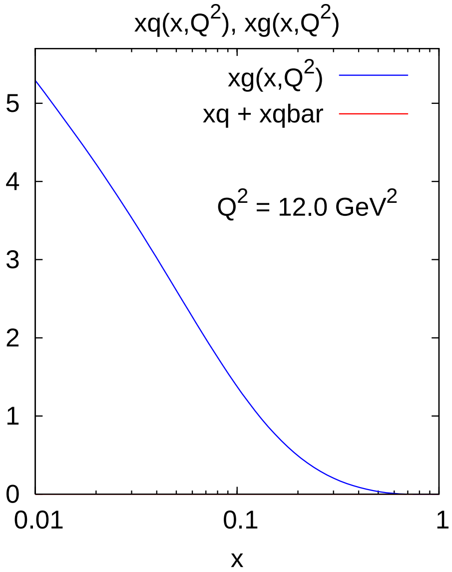

2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q \leftarrow g} \otimes g$$
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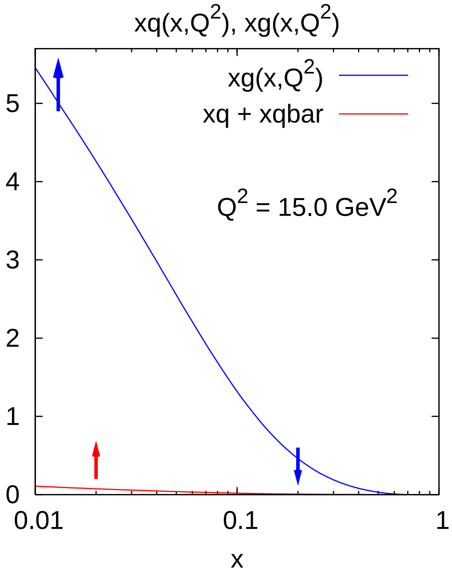

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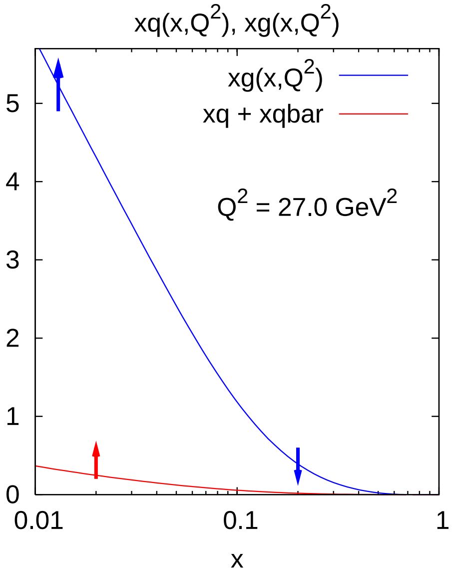

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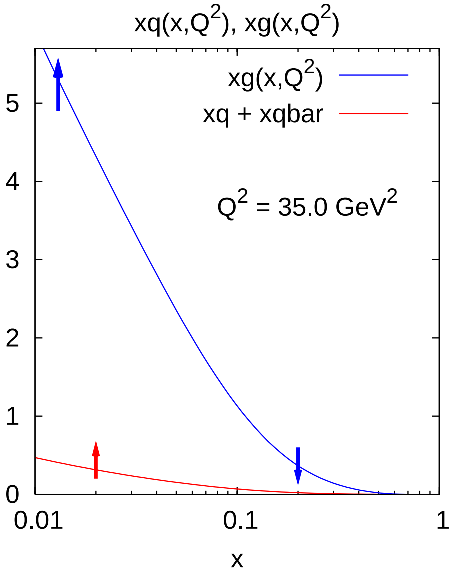

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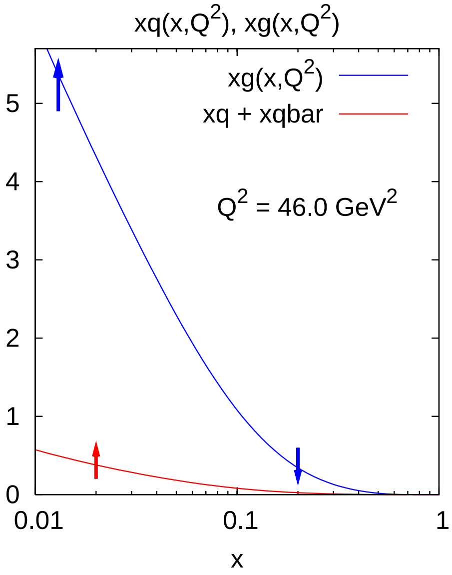

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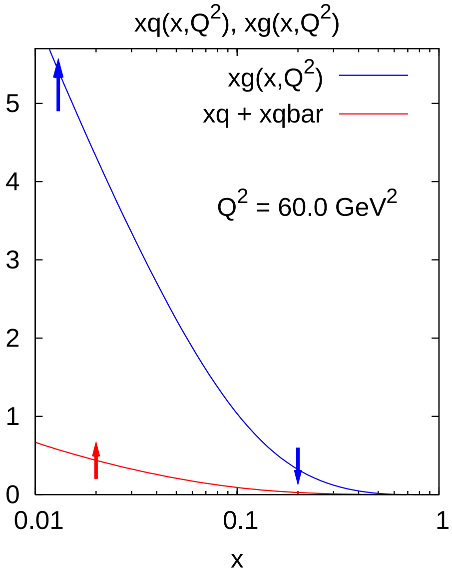

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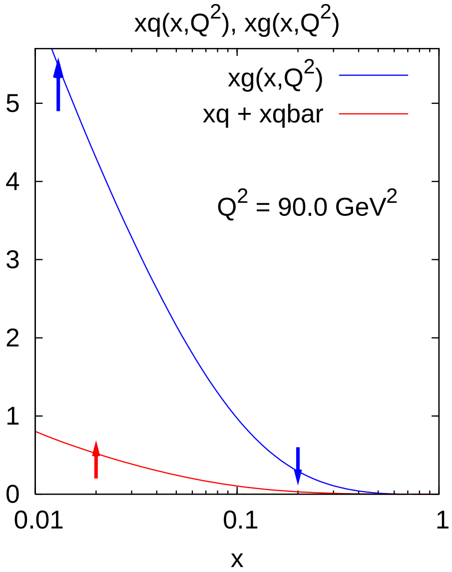

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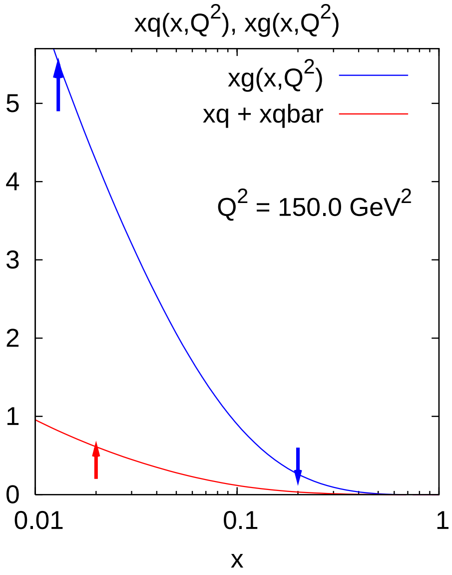

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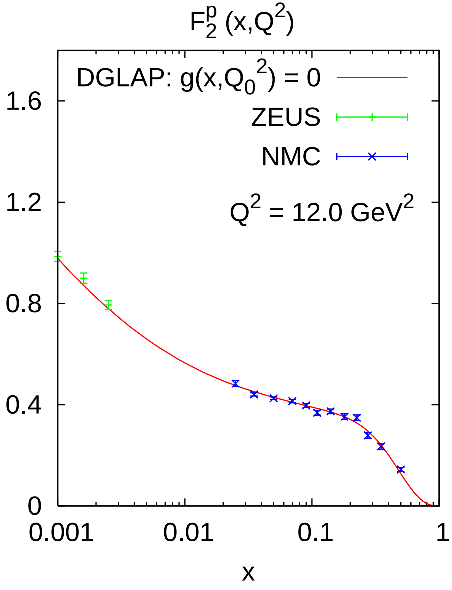
DGLAP evolution

- As Q^2 increases, partons lose longitudinal momentum; distributions all shift to lower x.
- gluons can be seen because they help drive the quark evolution.

Now consider data

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

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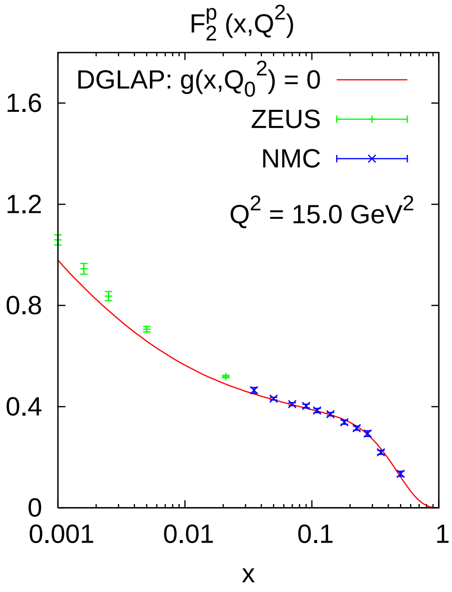

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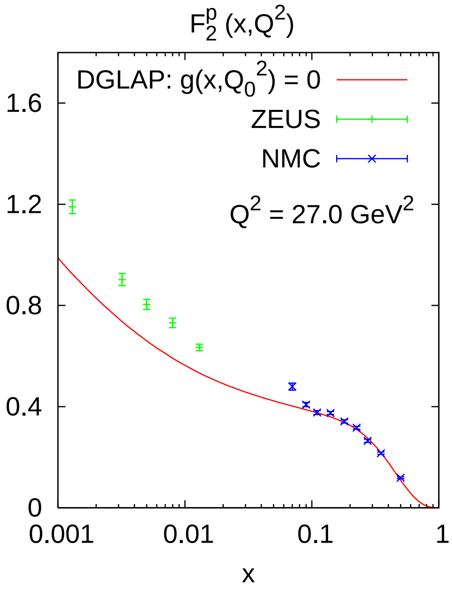

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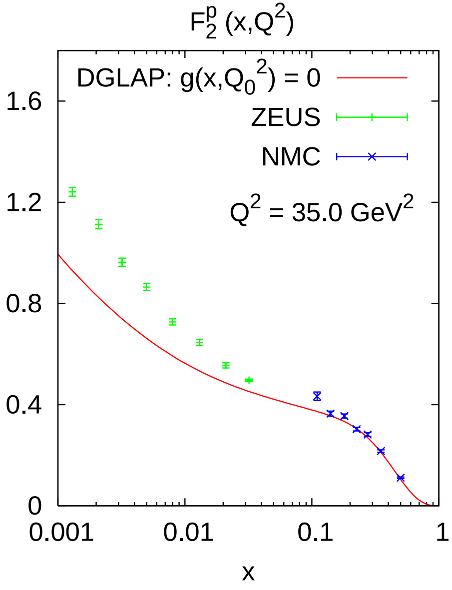

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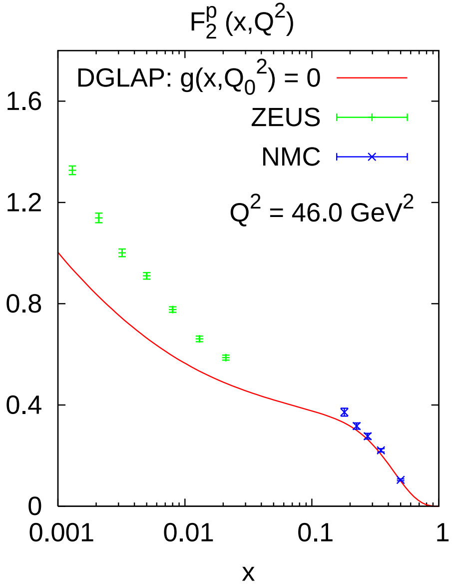

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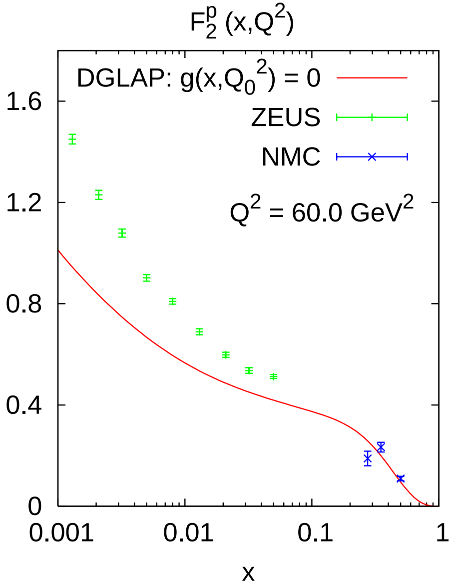

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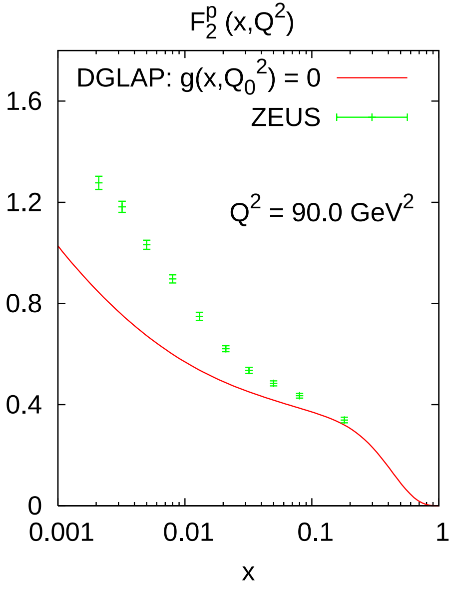

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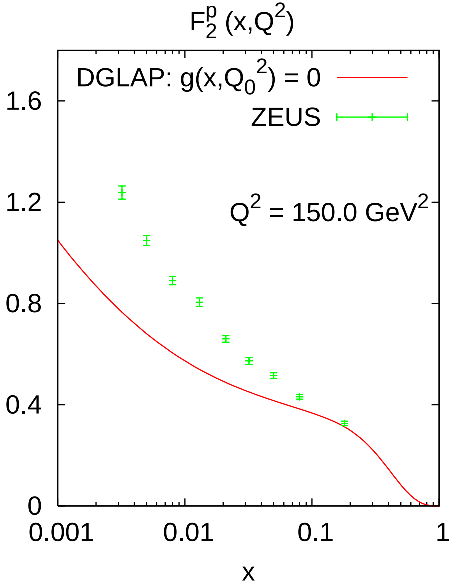

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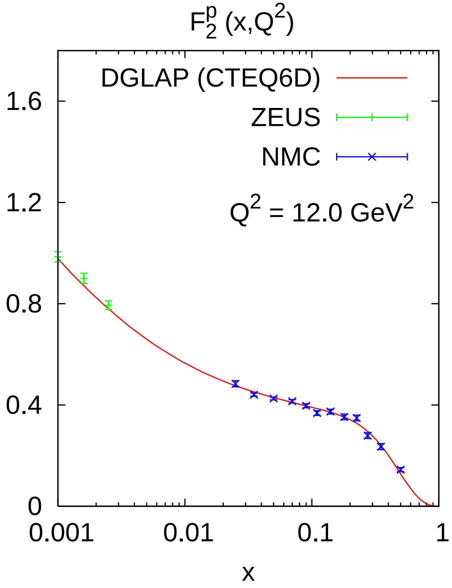

If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

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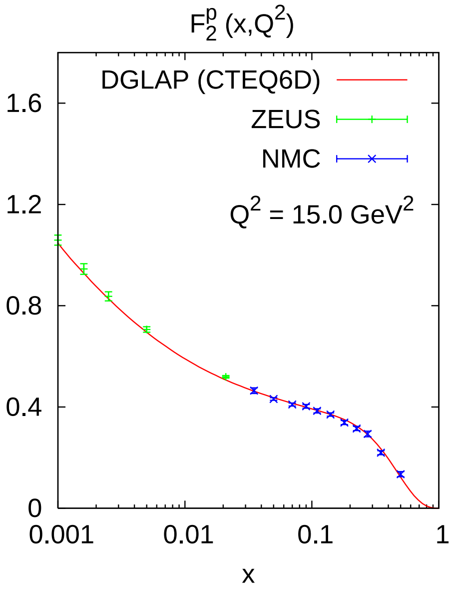

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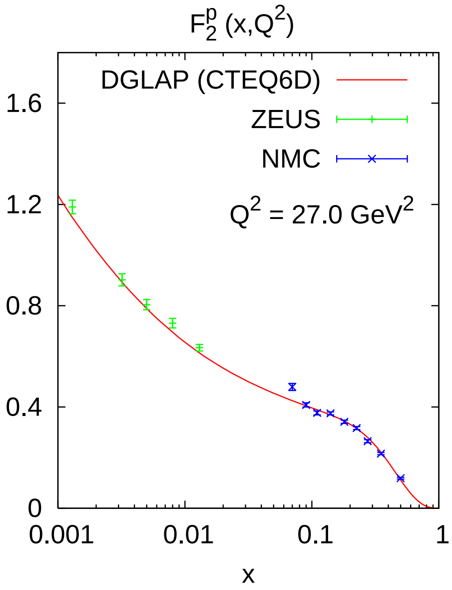

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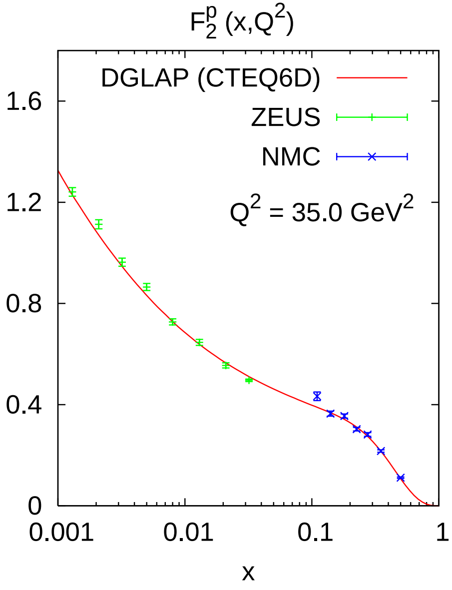

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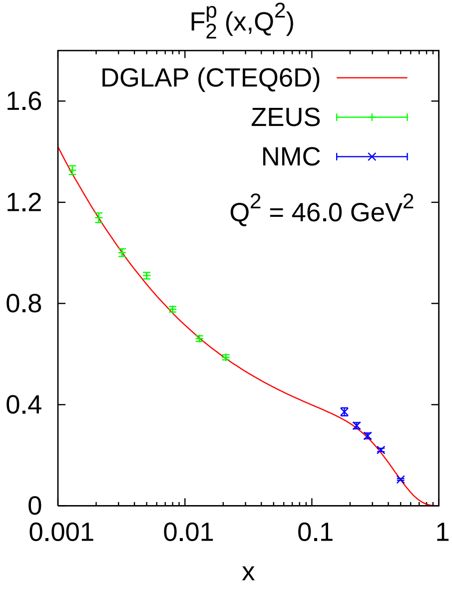

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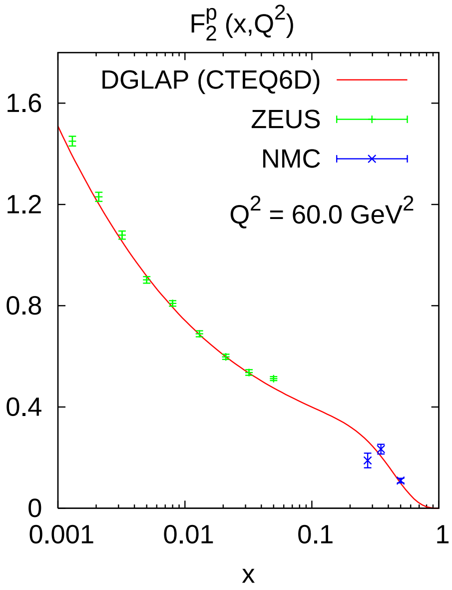

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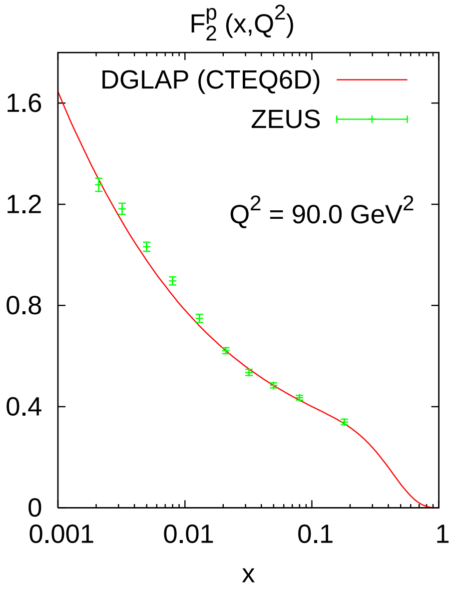

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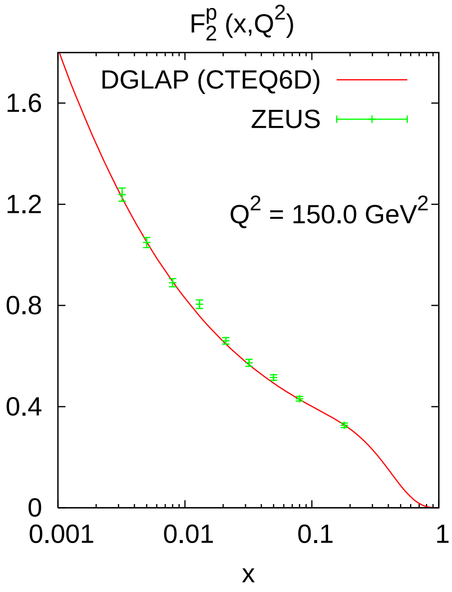

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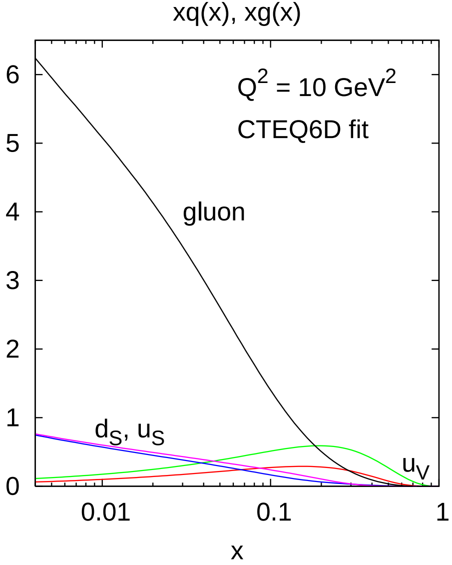
Gluon distribution



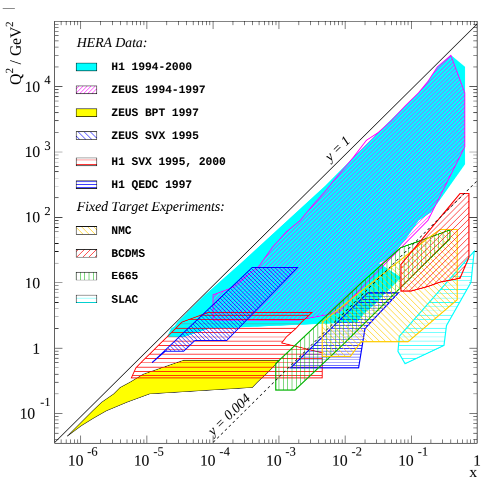
Gluon distribution is **HUGE!**

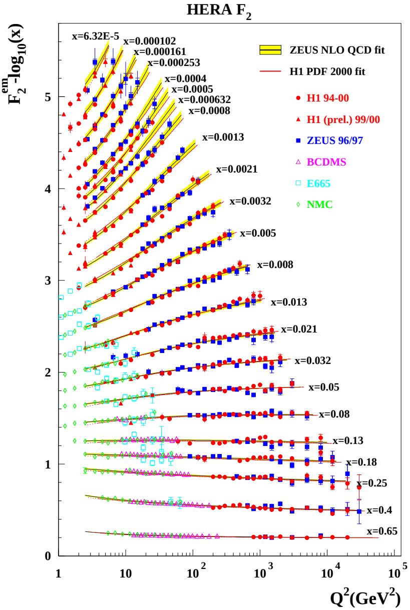
Can we really trust it?

- Consistency: momentum sum-rule is now *satisfied*.

  NB: gluon mostly at small x
- Agrees with vast range of data

DIS data and global fits





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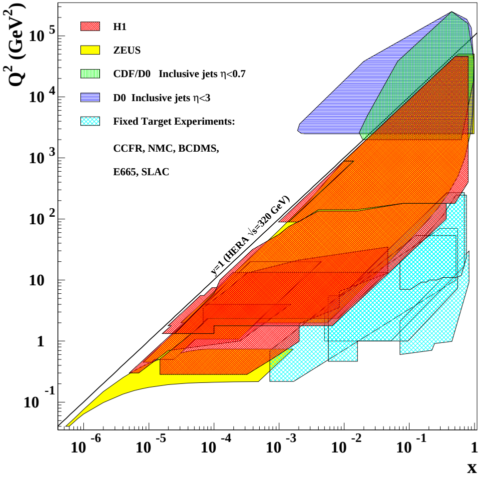
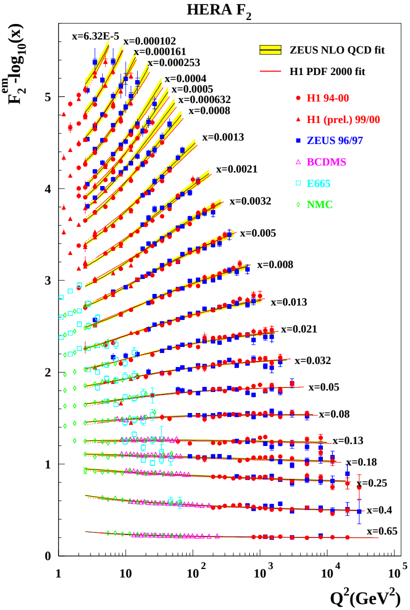
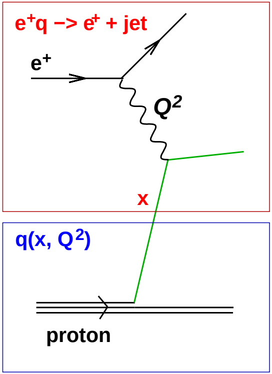


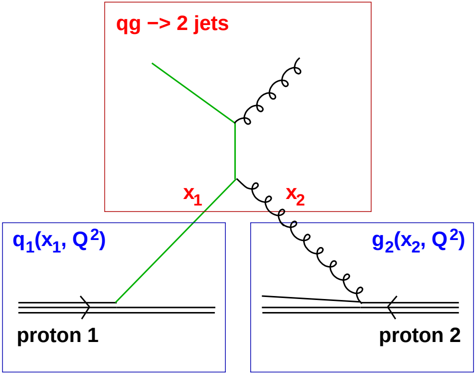


Crucial check: other processes

Factorization of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**



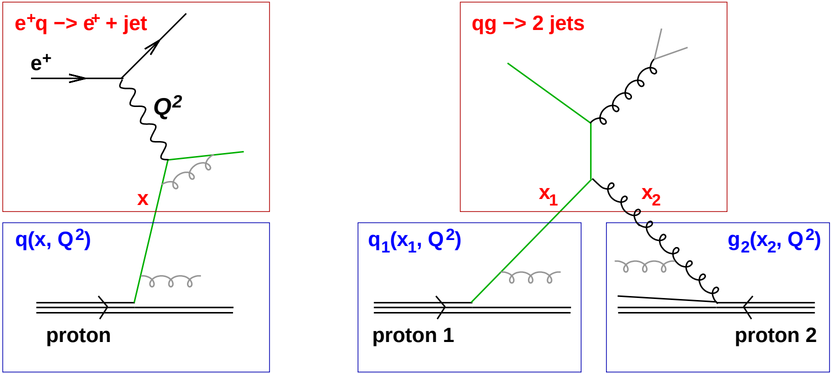


$$\sigma_{ep} = \sigma_{eq} \otimes q \qquad \sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

Crucial check: other processes

Factorization of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**



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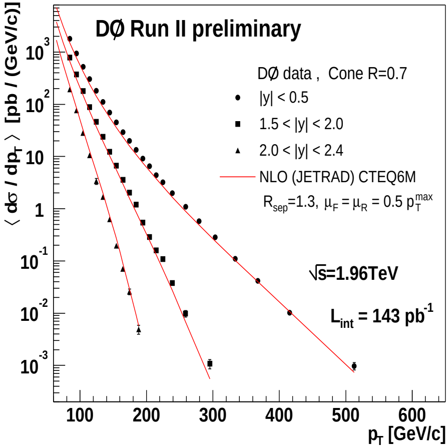
Jet production in $p\bar{p}$



Jet production in proton-antiproton collisions is *good test of large gluon distribution,* since there are large direct contributions from

$$gg \to gg\,, \qquad qg \to qg$$

NB: more complicated to interpret than DIS, since many channels, and x_1, x_2 dependence.

$p_T \sim \sqrt{x_1 x_2 s}$ jet transverse mom.

$\sim Q$

$$y \sim \frac{1}{2} \log \frac{x_1}{x_2}$$

$y = \log \tan \frac{\theta}{2}$

jet angle wrt $p\bar{p}$ beams

Good agreement confirms factorization

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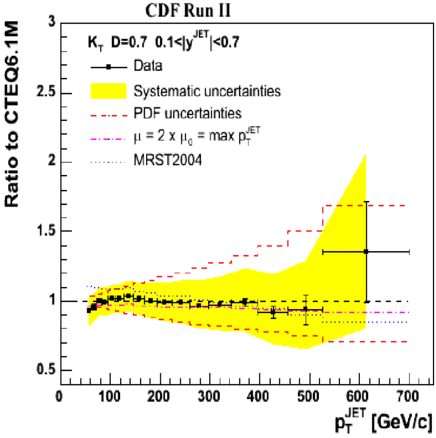

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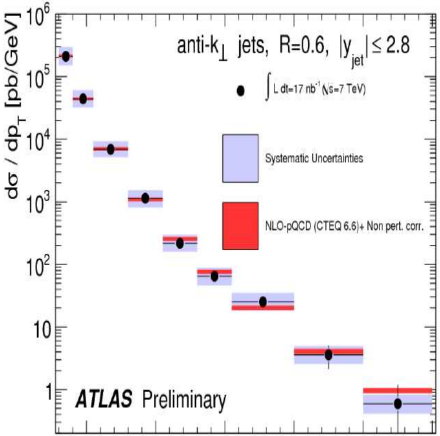

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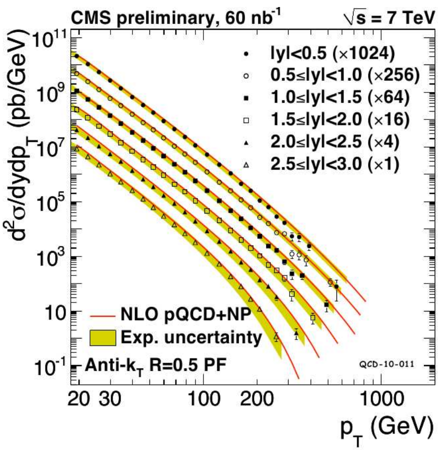

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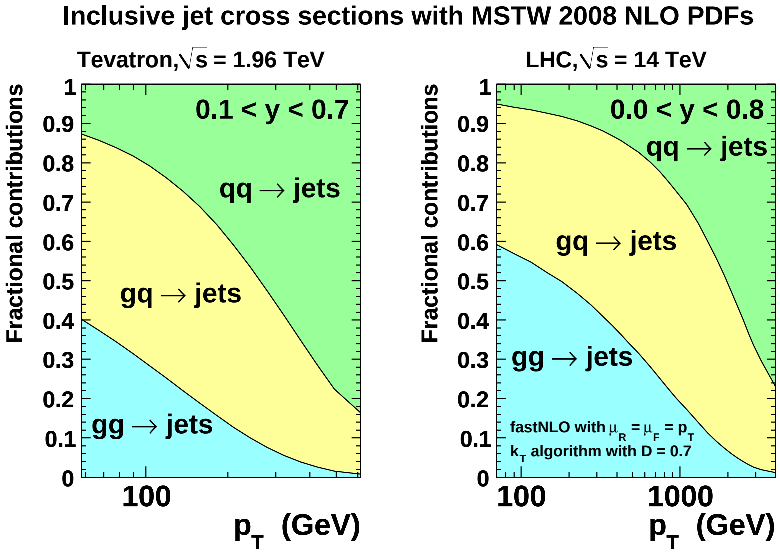
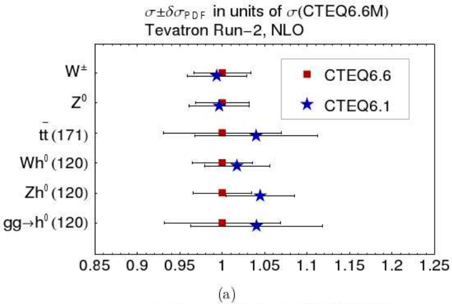
Which PDF channels contribute?

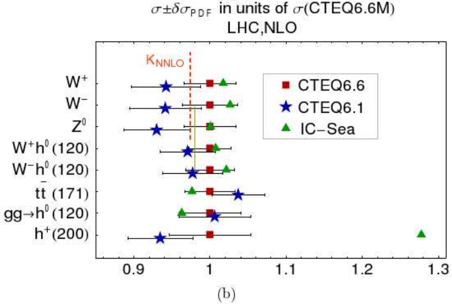


A large fraction of jets are gluon-induced

Uncertainties on predictions



(a)



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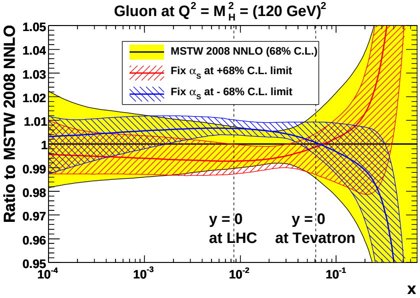


General message

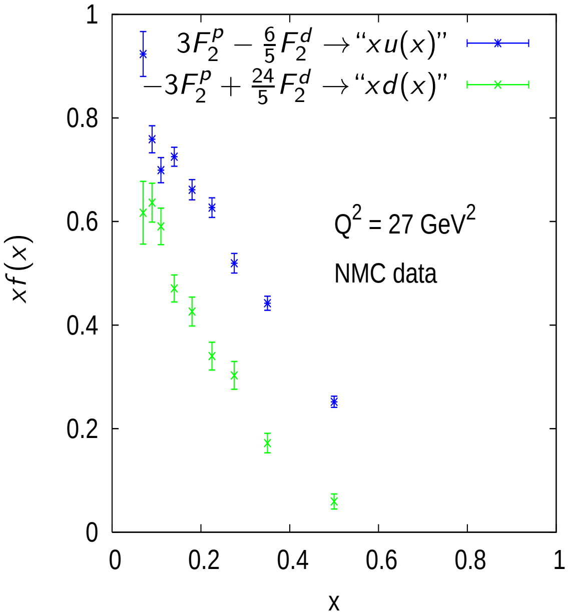
Data-related errors on PDFs are such that uncertainties are just a few % for many key Tevatron and LHC observables

Conclusions on PDFs

- Experiments tell us that proton really is what we expected (*uud*)
- Plus lots more: large number of 'sea quarks' ($q\bar{q}$), gluons (50% of momentum)
- *Factorization* is key to usefulness of PDFs
  - Non-trivial beyond lowest order
  - PDFs depend on factorization scale, evolve with *DGLAP equation*
  - Pattern of *evolution gives us info on gluon* (otherwise hard to measure)
  - PDFs really are universal!
- *Precision* of data & QCD calculations is striking.
- Crucial for understanding future signals of *new particles,* e.g. Higgs Boson production at LHC.

EXTRAS

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

How much u and d?

- Total $U = \int dx\, u(x)$
- $F_2 = x(\frac{4}{9}u + \frac{1}{9}d)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

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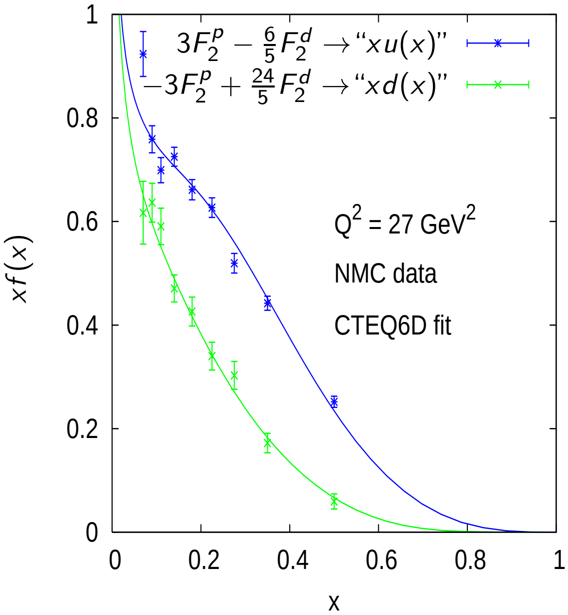

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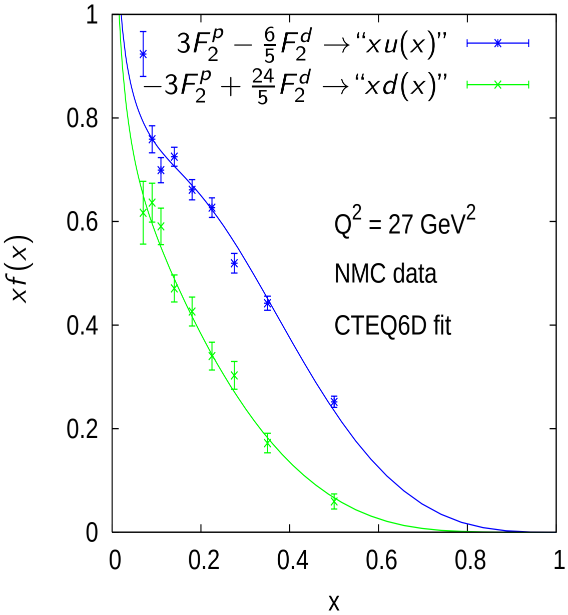

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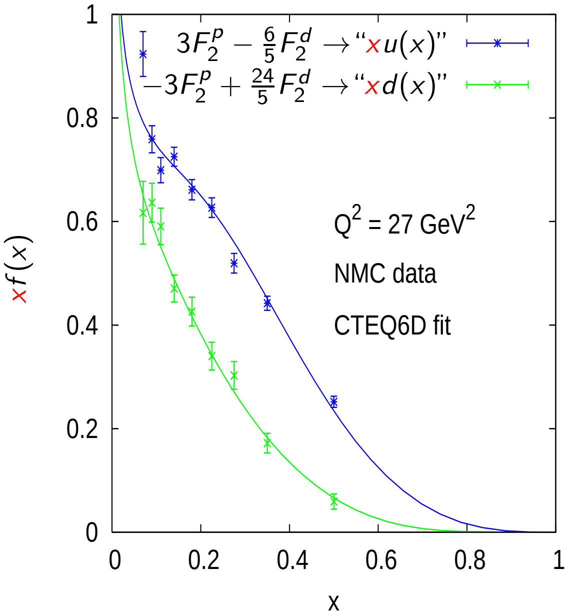

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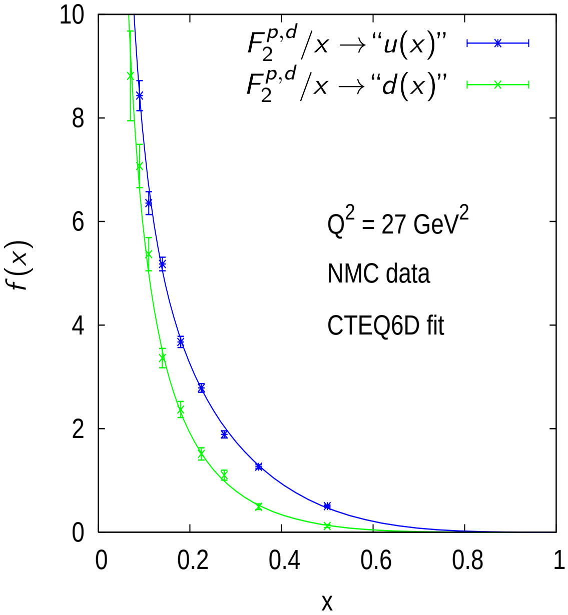

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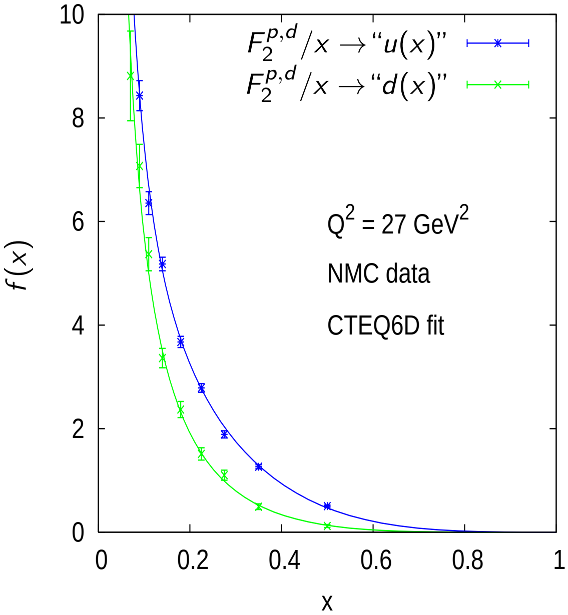

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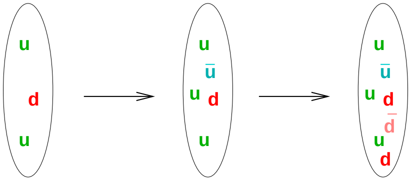
Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction ∼ square of charge → +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\, (u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

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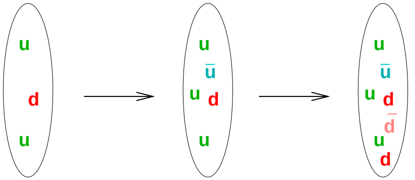

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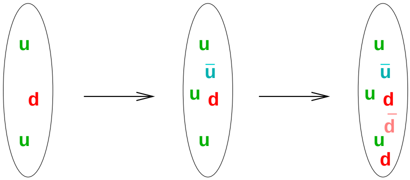

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"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both → no good. . .

Question: what interacts differently with particle & antiparticle?

Answer: W^+ *or* W^-

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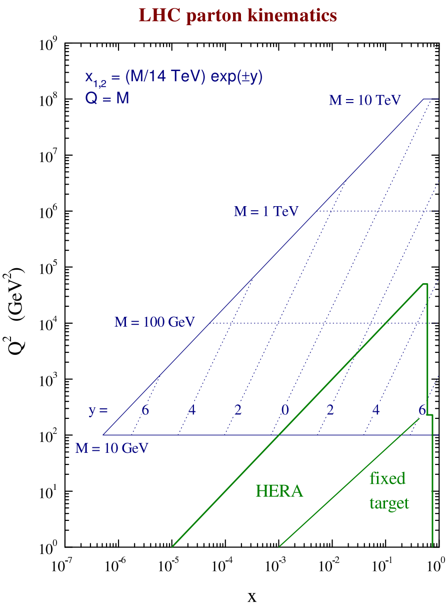
Taking PDFs from HERA to LHC



Suppose we produce a system of mass M at LHC from partons with momentum fractions x_1, x_2:

- $M = \sqrt{x_1 x_2 s}$
- rapidity $y = \dfrac{1}{2} \ln \dfrac{x_1}{x_2}$

pseudorapidity $\equiv \eta \equiv \ln \tan \frac{\theta}{2}$
$=$ rapidity for massless objects
$\lesssim 5$ at LHC

Are PDFs being used in region where measured?

Only partial kinematic overlap

- DGLAP evolution is **essential** for the prediction of PDFs in the LHC domain.

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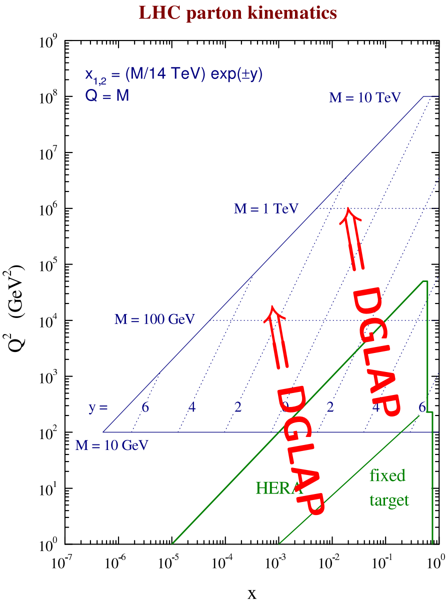

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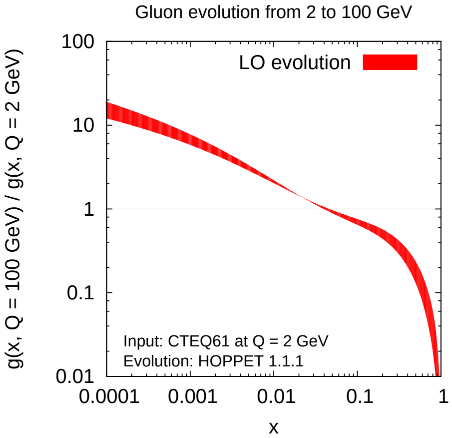
By how much do PDFs evolve?



Illustrate for the gluon distribution

Here using fixed Q scales

But for HERA $\rightarrow$ LHC

relevant Q range is x-dependent

- See factors $\sim 0.1 - 10$
- Remember: LHC involves product of two parton densities

It's crucial to get this right!

Without DGLAP evolution, you couldn't predict anything at LHC

It's not enough for data-related errors to be small.

DGLAP evolution must also be well constrained.

So evolution must be done with more than just leading-order DGLAP splitting functions

Higher-order calculations

Earlier, we saw leading order (LO) DGLAP splitting functions, $P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)}$:

$$P_{qq}^{(0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right] ,$$

$$P_{qg}^{(0)}(x) = T_R \left[x^2 + (1-x)^2 \right] ,$$

$$P_{gq}^{(0)}(x) = C_F \left[\frac{1+(1-x)^2}{x} \right] ,$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \frac{(11C_A - 4n_f T_R)}{6} .$$

Higher-order calculations

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski & Petronzio '80

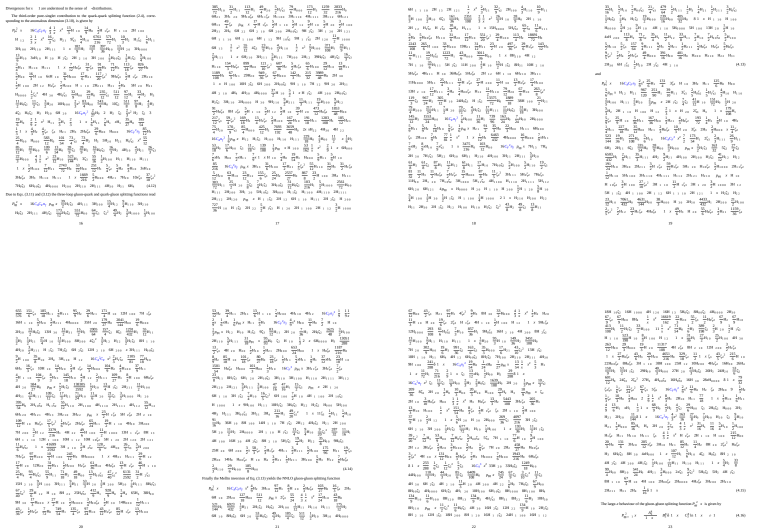
$$P^{(1)}_{\rm ps}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\right] + (1+x)\left[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right]\right)$$

$$\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)\mathrm{H}_{-1,0} - 2p_{\rm qg}(x)\mathrm{H}_{1,1} + x^2\left[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\right]\right.\\
&\left. +4(1-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\right] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\right) + 4\, C_F n_f \left(2p_{\rm qg}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2\right.\right.\\
&\left.\left. -\zeta_2\right] + 4x^2\left[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\right] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\right)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\right] - x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2\right.\\
&\left. -7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\left[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x)\mathrm{H}_{-1,0}\right) - 4\, C_F n_f \left(\frac{2}{3}x\right.\\
&\left. -p_{\rm gq}(x)\left[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\right]\right) + 4\, {C_F}^2 \left(p_{\rm gq}(x)\left[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\right] + (1+x)\left[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\right] - 3\mathrm{H}_{0,0}\right.\\
&\left. +1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x\right)
\end{aligned}$$

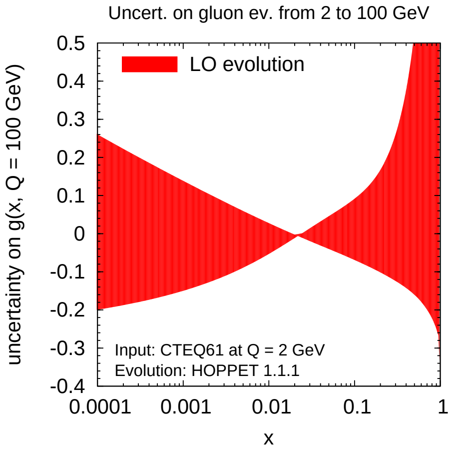
$$\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\, {C_A}^2 \left(27\right.\\
& +(1+x)\left[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12\mathrm{H}_0\\
& - \frac{44}{3}x^2\mathrm{H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\right] + \delta(1-x)\left.\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\, C_F n_f \left(2\mathrm{H}_0\right.\\
&\left. + \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right) .
\end{aligned}$$

NNLO splitting functions



NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

Evolution uncertainty

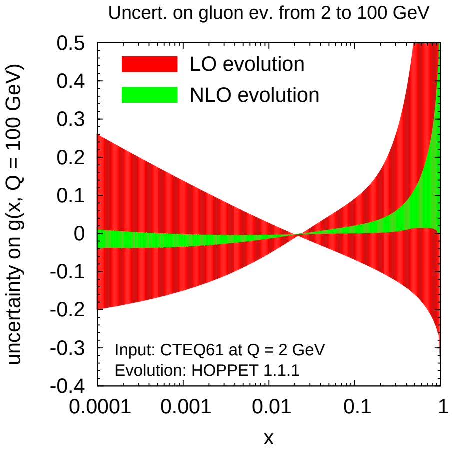


Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

Talk more about such tricks in next lecture

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\rightarrow 2\%$ Commensurate with data uncertainties

Evolution uncertainty



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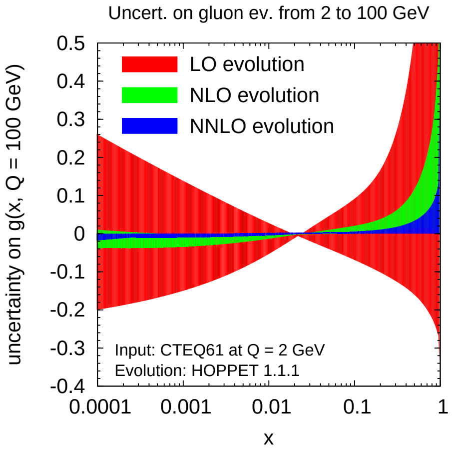
- NNLO $\rightarrow 2\%$ Commensurate with data uncertainties

Evolution uncertainty



Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

Talk more about such tricks in next lecture

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\to 2\%$ Commensurate with data uncertainties