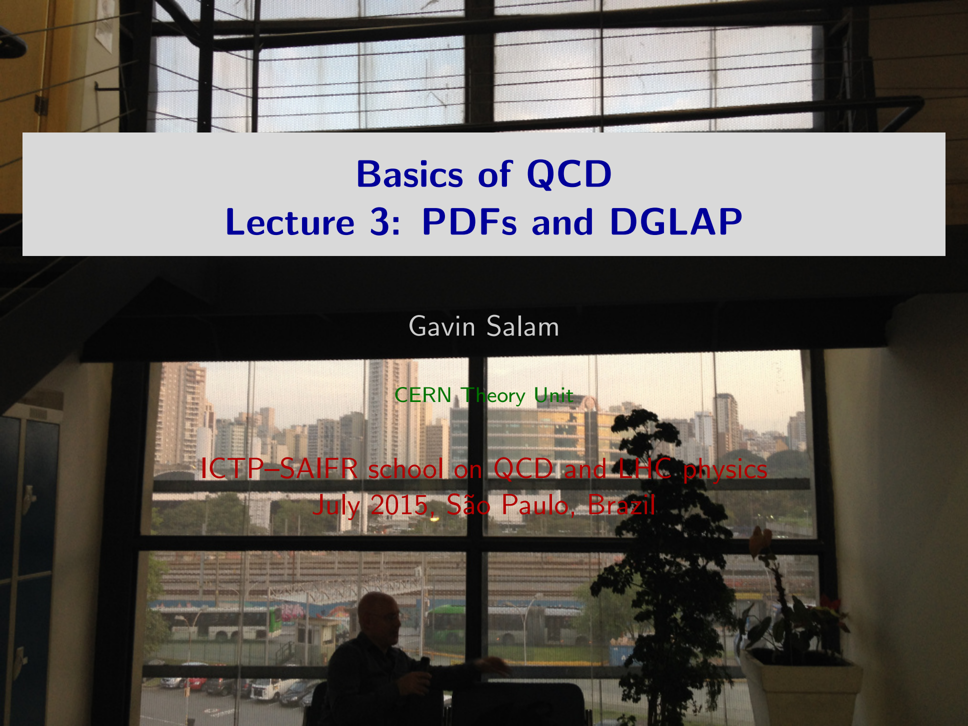


Basics of QCD
Lecture 3: PDFs and DGLAP

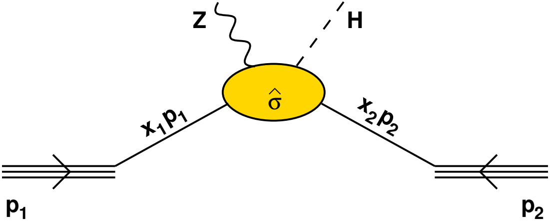
Gavin Salam

CERN Theory Unit

ICTP–SAIFR school on QCD and LHC physics
July 2015, São Paulo, Brazil

Factorization & parton distributions

Cross section for some hard process in hadron-hadron collisions



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- Total X-section is *factorized* into a 'hard part' $\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$ and 'normalization' from parton distribution functions (PDF).
- Measure total cross section ↔ *need to know PDFs* to be able to test hard part (e.g. Higgs electroweak couplings).
- Picture seems intuitive, but
  - **how can we determine the PDFs?** NB: non-perturbative
  - **does picture really stand up to QCD corrections?**

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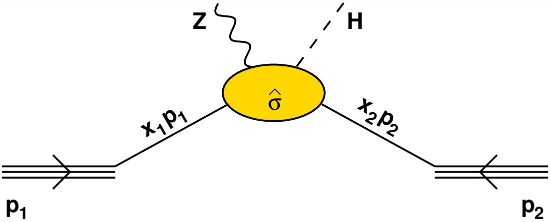

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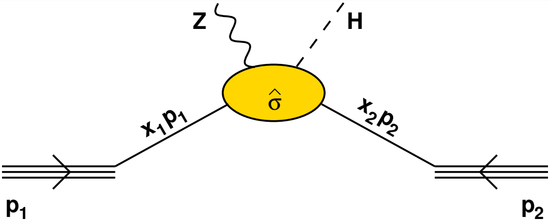

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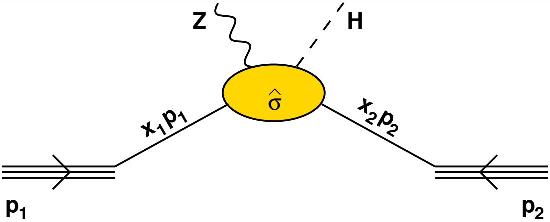

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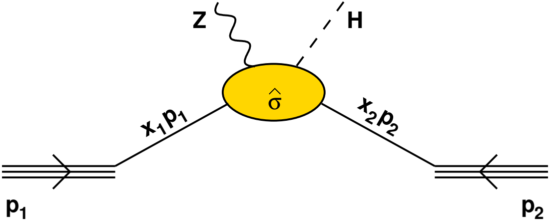

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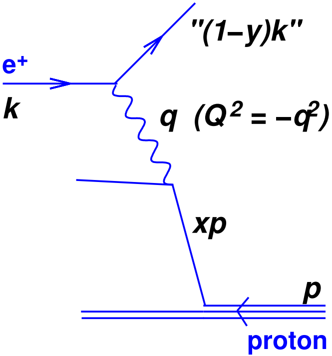
Deep Inelastic Scattering: kinematics

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



Kinematic relations:

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality ↔ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
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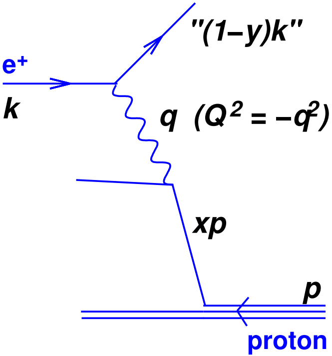

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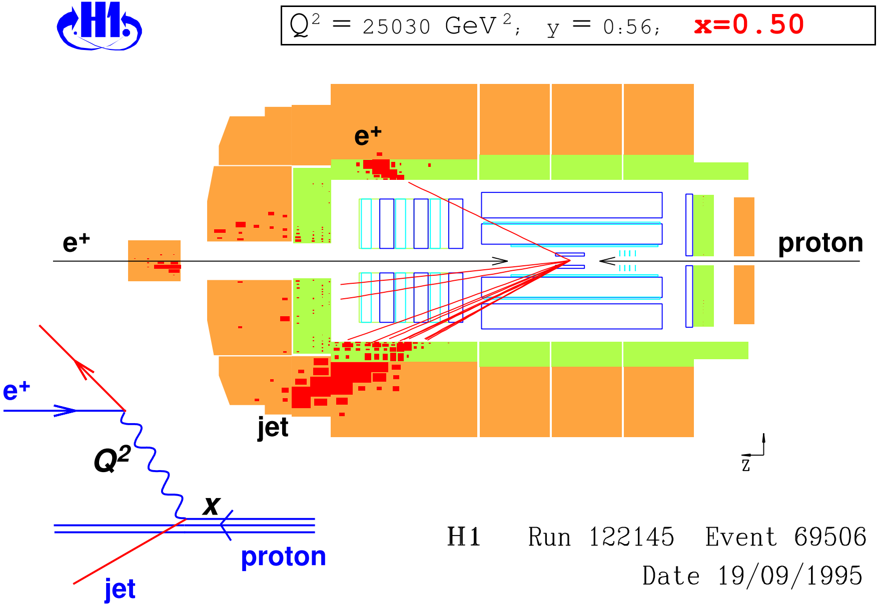
Deep Inelastic scattering (DIS): example



E.g.: extracting u & d distributions

Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

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F_2 gives us *combination* of u and d.
How can we extract them separately?

Extracting full flavour structure?

- Using neutrons and **isospin**

$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x)$$

- Using charged-current ($W^\pm$) scattering

  [neutrinos instead of electrons in initial or final-state]

  - W^+ interacts only with $d, \bar{u}$
  - angular structure of interaction differs between d and $\bar{u}$

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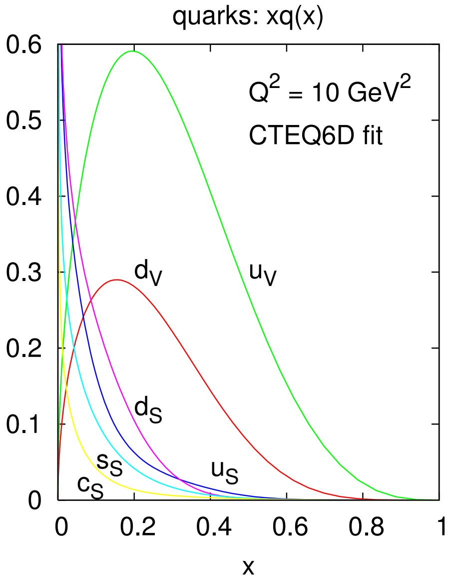

These & other methods $\rightarrow$ whole set of quarks & antiquarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*
  $x \rightarrow 1 : xq_V(x) \sim (1-x)^3$
  quark counting rules
  $x \rightarrow 0 : xq_V(x) \sim x^{0.5}$
  Regge theory
- sea quarks ($u_S = 2\bar{u}, \ldots$) fairly *soft* (low-momentum)
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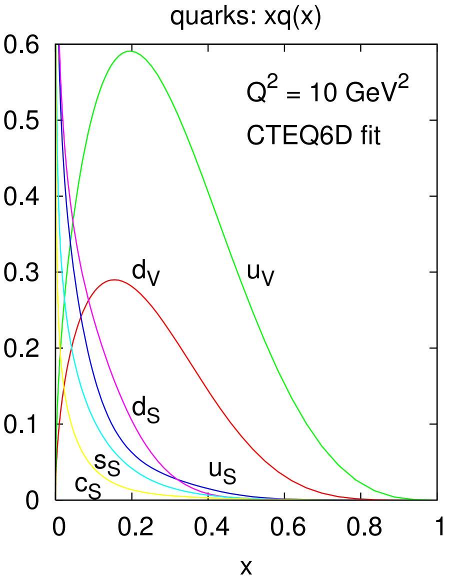

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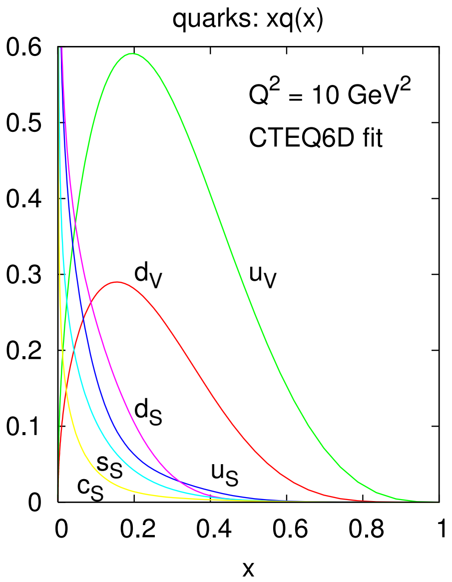

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Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Only parton type we've neglected so far is the

gluon

Not directly probed by photon or $W^\pm$.

NB: it's crucial to know it for $gg \to H$

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD splitting. . .

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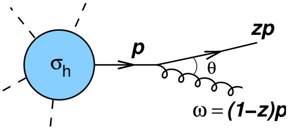
Recall final-state splitting

Yesterday: calculated $q \to qg$ $(\theta \ll 1,\ E \ll p)$ for final state of arbitrary hard process (σ_h):

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta^2}{\theta^2}$$



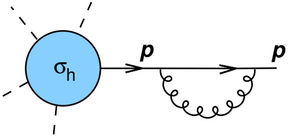
Rewrite with different kinematic variables

$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

$$E = (1-z)p$$
$$k_t = E\sin\theta \simeq E\theta$$

If we avoid distinguishing $q+g$ final state from q (infrared-collinear safety), then divergent real and virtual corrections *cancel*

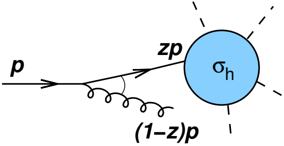
$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



Initial-state splitting

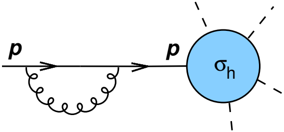
For initial state splitting, hard process occurs *after splitting*, and momentum entering hard process is modified: $p \to zp$.

$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



For virtual terms, momentum entering hard process is unchanged

$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



Total cross section gets contribution with *two different hard X-sections*

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\int \frac{dk_t^2}{k_t^2}\frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]$$

NB: We assume σ_h involves momentum transfers $\sim Q \gg k_t$, so ignore extra transverse momentum in σ_h

Initial-state collinear divergence

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles
So how do we do QCD calculations in such cases?

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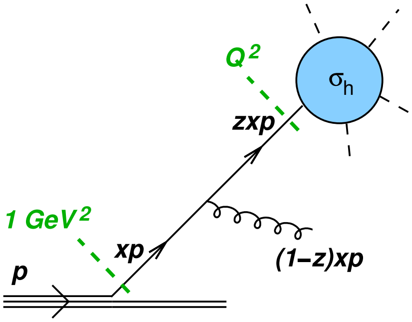
Collinear cutoff



By what right did we go to $k_t = 0$?

We assumed pert. QCD to be valid for all scales, but *below* 1 GeV *it becomes non-perturbative*.

Cut out this divergent region, & instead put non-perturbative quark distribution in proton.

$$\sigma_0 = \int dx\; \sigma_h(xp)\; q(x, 1\text{ GeV}^2)$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{1\text{ GeV}^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\, dz}{1-z} \left[\sigma_h(zxp) - \sigma_h(xp)\right] q(x, 1\text{ GeV}^2)}_{\text{finite}}$$

In general: replace 1 GeV^2 cutoff with arbitrary *factorization scale* μ_F^2.

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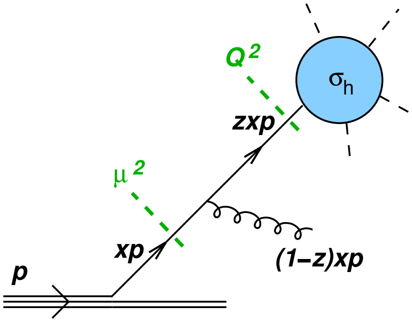

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- Collinear divergence for incoming partons *not cancelled* by virtuals.

  Real and virtual have different longitudinal momenta

- Situation analogous to renormalization: need to *regularize* (but in IR instead of UV).

  Technically, often done with dimensional regularization

- Physical sense of regularization is to separate *(factorize)* proton non-perturbative dynamics from perturbative hard cross section.

  Choice of factorization scale, μ^2, is arbitrary between 1 GeV^2 and Q^2

- In analogy with running coupling, we can *vary factorization scale* and get a *renormalization group equation* for parton distribution functions.

  Dokshizer Gribov Lipatov Altarelli Parisi equations (DGLAP)

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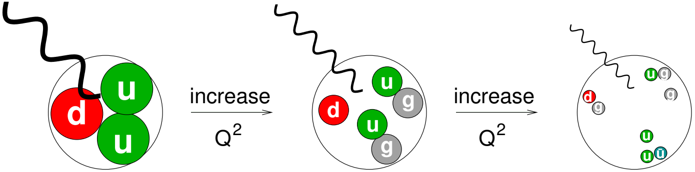
- Physical sense of regularization is to separate *(factorize)* proton non-perturbative dynamics from perturbative hard cross section.

  Choice of factorization scale, μ^2, is arbitrary between 1 GeV^2 and Q^2

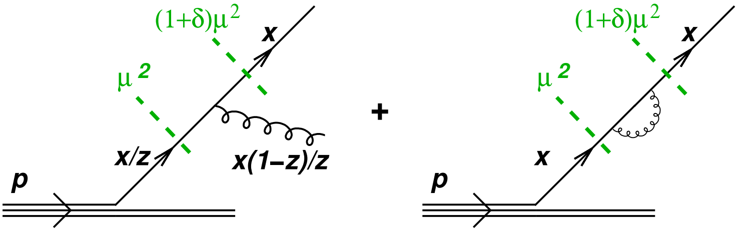
- In analogy with running coupling, we can *vary factorization scale* and get a *renormalization group equation* for parton distribution functions.

  Dokshizer Gribov Lipatov Altarelli Parisi equations (DGLAP)



DGLAP equation ($q \leftarrow q$)

Change convention: (a) now *fix outgoing* longitudinal momentum x; (b) *take derivative* wrt factorization scale μ^2



$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\, \frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F \frac{1+z^2}{1-z}$

Until now we approximated it in soft ($z \to 1$) limit, $p_{qq} \simeq \frac{2C_F}{1-z}$

DGLAP rewritten

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi} \underbrace{\int_x^1 dz\, P_{qq}(z)\, \frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_0^1 dz\, [g(z)]_+\, f(z) = \int_0^1 dz\, g(z)\, f(z) - \int_0^1 dz\, g(z)\, f(1)$$

$z=1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z=1$

DGLAP flavour structure

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

2015 EPS HEP prize to Bjorken, Altarelli, Dokshitzer, Lipatov & Parisi

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Higher-order calculations

NLO:

$$P_{\rm ps}^{(1)}(x) = 4\,C_F n_f\left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\right] + (1+x)\left[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right]\right)$$

$$P_{\rm qg}^{(1)}(x) = 4\,C_A n_f\left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)\mathrm{H}_{-1,0} - 2p_{\rm qg}(x)\mathrm{H}_{1,1} + x^2\left[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\right]\right.$$
$$\left. +4(1-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\right] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\right) + 4\,C_F n_f\left(2p_{\rm qg}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2\right.\right.$$
$$\left.\left. -\zeta_2\right] + 4x^2\left[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\right] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\right)$$

$$P_{\rm gq}^{(1)}(x) = 4\,C_A C_F\left(\frac{1}{x} + 2p_{\rm gq}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\right] - x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2\right.$$
$$\left. -7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\left[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x)\mathrm{H}_{-1,0}\right) - 4\,C_F n_f\left(\frac{2}{3}x\right.$$
$$\left. -p_{\rm gq}(x)\left[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\right]\right) + 4\,C_F^{\,2}\left(p_{\rm gq}(x)\left[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\right] + (1+x)\left[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\right] - 3\mathrm{H}_{0,0}\right.$$
$$\left. +1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x\right)$$

$$P_{\rm gg}^{(1)}(x) = 4\,C_A n_f\left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\,C_A^{\,2}\left(27\right.$$
$$+(1+x)\left[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12\mathrm{H}_0$$
$$\left. -\frac{44}{3}x^2\mathrm{H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\,C_F n_f\left(2\mathrm{H}_0\right.$$
$$\left. +\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right)\,.$$

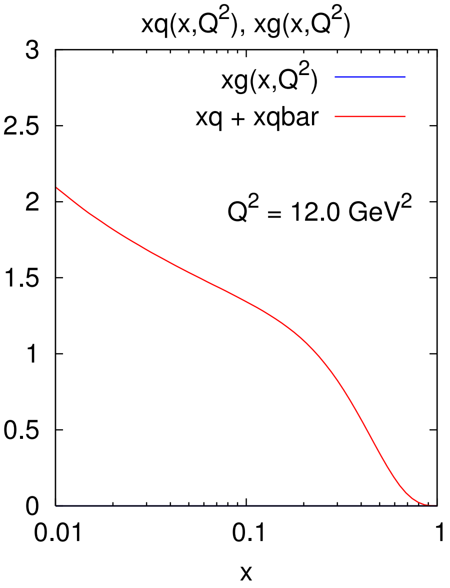
$$P_{ab} = \frac{\alpha_s}{2\pi}P^{(0)} + \frac{\alpha_s^2}{16\pi^2}P^{(1)}$$

Curci, Furmanski & Petronzio '80

NNLO splitting functions

NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

Effect of (LO) DGLAP: initial quarks



Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

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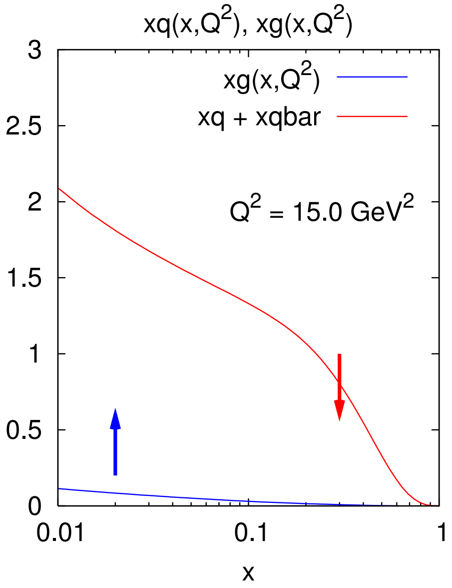

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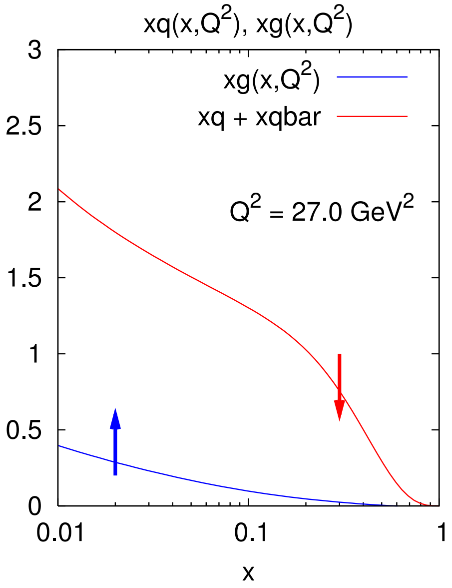

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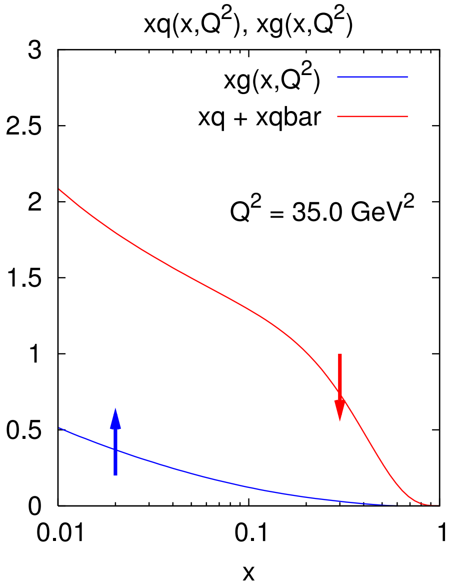

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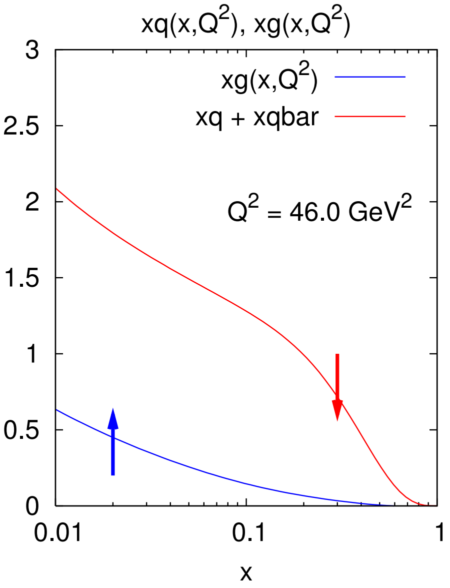

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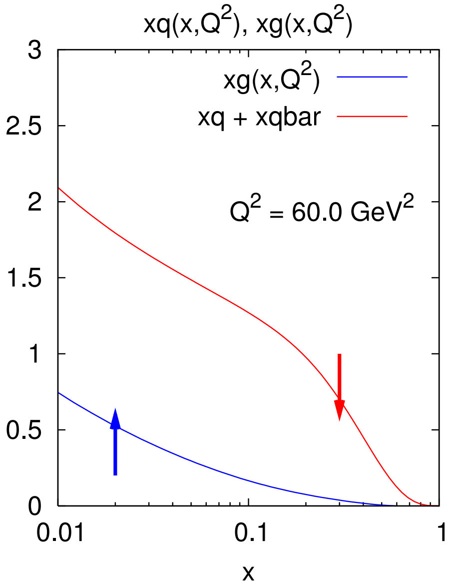

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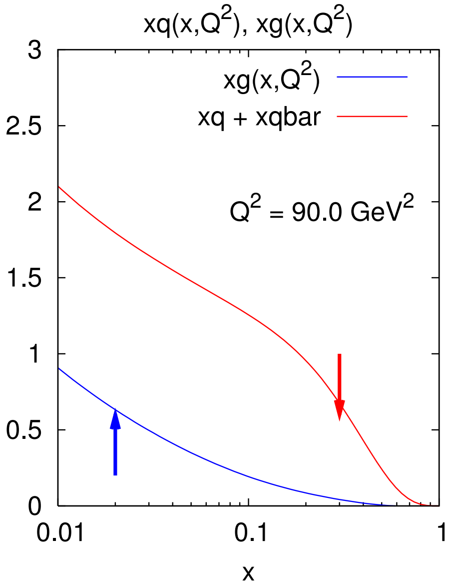

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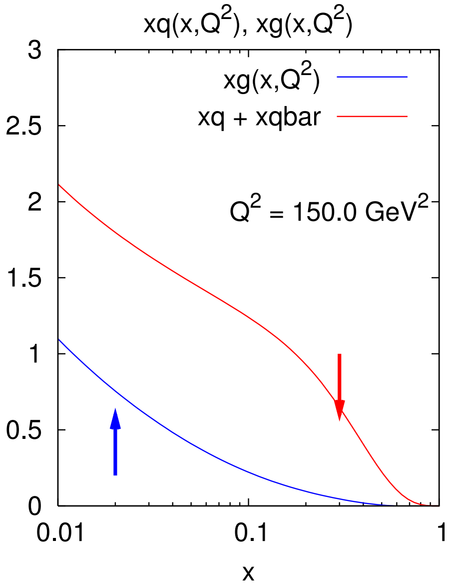

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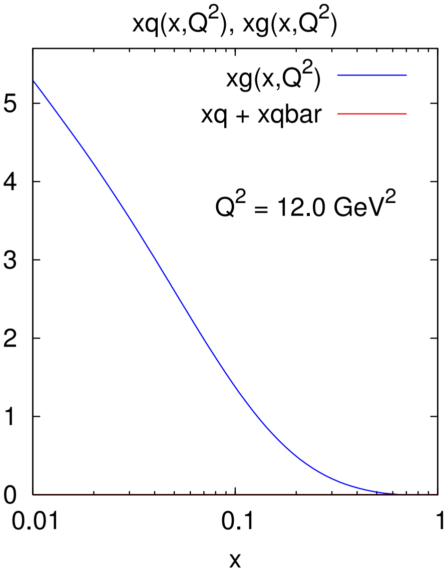

2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{q\leftarrow g} \otimes g$$
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- high-x gluon feeds growth of small x gluon & quark.

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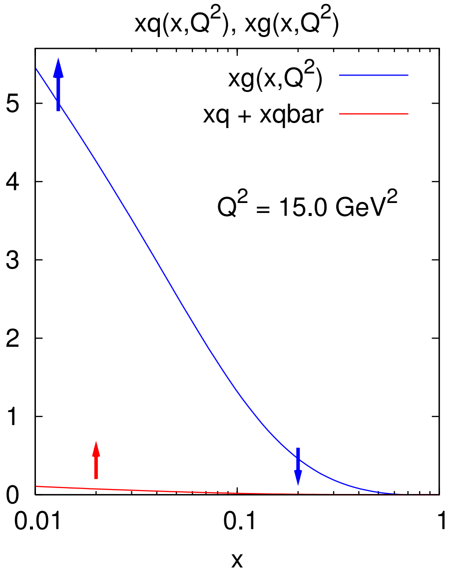

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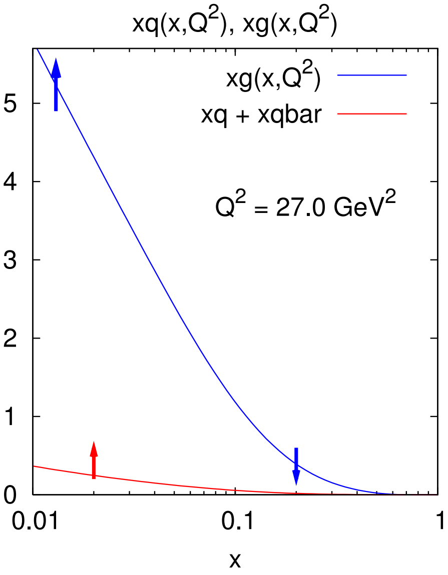

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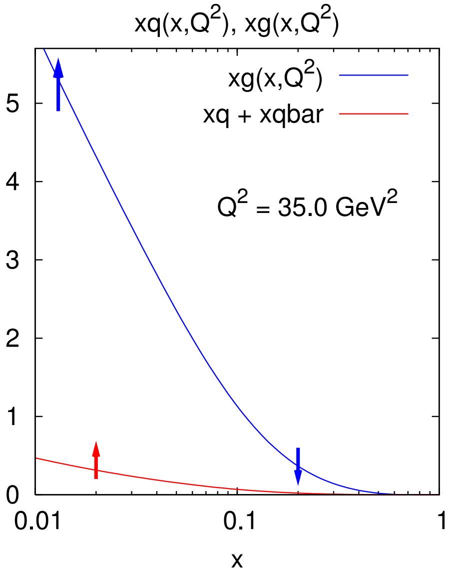

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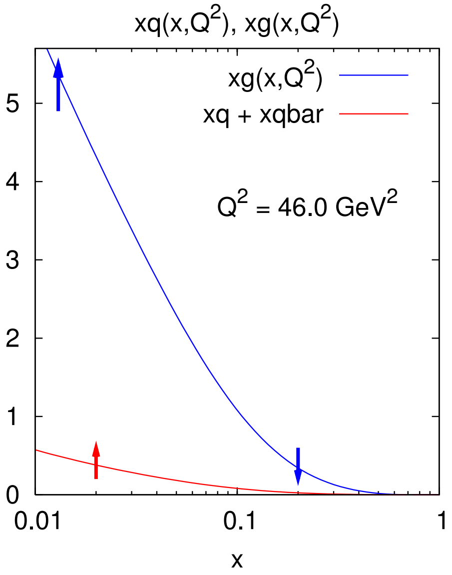

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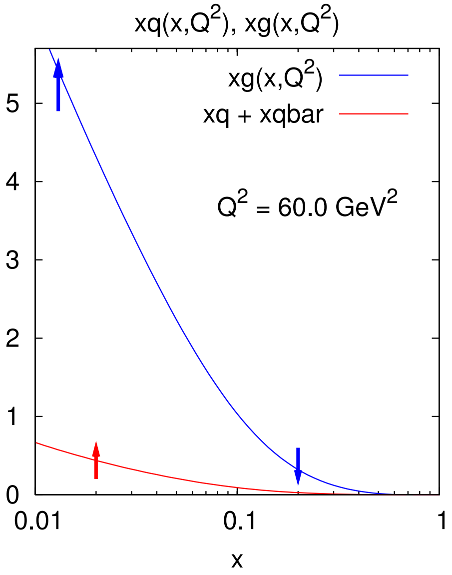

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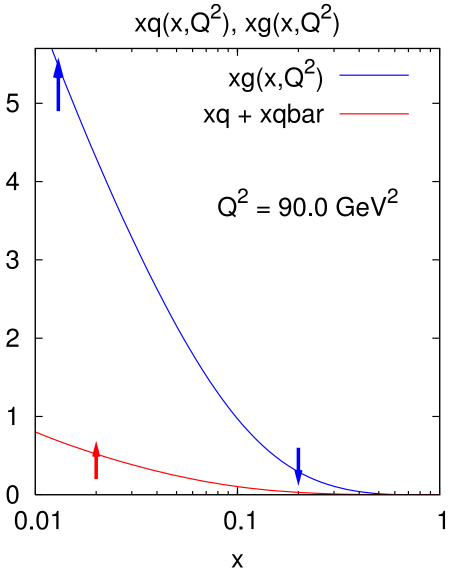

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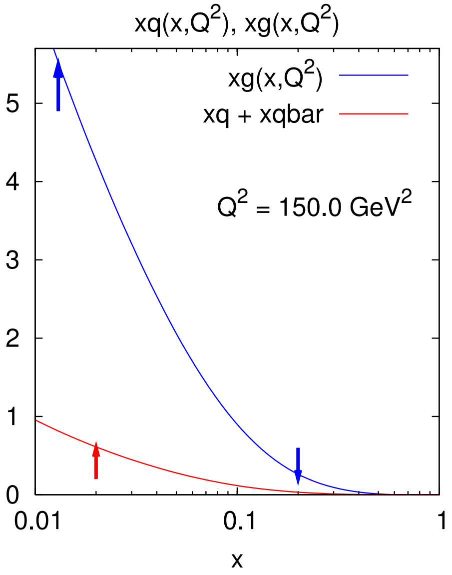

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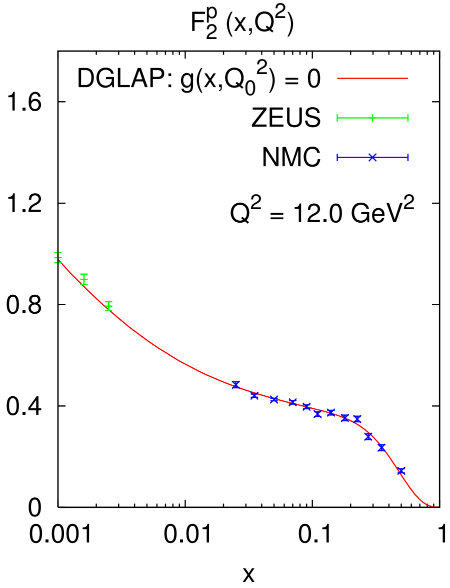
DGLAP evolution

- As Q^2 increases, partons lose longitudinal momentum; distributions all shift to lower x.
- gluons can be seen because they help drive the quark evolution.

Now consider data

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

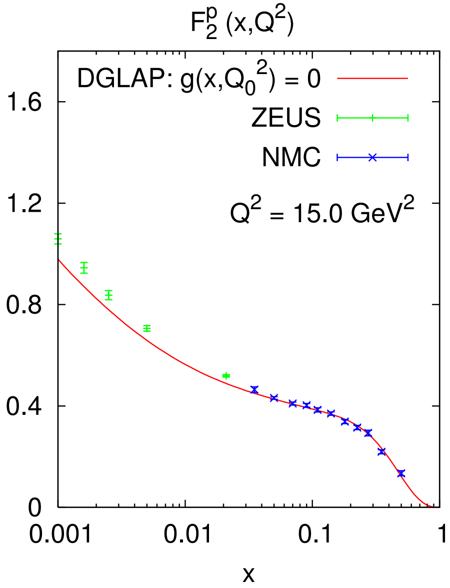
NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

DGLAP with initial gluon = 0



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Complete failure!

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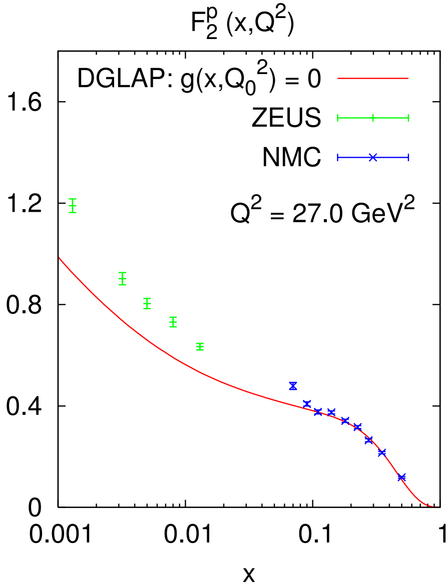

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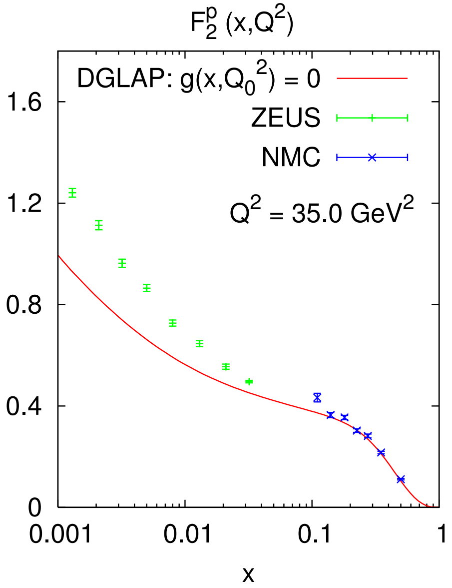

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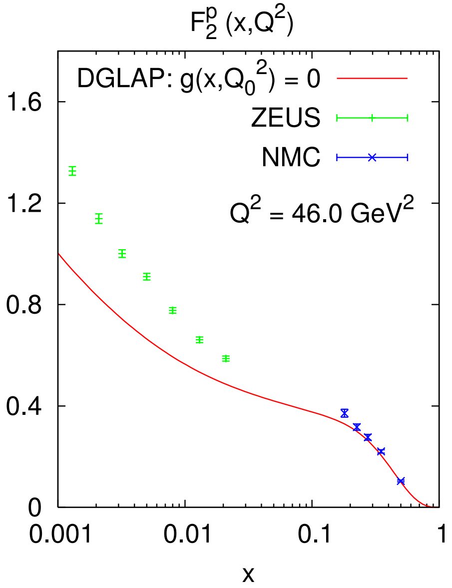

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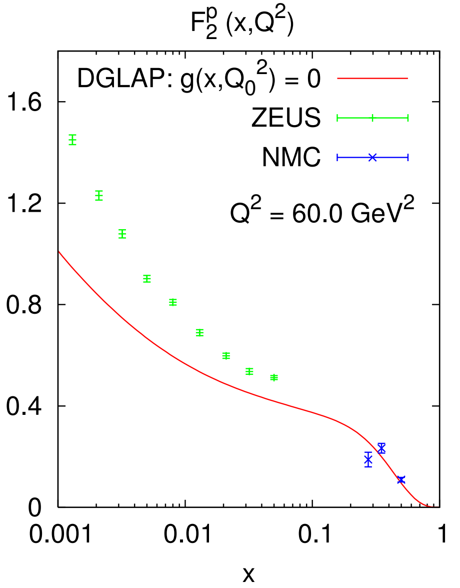

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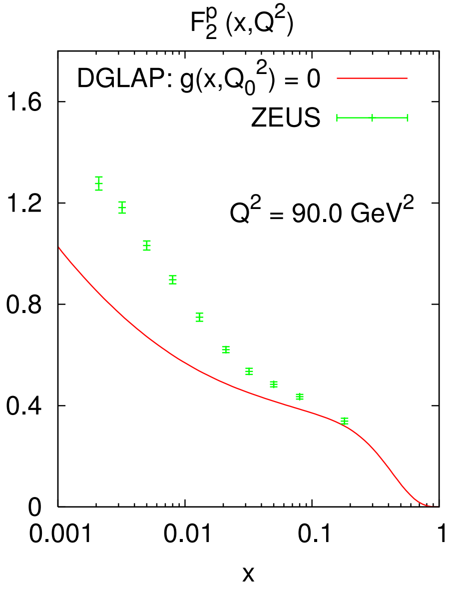

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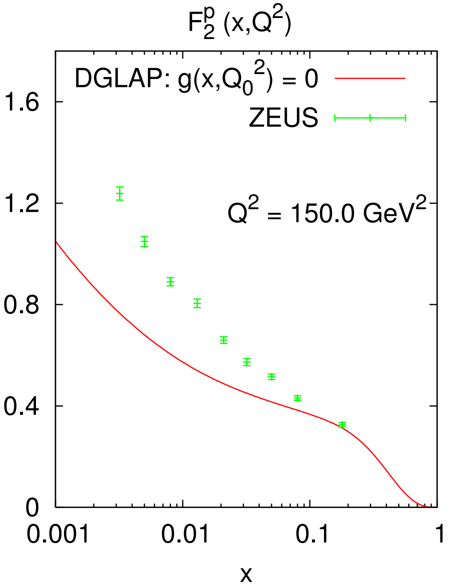

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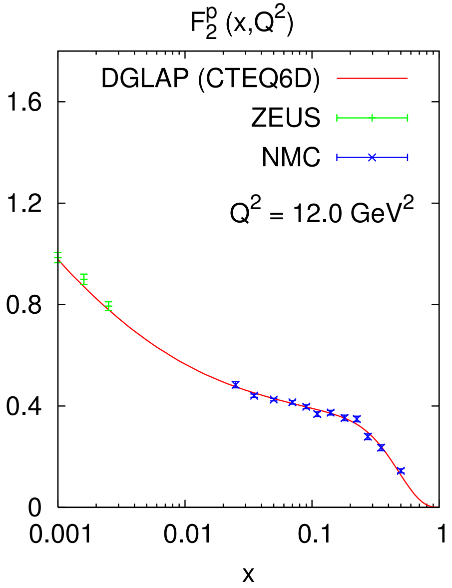
Complete failure!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

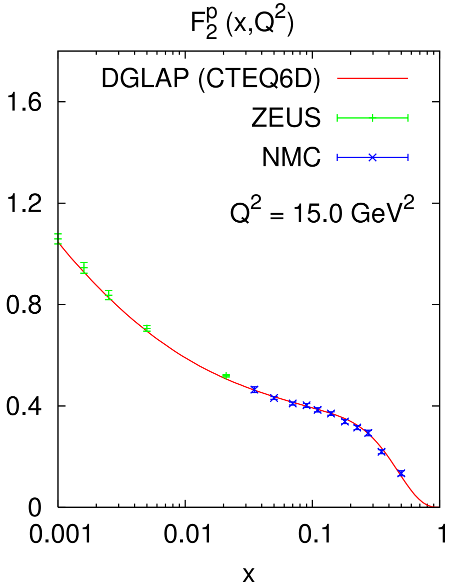
➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

Success!

DGLAP with initial gluon $\neq 0$



If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➡ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . .
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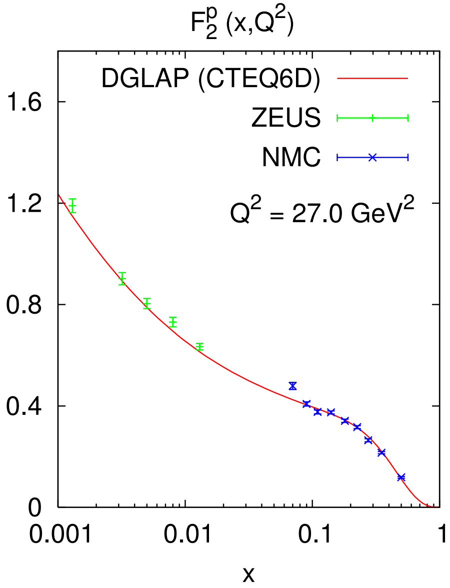

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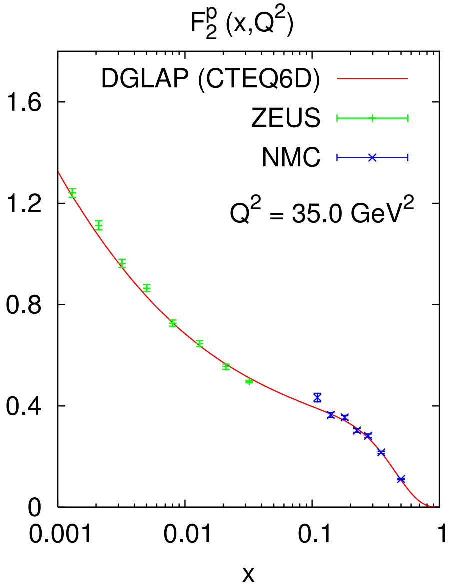

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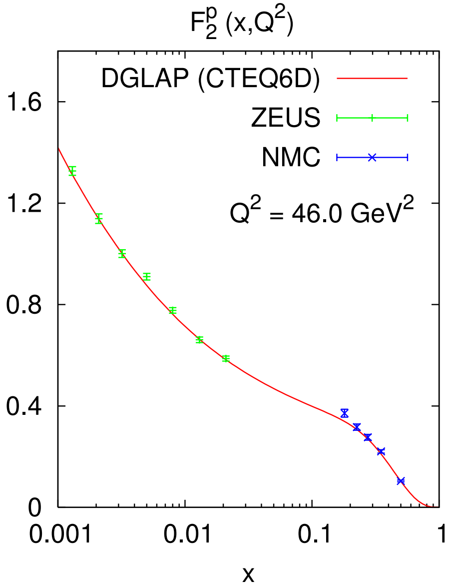

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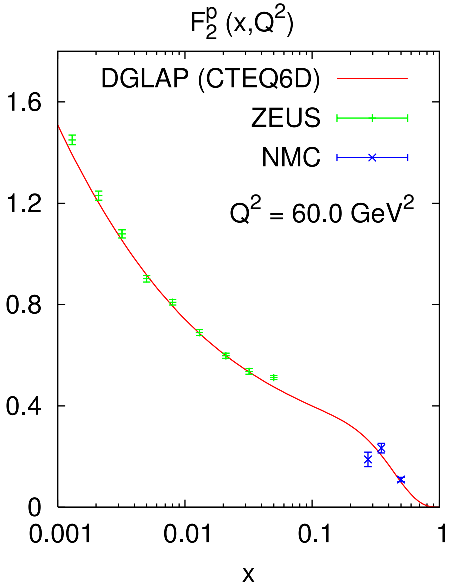

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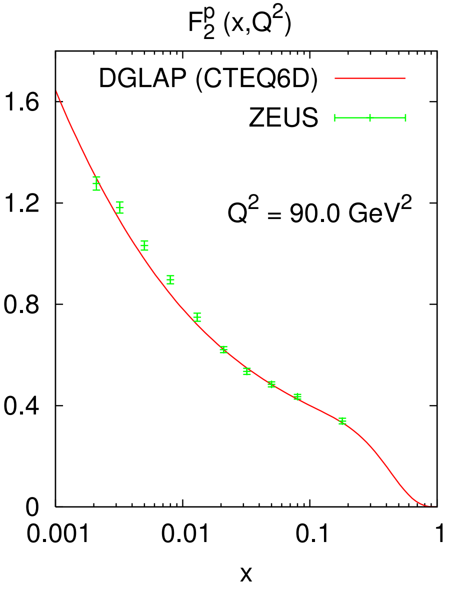

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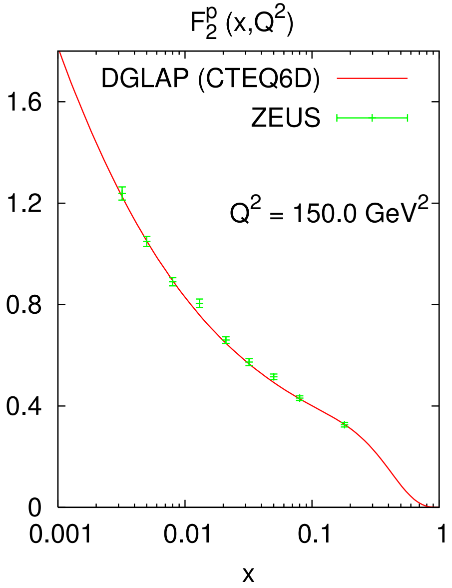

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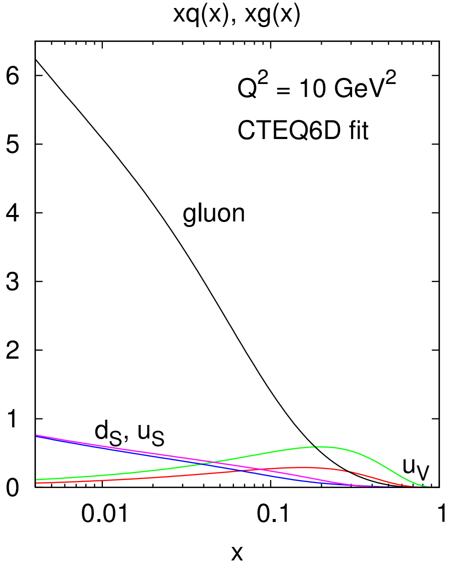
Gluon distribution



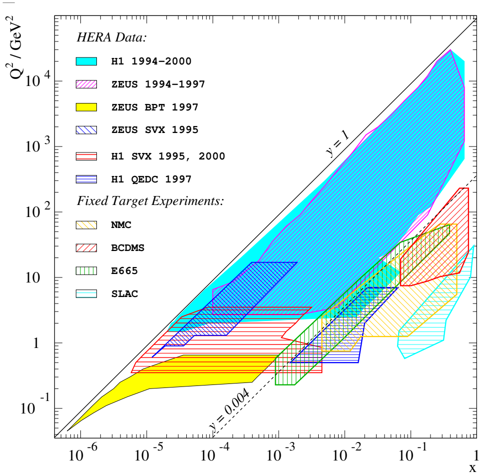
Gluon distribution is **HUGE!**

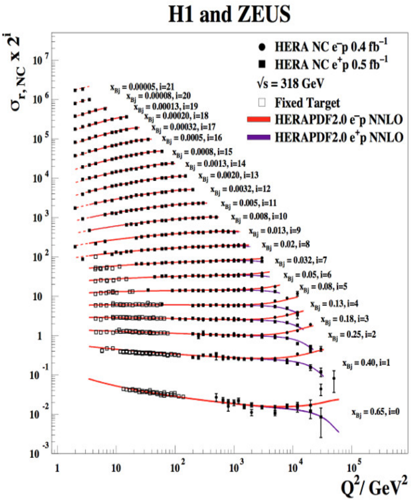
Can we really trust it?

- Consistency: momentum sum-rule is now *satisfied*.

  NB: gluon mostly at small x
- Agrees with vast range of data

DIS data and global fits





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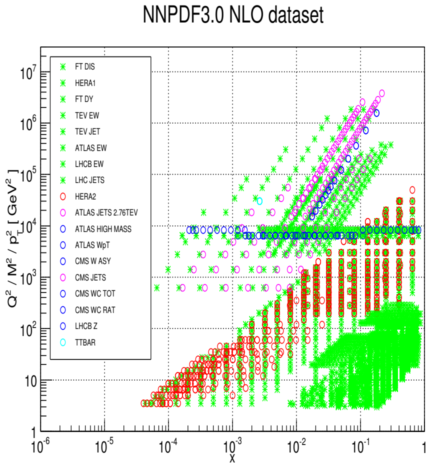
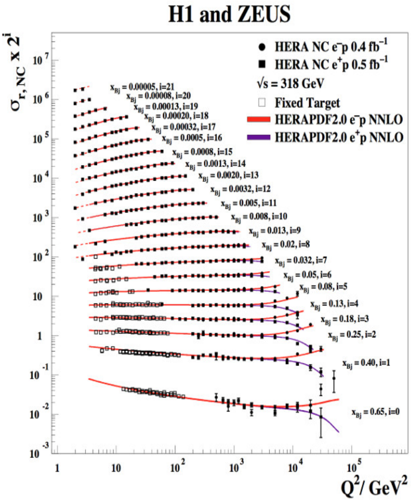
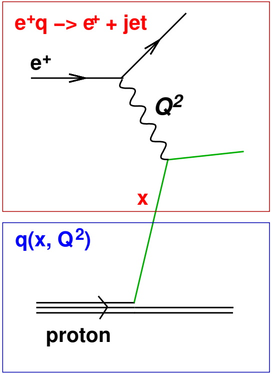


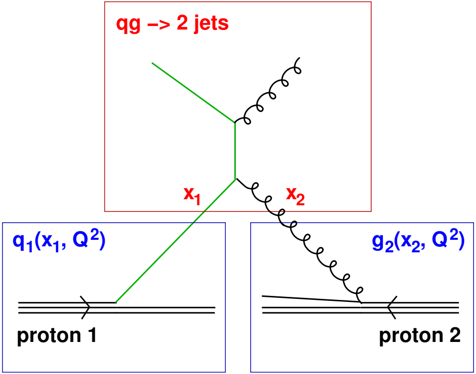


Other processes

Factorization of QCD cross-sections into convolution of:

- hard (perturbative) process-dependent **partonic subprocess**
- non-perturbative, process-independent **parton distribution functions**





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$$\sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

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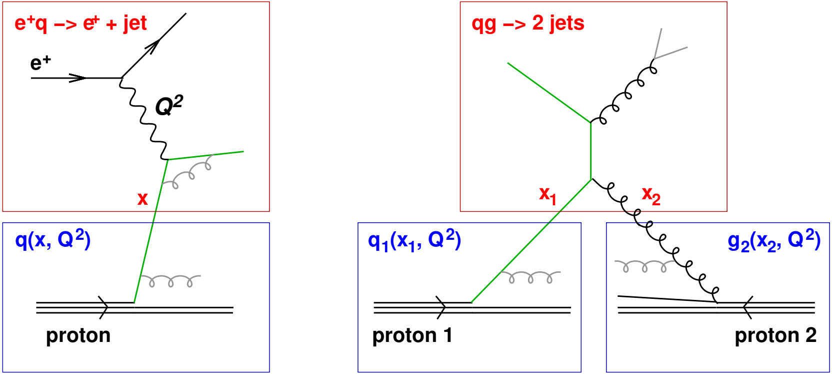

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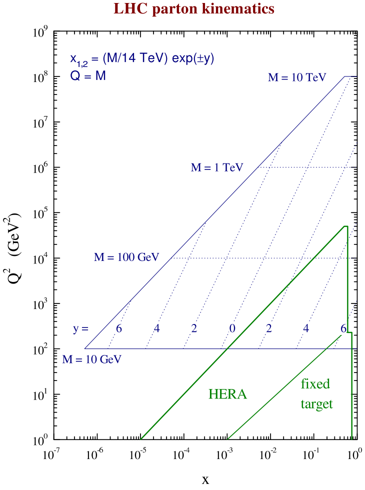
Taking PDFs from HERA to LHC



Suppose we produce a system of mass M at LHC from partons with momentum fractions x_1, x_2:

- $M = \sqrt{x_1 x_2 s}$
- rapidity $y = \frac{1}{2} \ln \frac{x_1}{x_2}$

pseudorapidity $\equiv \eta \equiv \ln \tan \frac{\theta}{2}$
$=$ rapidity for massless objects
$\lesssim 5$ at LHC

Are PDFs being used in region where measured?

Only partial kinematic overlap

- DGLAP evolution is **essential** for the prediction of PDFs in the LHC domain.

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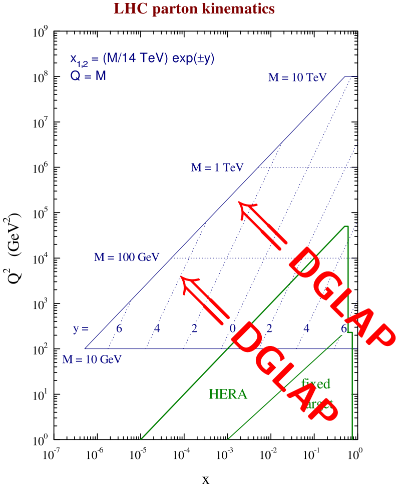

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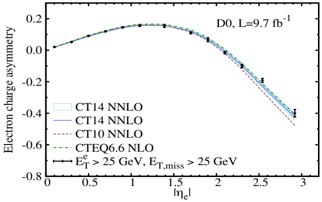
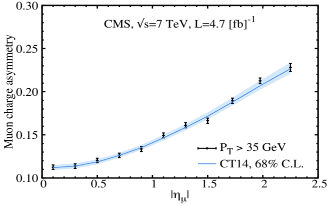
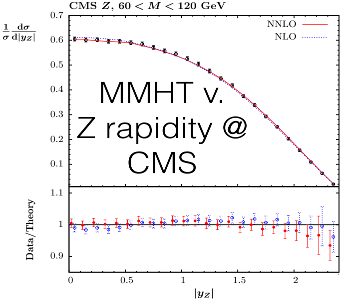
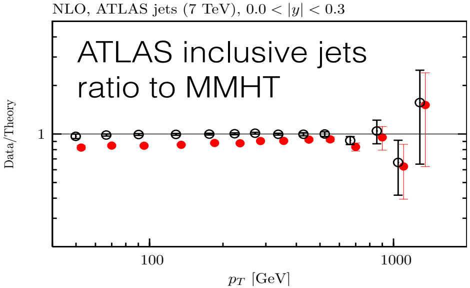
Comparisons to hadron-collider data

Lepton charge asym. v. CT14 @ D0 & CMS

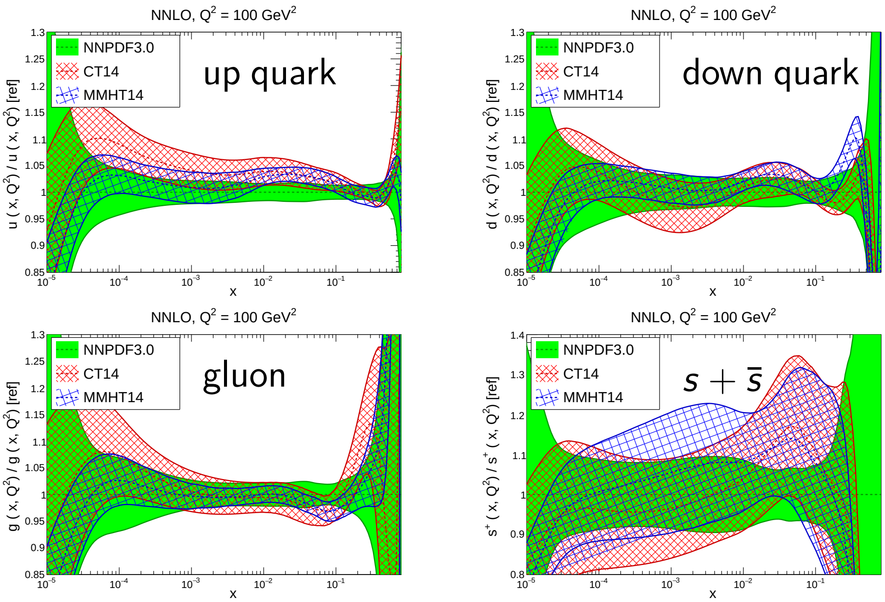








Precision of today's PDFs (from PDF4LHC)



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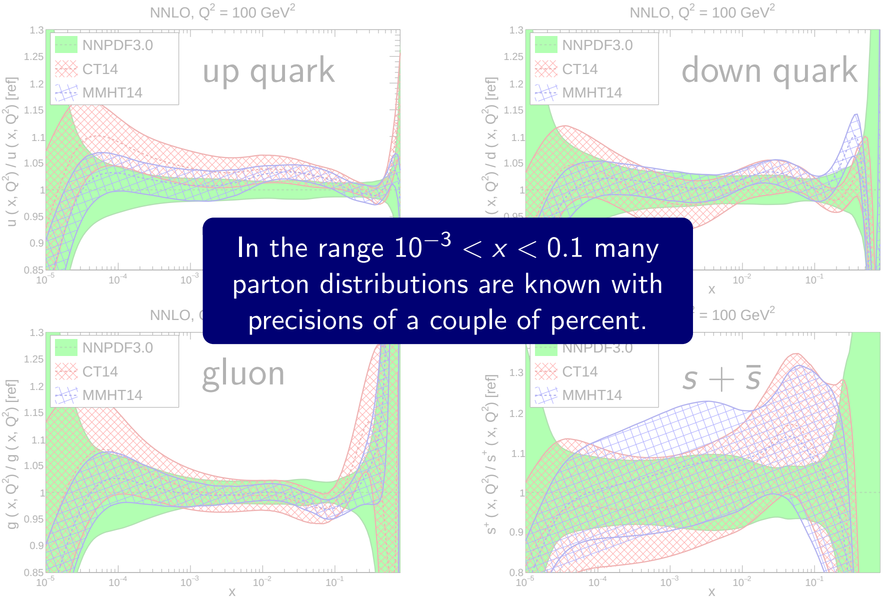


In the range $10^{-3} < x < 0.1$ many parton distributions are known with precisions of a couple of percent.

Conclusions on PDFs

- Factorization is key to our ability carry out calculations for hadron colliders
- Beyond leading order, factorization implies shuffling of divergences between the PDFs and the perturbative part of the calculation
- Equation for (factorisation) scale dependence of PDFs is DGLAP
- Our knowledge of PDFs comes both from the direct DIS measurements (for quarks) and from the scale-dependence of those measurements (for gluons), as well as Tevatron & LHC data.
- Today's precision on (some) PDFs approaches the percent level, which is crucial for Higgs precision prospects.

PDF exercise 1

Check that LO DGLAP evolution preserves the momentum sum rule unchanged.

Hints:

- One way of doing this is to use Mellin moments: $f_{i/p}^{(N)}(\mu^2) = \int_0^1 dx\, x^N\, f_{i/p}(x,\mu^2)$
- Which Mellin moment tells you about the momentum carried by parton flavour i?
- Show that the N^{th} Mellin moment of the DGLAP convolution, i.e. $\int_0^1 dx\, x^N \int_x^1 \frac{dz}{z} P(z) f(x/z)$ is given by the product of the N^{th} Mellin moments of P and f, i.e. $P^{(N)} f^{(N)} \equiv \int_0^1 dz\, z^N P(z) \times \int_0^1 dx\, x^N f(x)$.
- Then work out the appropriate Mellin moments and you should be able to prove momentum conservation.

PDF exercise 2

Show that at asymptotically large Q^2, the fraction of a hadron's momentum that is contained in the gluons tends to the value

$$\frac{2C_F}{2C_F + n_f T_R}$$

(and compare this to the result shown earlier in the slides)

Hints:

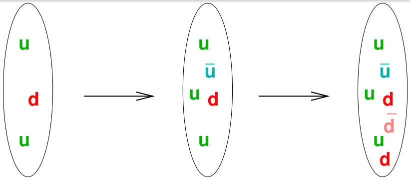
- write the evolution equation for the momentum in the quarks and gluons in terms of Mellin moments as a differential matrix equation:

$$\partial_{\ln \mu^2} \begin{pmatrix} \Sigma^{(N)}(\mu^2) \\ g^{(N)}(\mu^2) \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{\Sigma\Sigma}^{(N)} & P_{\Sigma g}^{(N)} \\ P_{g\Sigma}^{(N)} & P_{gg}^{(N)} \end{pmatrix} \begin{pmatrix} \Sigma^{(N)}(\mu^2) \\ g^{(N)}(\mu^2) \end{pmatrix}$$

  where Σ corresponds to a sum over all quark flavours. Watch out for the factors of 2 and n_f in $P_{\Sigma g}^{(N)}$.
- Examine the asymptotic solutions of this equation

EXTRAS

Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wavefunction *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Antiquarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge $\to$ +ve

- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite*, so

$$\int dx\, (u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

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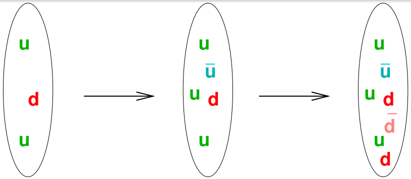

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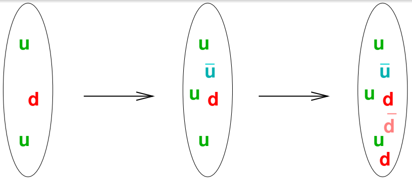

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"Valence" quarks

When we say proton has 2 up quarks & 1 down quark we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

How do we measure *difference* between u and $\bar{u}$? Photon interacts identically with both → no good...

Question: what interacts differently with particle & antiparticle?

Answer: W^+ *or* W^-

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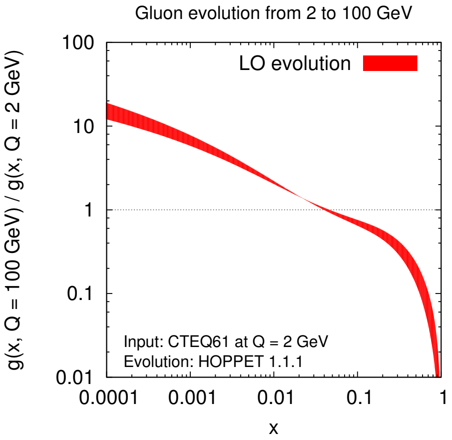
By how much do PDFs evolve?



Illustrate for the gluon distribution

Here using fixed Q scales
But for HERA $\to$ LHC
relevant Q range is x-dependent

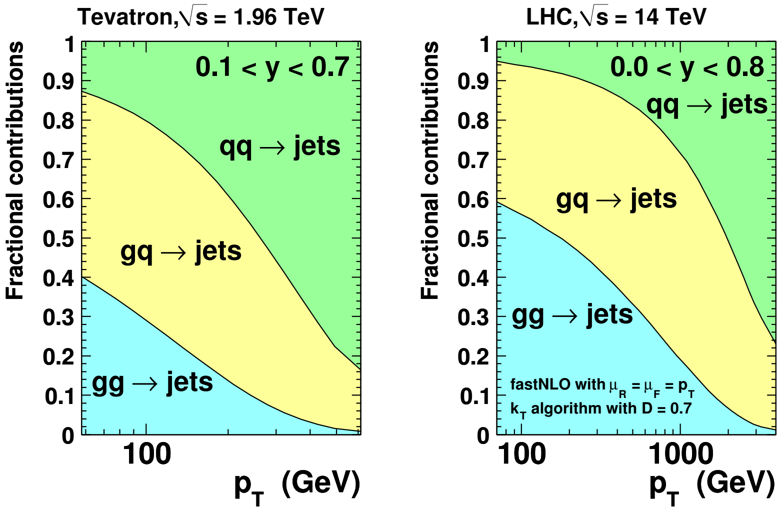
- See factors $\sim 0.1 - 10$
- Remember: LHC involves product of two parton densities

It's crucial to get this right!

Without DGLAP evolution, you couldn't predict anything at LHC

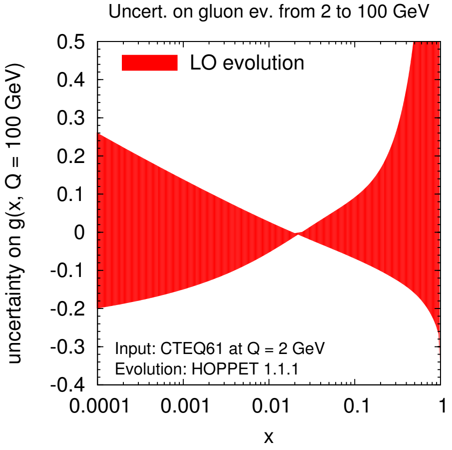
Which PDF channels contribute?

Inclusive jet cross sections with MSTW 2008 NLO PDFs



A large fraction of jets are gluon-induced

Evolution uncertainty

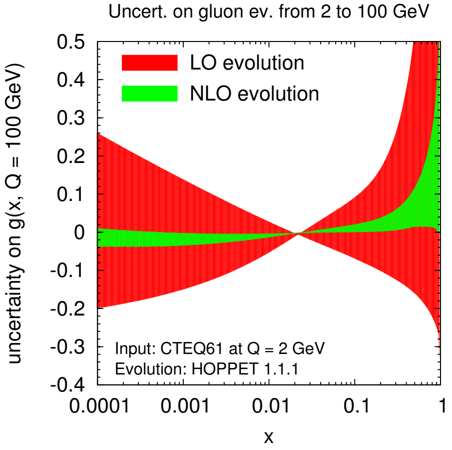


Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

Talk more about such tricks in next lecture

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\rightarrow 2\%$ Commensurate with data uncertainties

Evolution uncertainty



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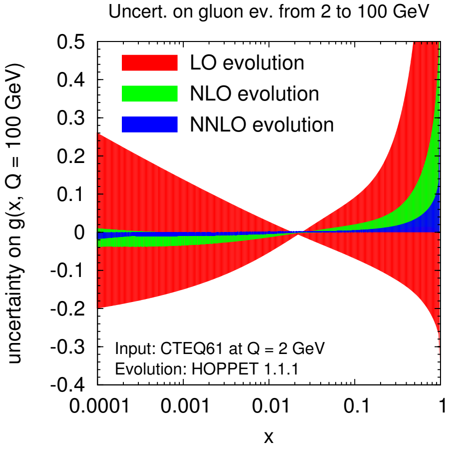

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