QCD (for Colliders) Lecture 2

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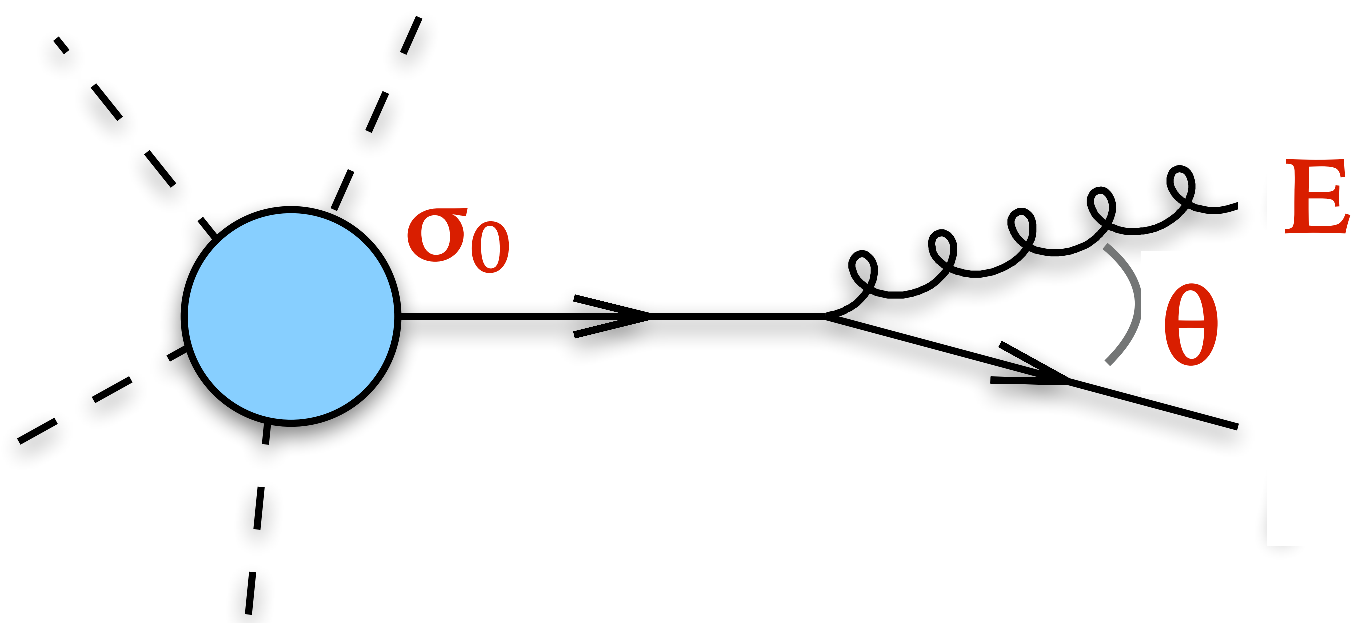
Yesterday:

- QCD Lagrangian
- Running coupling
- Soft gluon emission & its divergences

Today

- Real–virtual cancellation
- Factorisation
- Parton Distribution Functions (PDFs)
- Total cross sections & their perturbative series

GLUON EMISSION FROM A QUARK



Consider an emission with

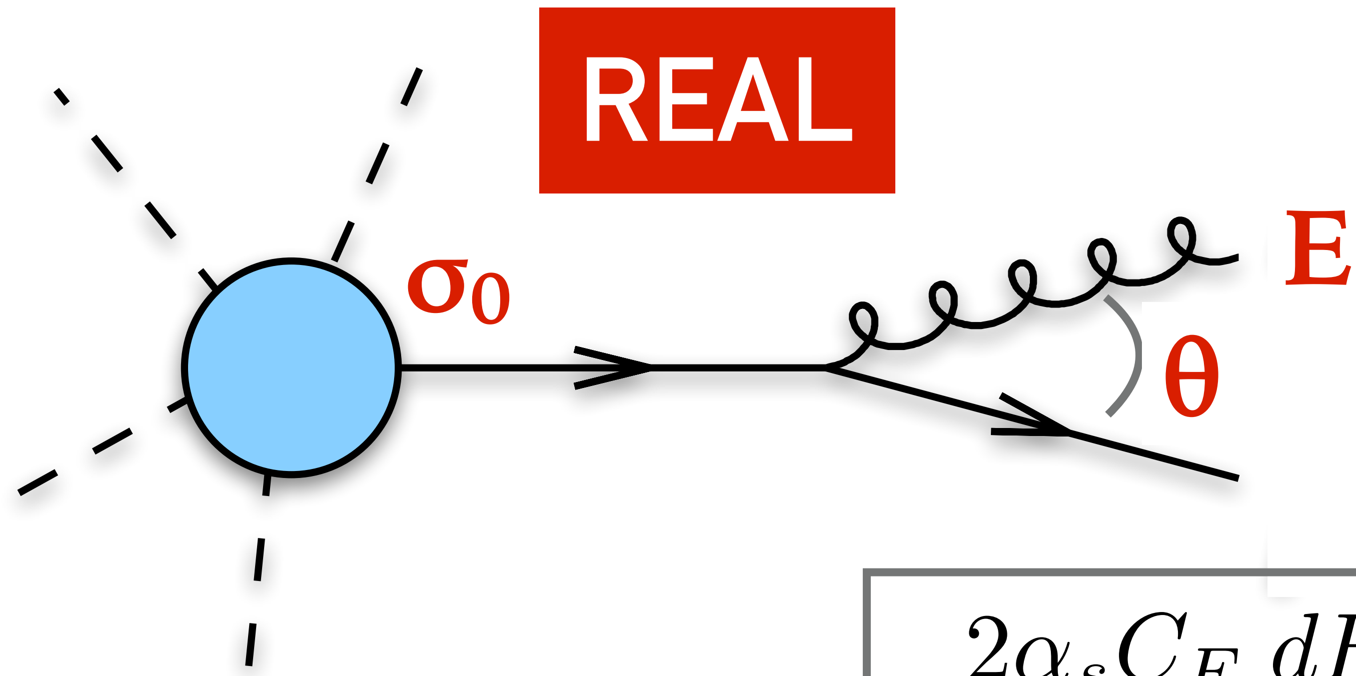
- energy **E ≪ √s** (“soft”)
- angle **θ ≪ 1** (“collinear” wrt quark)

Examine correction to some hard process with cross section σ_0

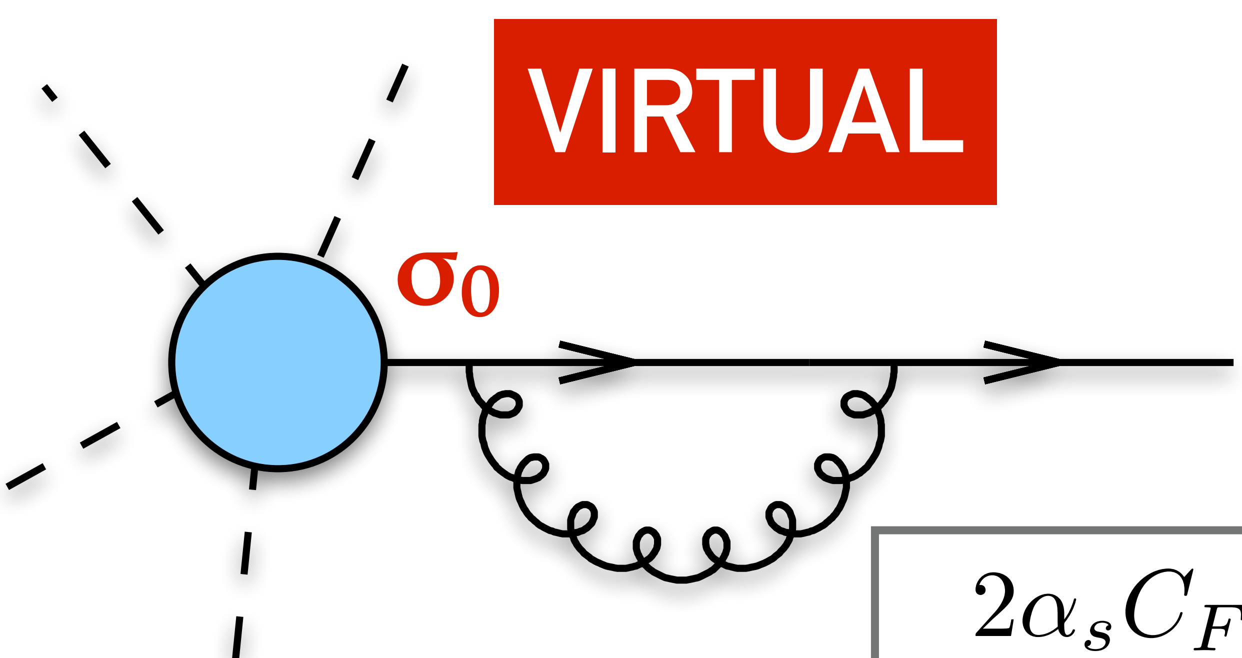
$$d\sigma \simeq \sigma_0 \times \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

This has a divergence when E→0 or θ→0

[in some sense because of quark propagator going on-shell]



$$+\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$



$$-\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

Divergences are present in both real and virtual diagrams.

If you are "**inclusive**", i.e. your measurement doesn't care whether a soft/collinear gluon has been emitted then the **real and virtual divergences cancel.**

Beyond inclusive cross sections: infrared and collinear (IRC) safety

> *For an observable's distribution to be calculable in [fixed-order] perturbation theory, the observable should be infra-red safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching*
>
> $$\vec{p}_i \to \vec{p}_j + \vec{p}_k$$
>
> *whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel [collinear] or one of them is small [infrared].*
>
> [QCD and Collider Physics (Ellis, Stirling & Webber)]

Examples

Multiplicity of gluons is not IRC safe

[modified by soft/collinear splitting]

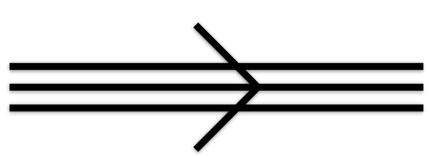
Energy of hardest particle is not IRC safe

[modified by collinear splitting]

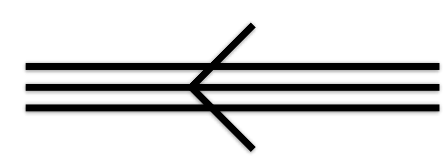
Energy flow into a cone is IRC safe

[soft emissions don't change energy flow,
collinear emissions don't change its direction]

A proton-proton collision: INITIAL STATE

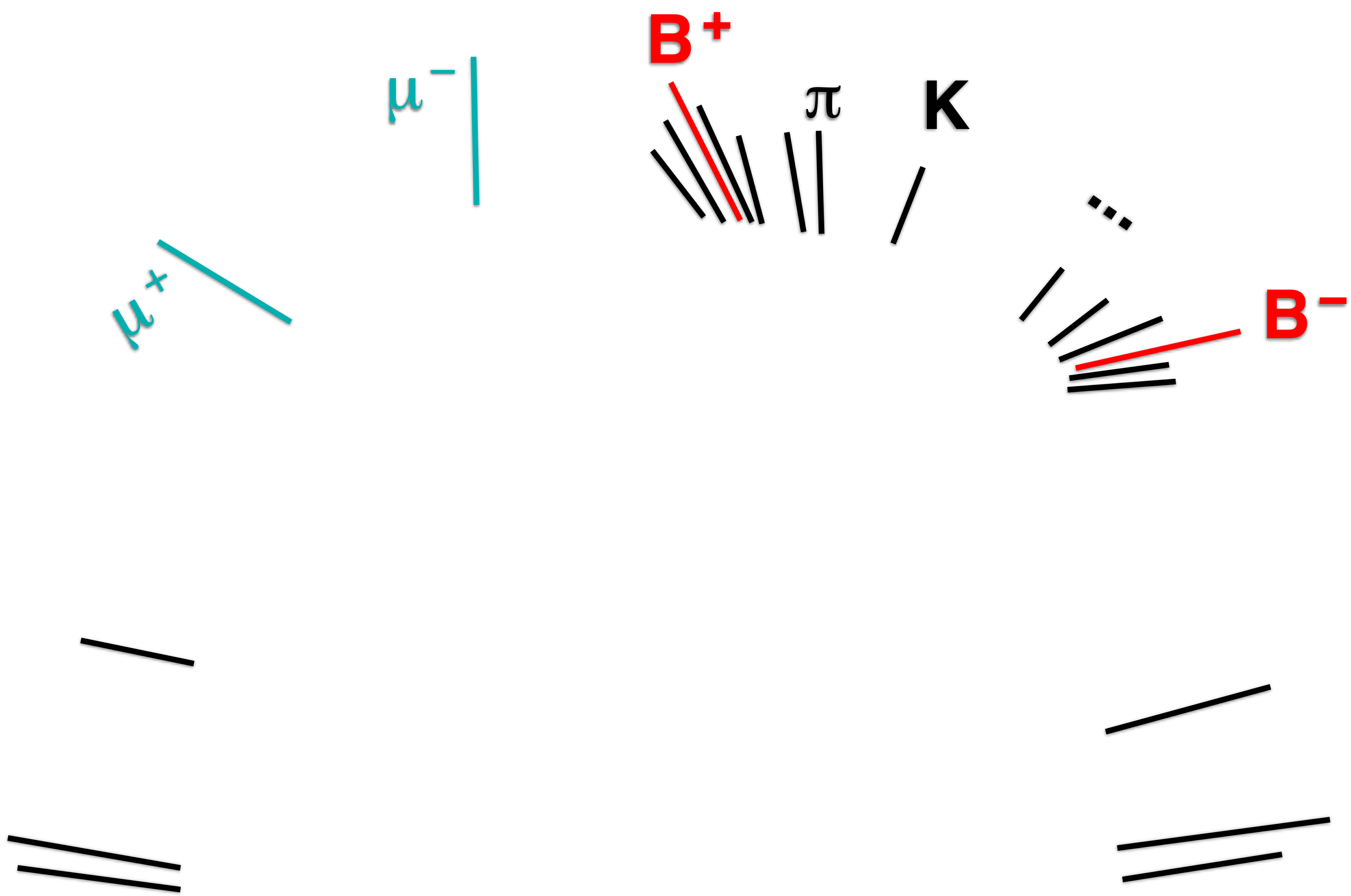


proton



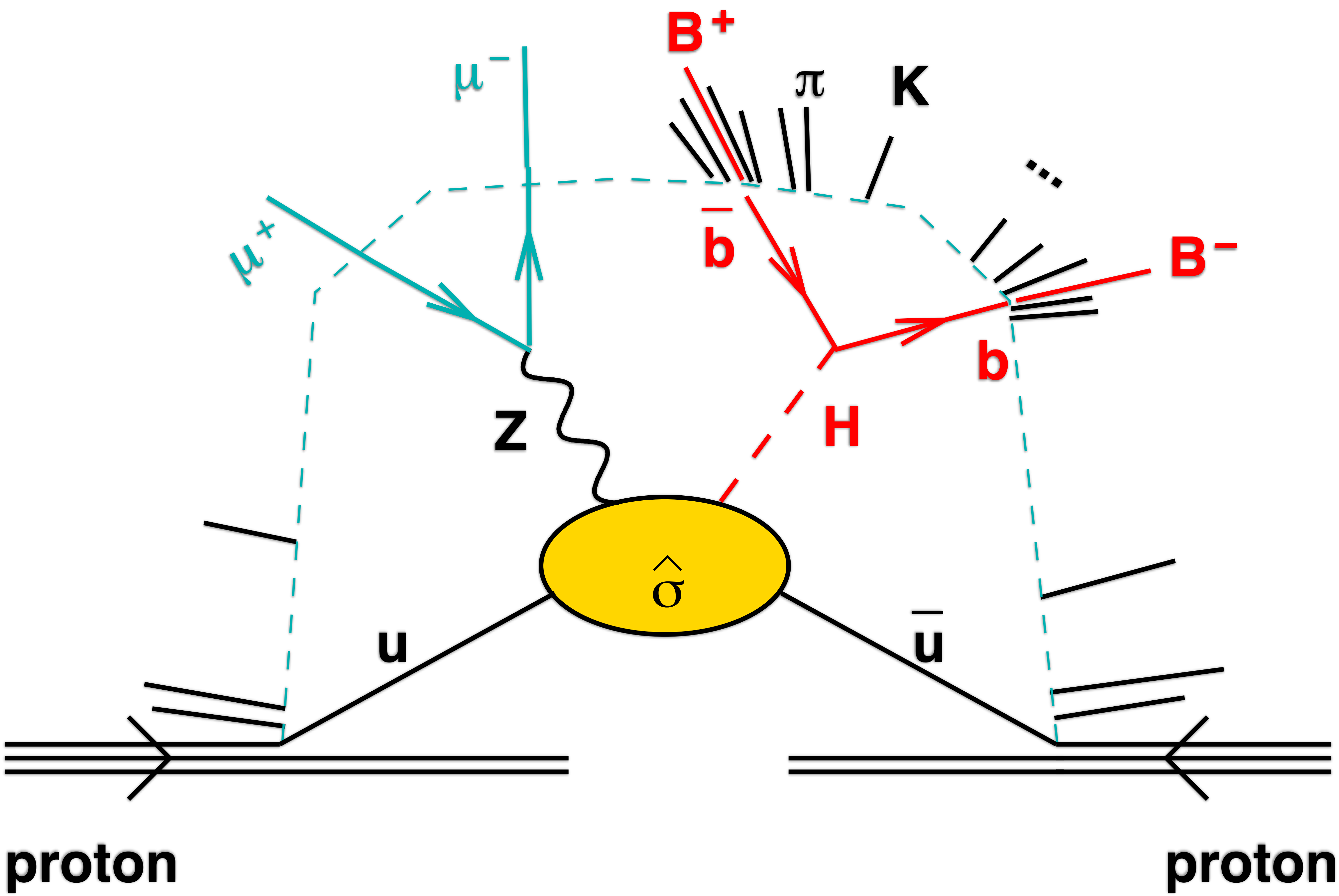
proton

A proton-proton collision: FINAL STATE

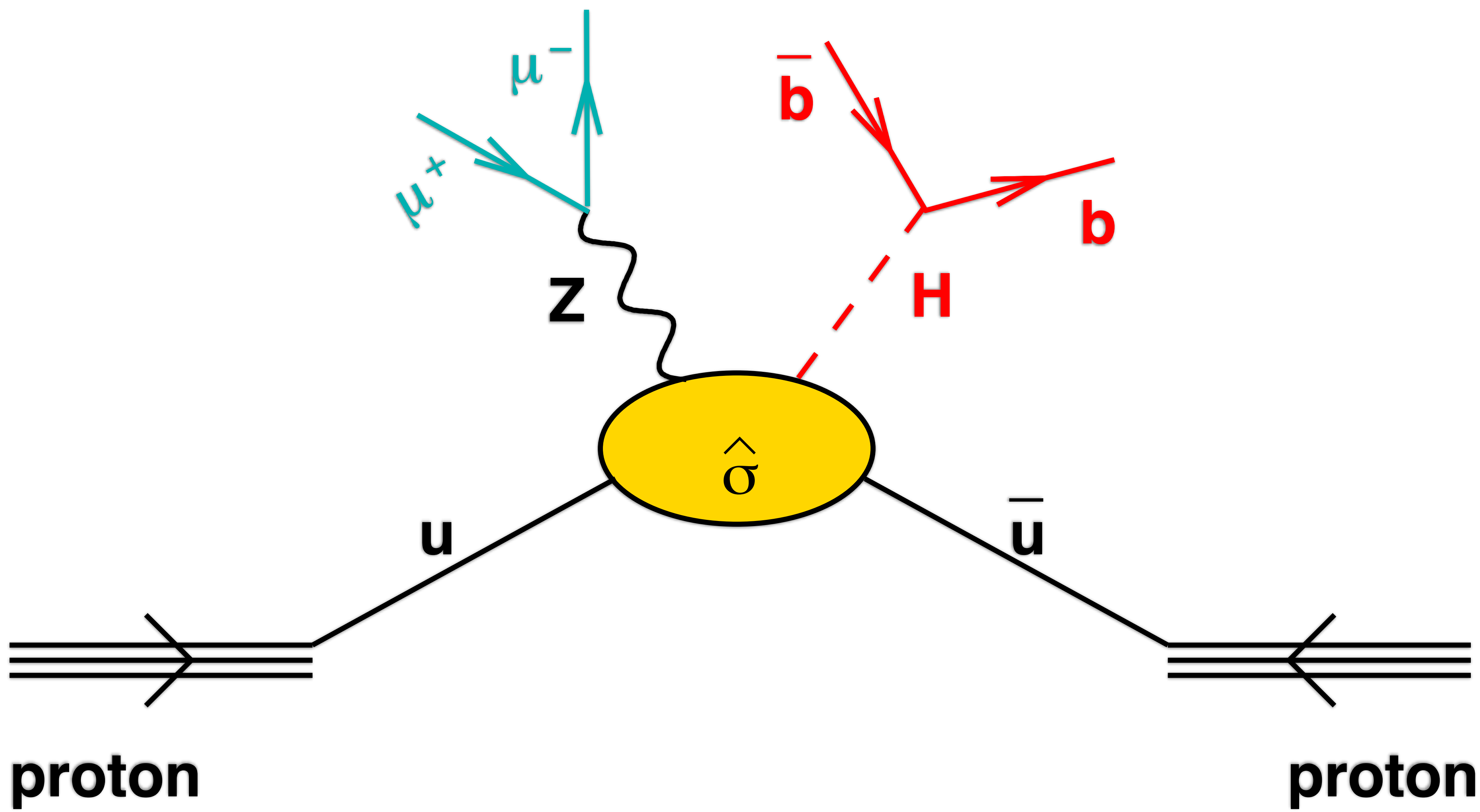


(actual final-state multiplicity ~ several hundred hadrons)

A proton-proton collision: FILLING IN THE PICTURE



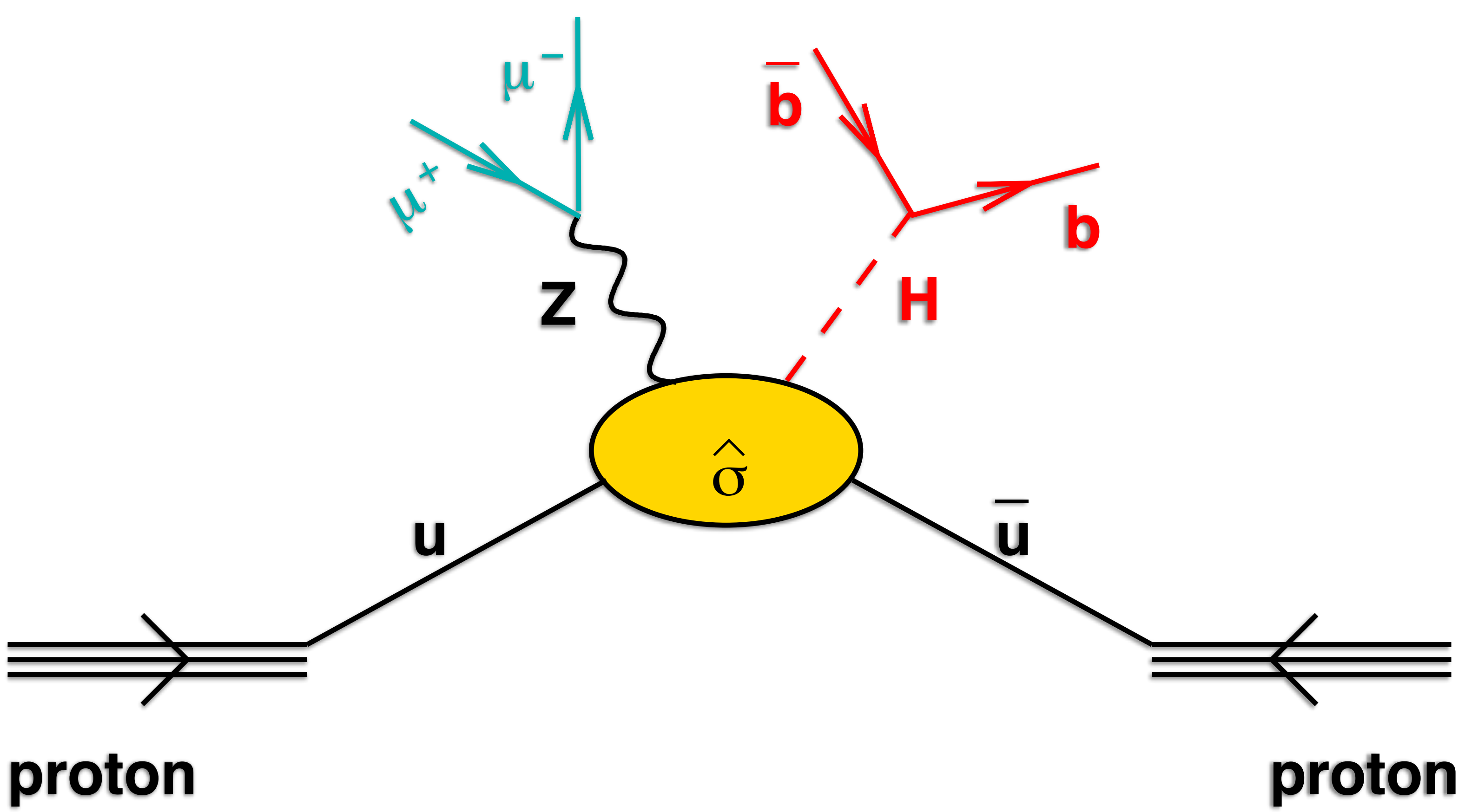
A proton-proton collision: SIMPLIFYING IN THE PICTURE



THE MASTER EQUATION — FACTORISATION

$$\sigma\left(h_1 h_2 \to ZH + X\right) = \sum_{n=0}^{\infty} \alpha_s^n\left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right)$$

$$\times\, \hat{\sigma}^{(n)}_{ij \to ZH+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$



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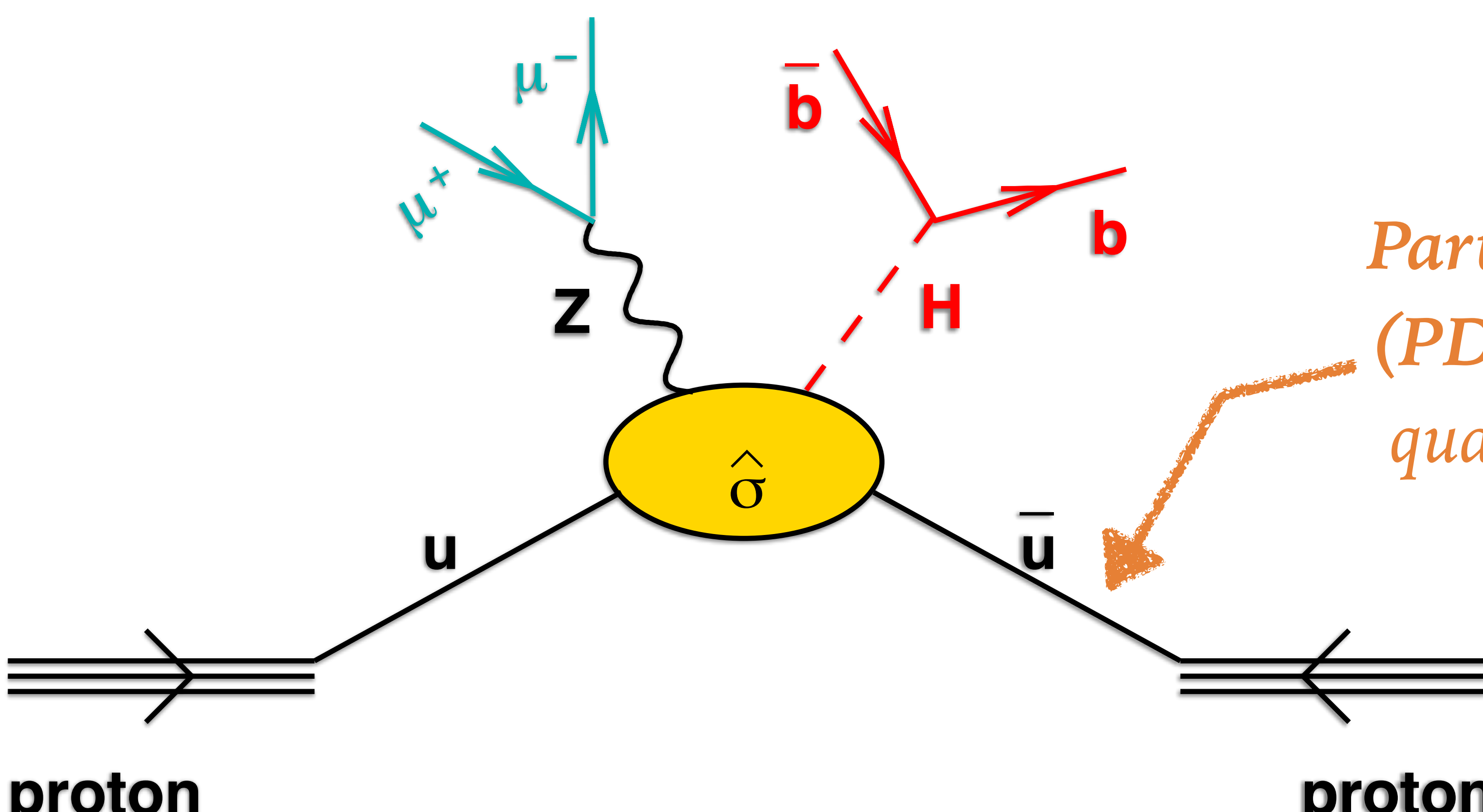

Parton distribution function (PDF): e.g. number of up anti-quarks carrying fraction x_2 of proton's momentum

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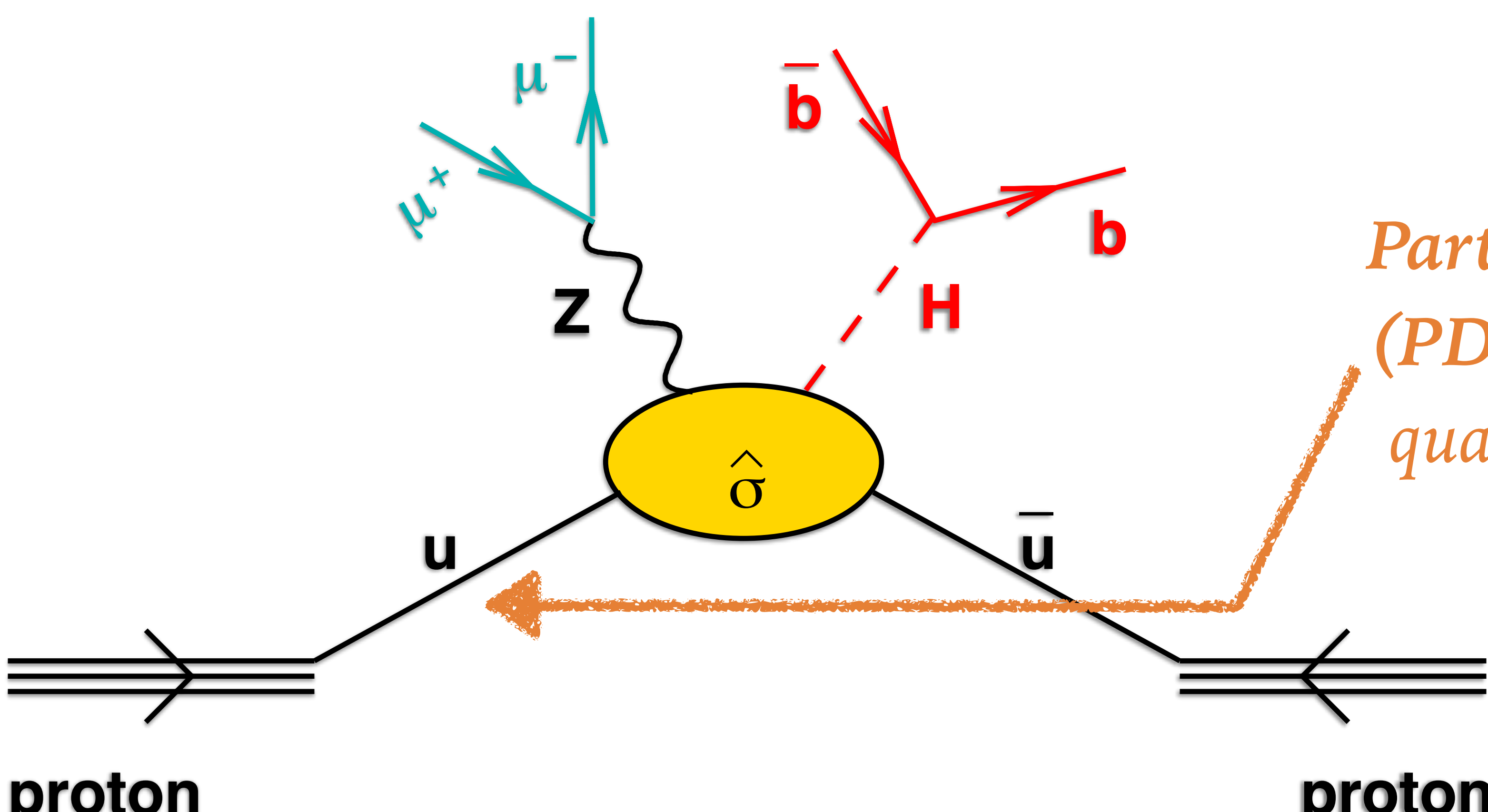

Parton distribution function (PDF): e.g. number of up quarks carrying fraction x_1 of proton's momentum

THE MASTER EQUATION — FACTORISATION

Perturbative sum over powers of the strong coupling: typically we know first 2-4 orders

$$\sigma\left(h_1 h_2 \to ZH + X\right) = \sum_{n=0}^{\infty} \alpha_s^n\left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right)$$

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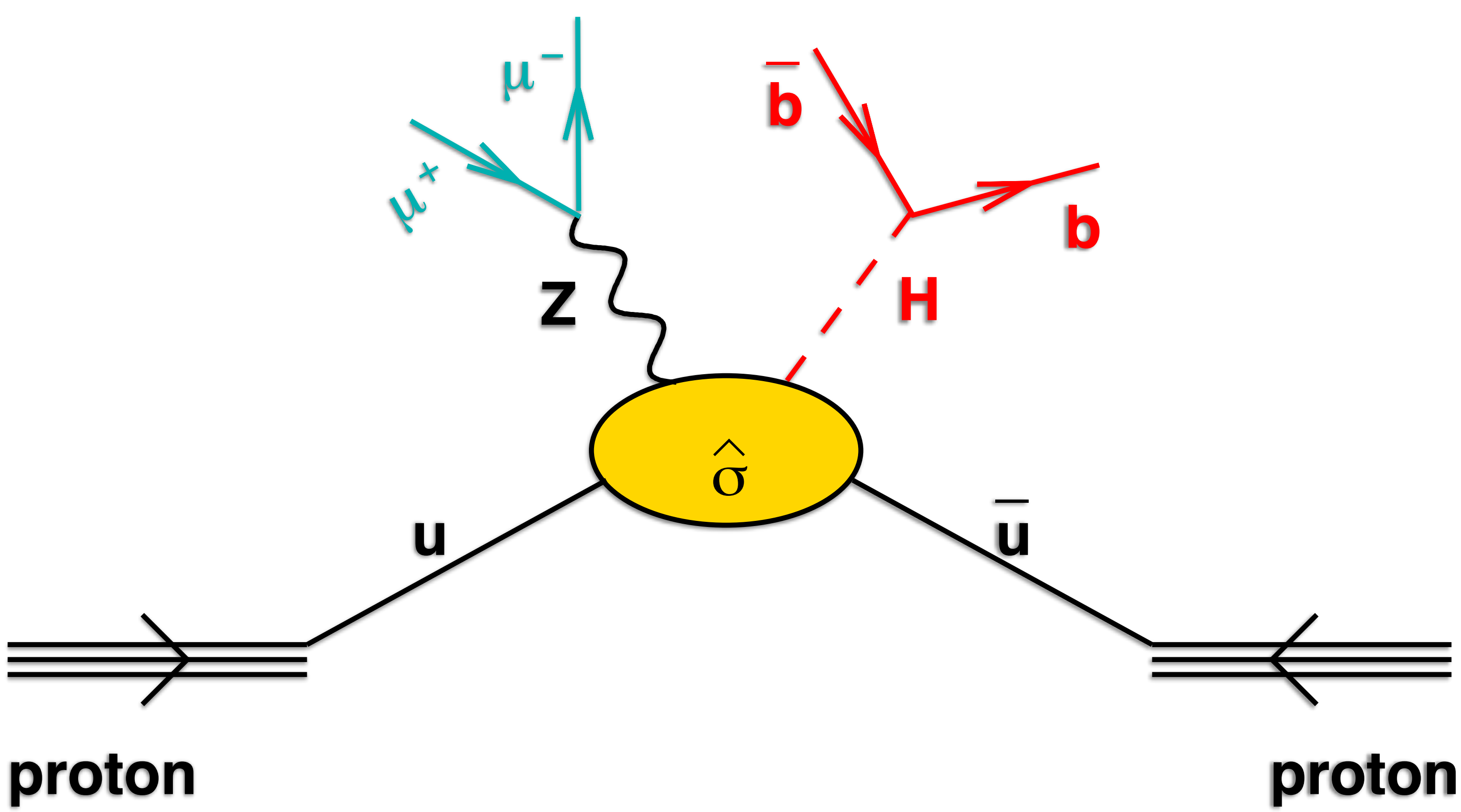

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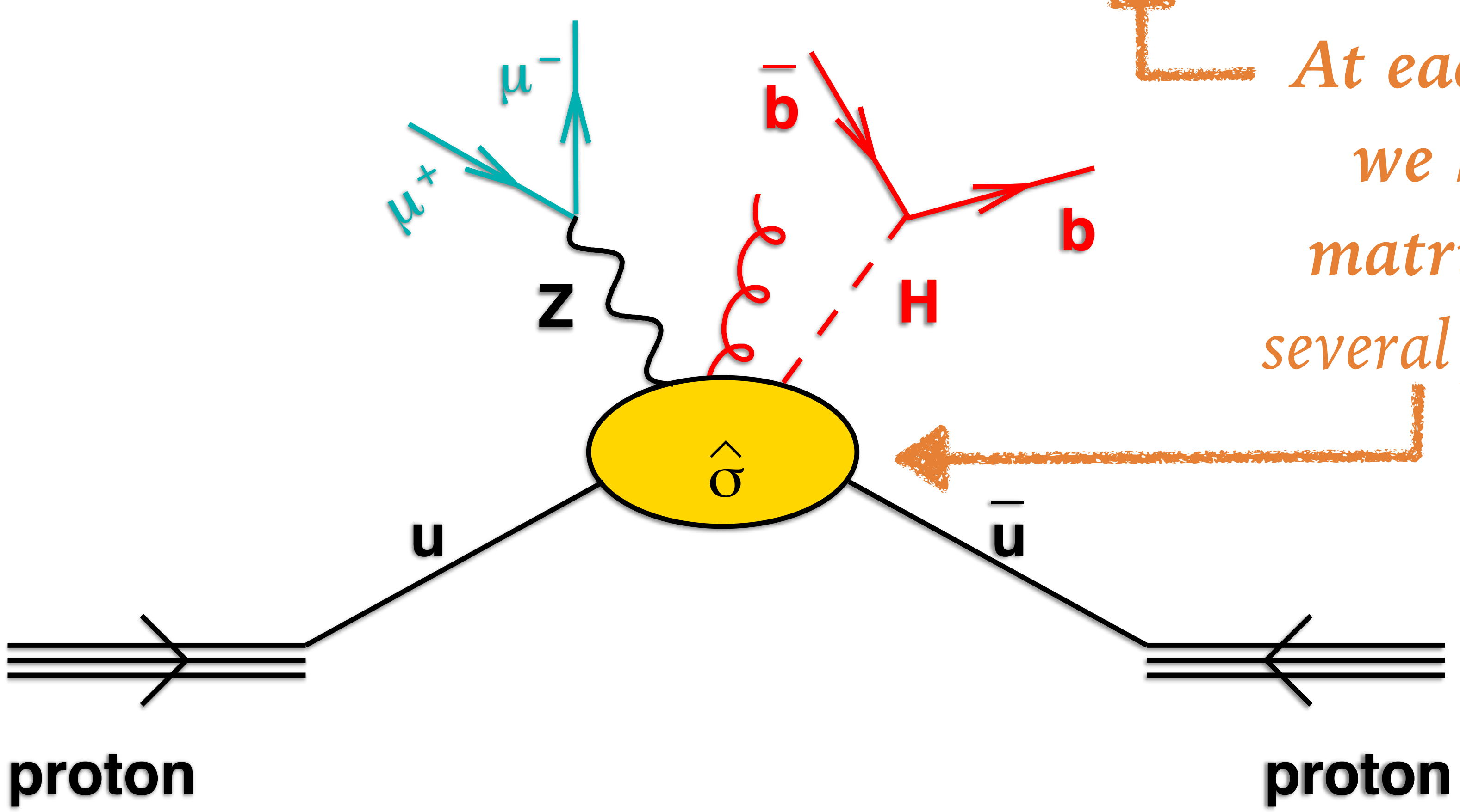
At each perturbative order n we have a specific "hard matrix element" (sometimes several for different subprocesses)



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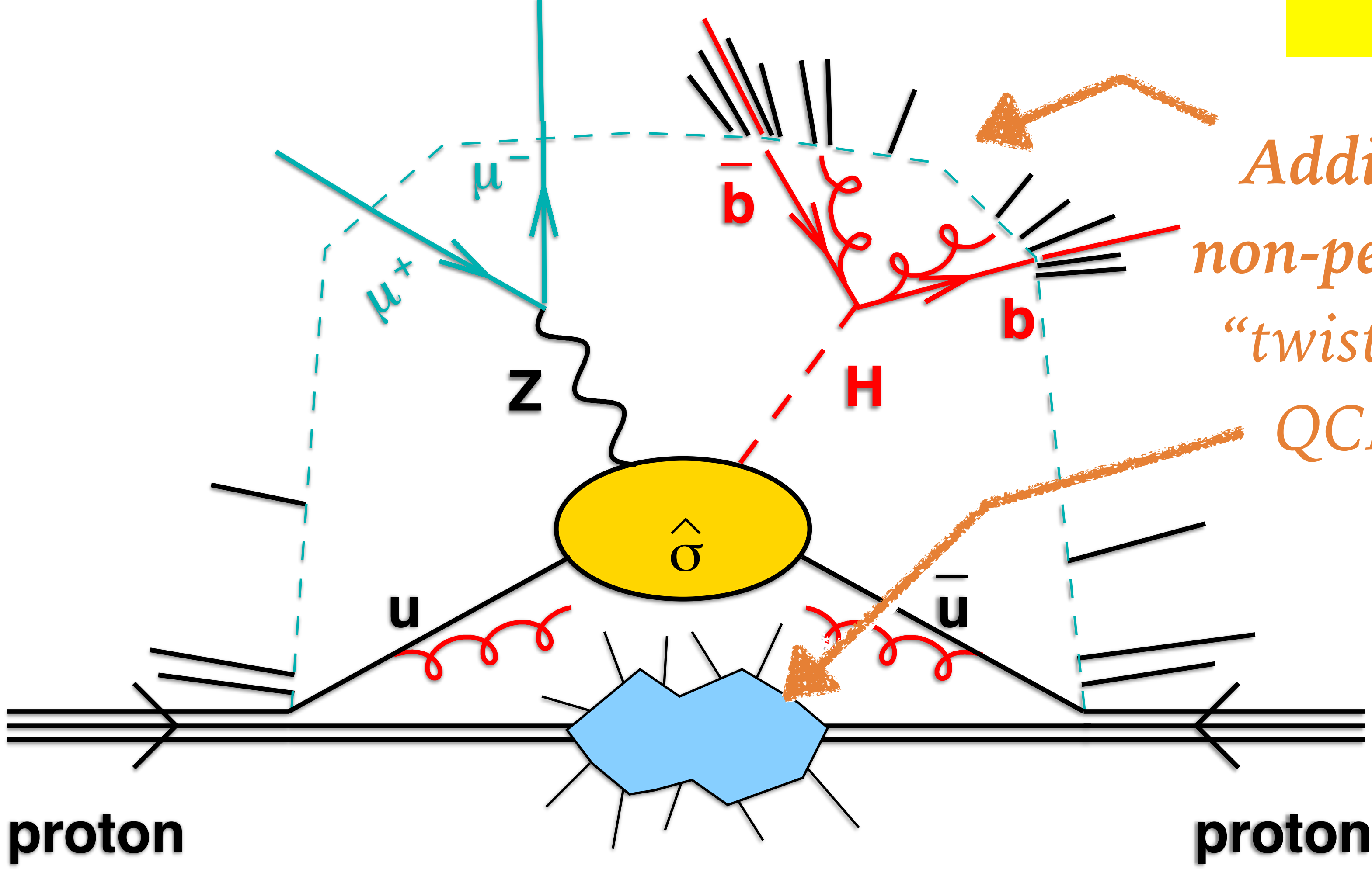
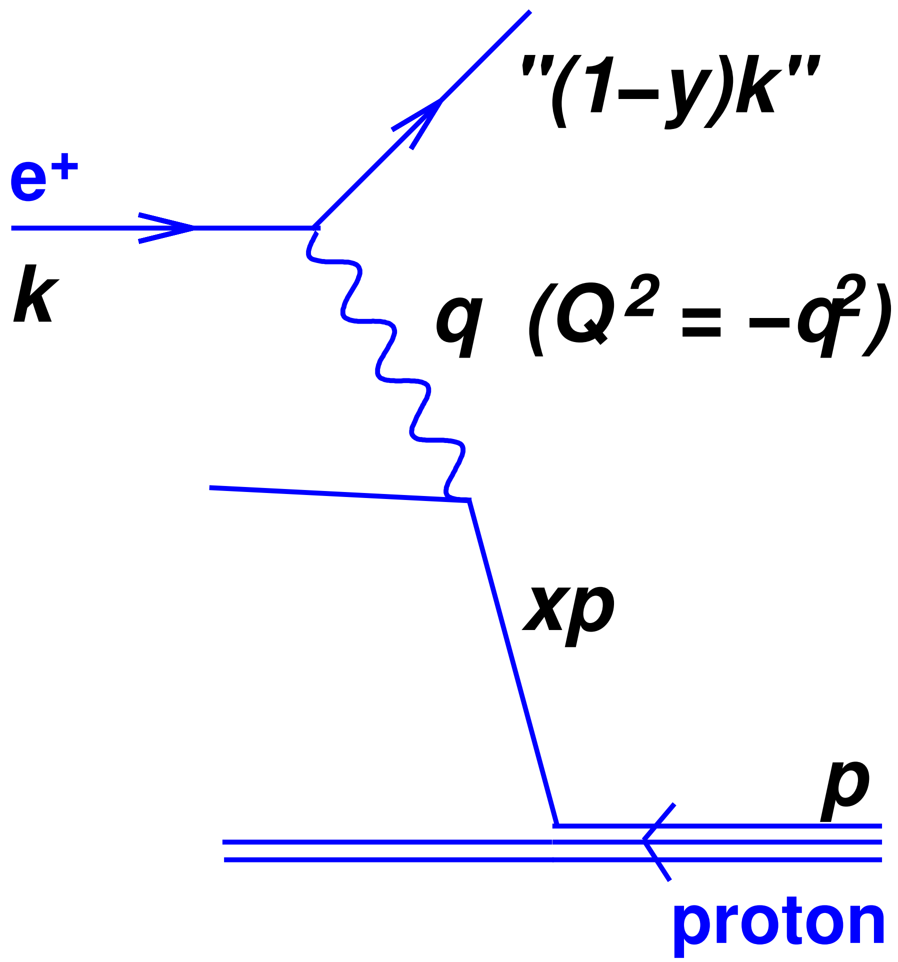


Additional corrections from non-perturbative effects (higher "twist", suppressed by powers of QCD scale (Λ) / hard scale)

PARTON DISTRIBUTION FUNCTIONS (PDFs)

Hadron-hadron is complex because of two incoming partons — so start with simpler Deep Inelastic Scattering (DIS).



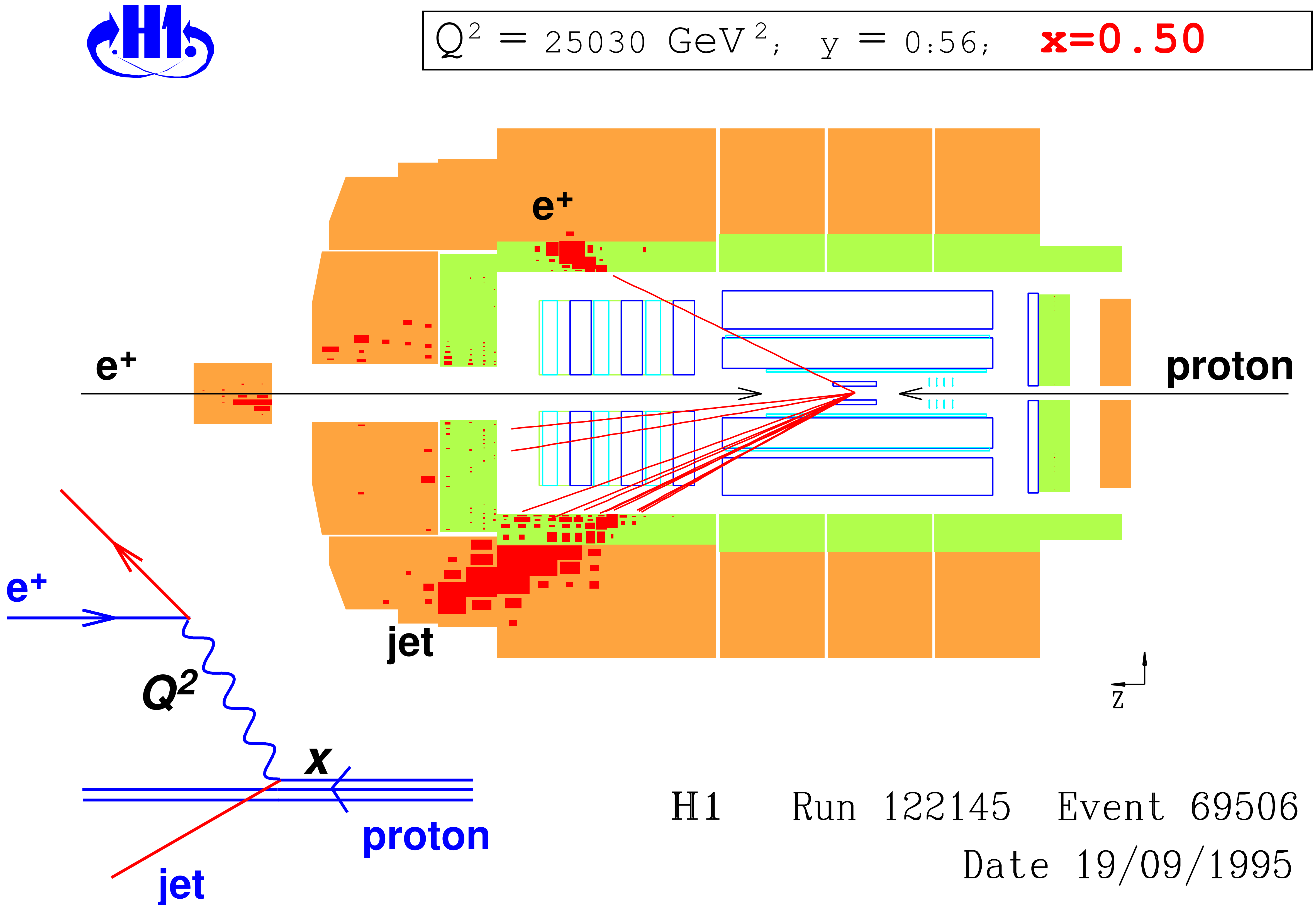
Kinematic relations:

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ = c.o.m. energy

- Q^2 = photon virtuality $\leftrightarrow$ *transverse resolution* at which it probes proton structure
- x = *longitudinal momentum fraction* of struck parton in proton
- y = momentum fraction lost by electron (in proton rest frame)

DEEP INELASTIC SCATTERING



Write DIS X-section to zeroth order in α_s ('quark parton model'):

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

$\propto F_2^{em}$ [structure function]

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

[$u(x)$, $d(x)$: parton distribution functions (PDF)]

NB:

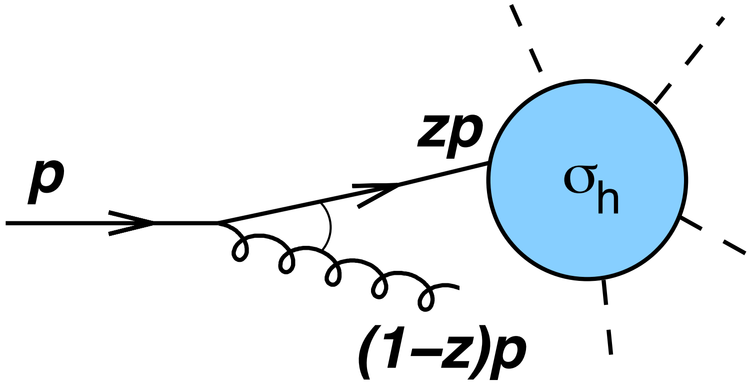
- use perturbative language for interactions of up and down quarks
- but distributions themselves have a *non-perturbative* origin.

Higher order corrections from initial state splittings?

not in handout

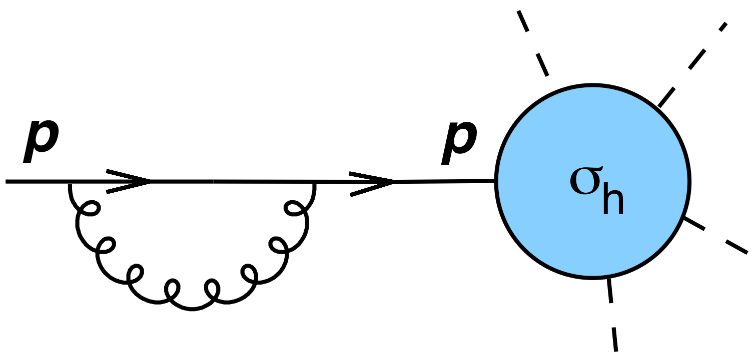
For initial state splitting, hard process occurs *after splitting*, and momentum entering hard process is modified: $p \to zp$.

$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



For virtual terms, momentum entering hard process is unchanged

$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



Total cross section gets contribution with *two different hard X-sections*

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\int \frac{dk_t^2}{k_t^2}\frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]$$

NB: We assume σ_h involves momentum transfers $\sim Q \gg k_t$, so ignore extra transverse momentum in σ_h

not in handout

Higher order corrections from initial state splittings?

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

- In soft limit ($z \to 1$), $\sigma_h(zp) - \sigma_h(p) \to 0$: *soft divergence cancels*.
- For $1 - z \neq 0$, $\sigma_h(zp) - \sigma_h(p) \neq 0$, so z integral is non-zero but finite.

BUT: k_t integral is just a factor, and is *infinite*

This is a collinear ($k_t \to 0$) divergence.

Cross section with incoming parton is not collinear safe!

This always happens with coloured initial-state particles

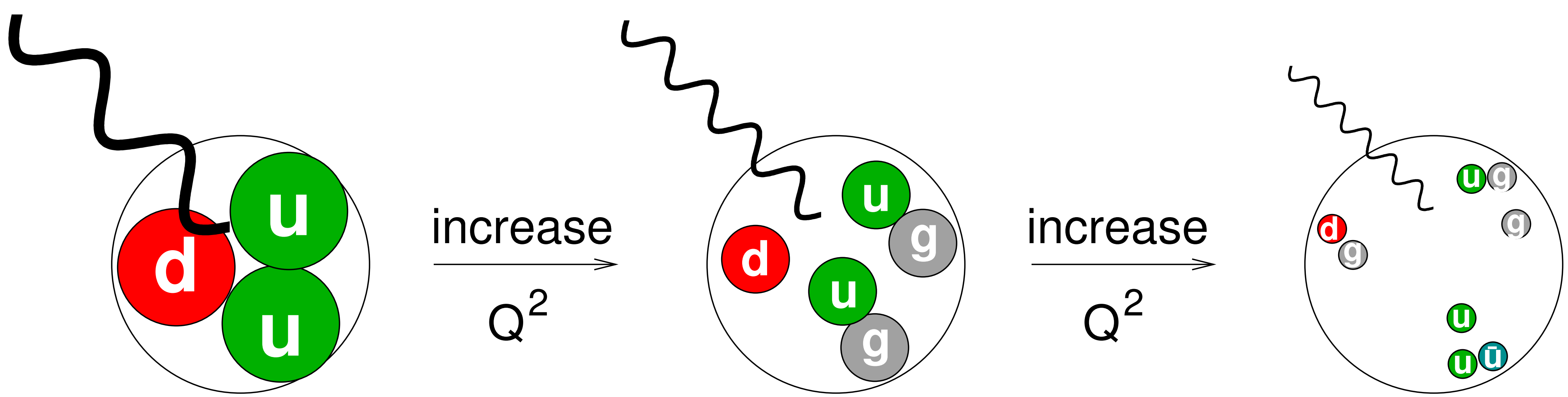
So how do we do QCD calculations in such cases?

Parton distributions and DGLAP

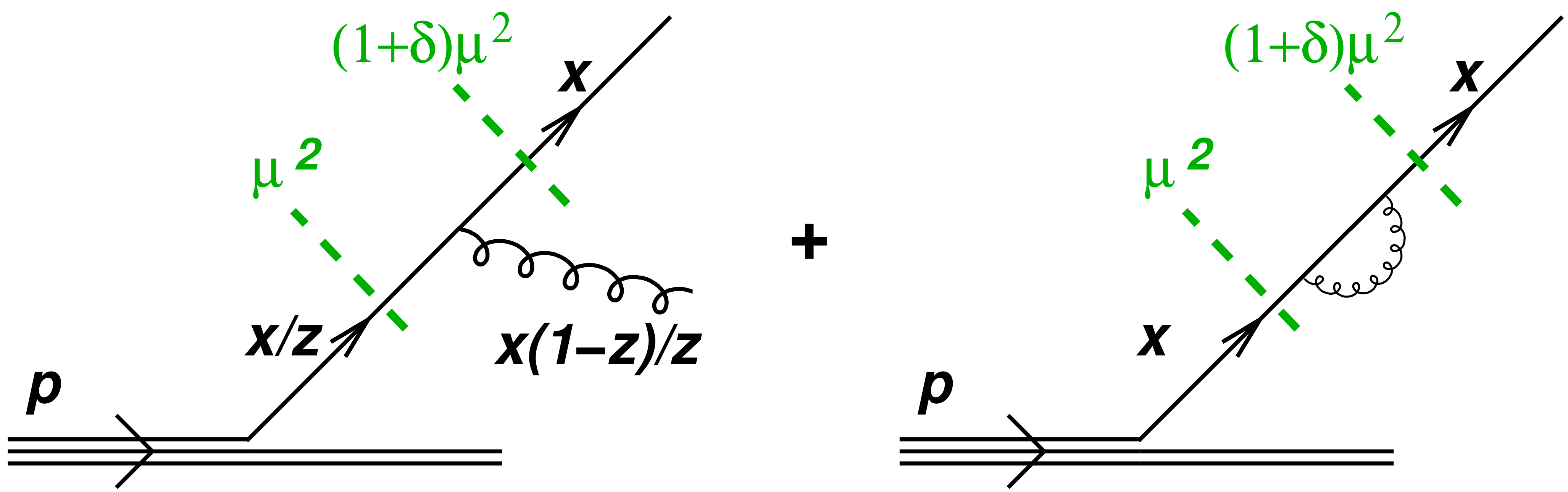
- Write up-quark distribution in proton as

$$u(x, \mu_F^2)$$

- Perturbative collinear (IR) divergence absorbed into the parton distribution (NB divergence not physical: non-perturbative physics provides a physical cutoff)
- μ_F is the **factorisation scale** — a bit like the renormalisation scale (μ_R) for the running coupling.
- As you vary the factorisation scale, the parton distributions evolve with a renormalisation-group type equation



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations *take derivative* wrt factorization scale μ^2



$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\, \frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F \frac{1+z^2}{1-z}$

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_0^1 dz\,[g(z)]_+\,f(z) = \int_0^1 dz\,g(z)\,f(z) - \int_0^1 dz\,g(z)\,f(1)$$

$z=1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z=1$

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

$$P_{qg}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \delta(1-z)\frac{(11C_A - 4n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$* (except virtuals)
- P_{qq}, P_{gg}: *diverge for $z \to 1$* soft gluon emission
- P_{gg}, P_{gq}: *diverge for $z \to 0$* Implies PDFs grow for $x \to 0$

2015 EPS HEP prize to Bjorken, Altarelli, Dokshitzer, Lipatov & Parisi

NLO DGLAP

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P^{(0)} + \frac{\alpha_s^2}{16\pi^2} P^{(1)}$$

Curci, Furmanski & Petronzio '80

$$P^{(1)}_{\rm ps}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2\left[\frac{8}{3}{\rm H}_0 - \frac{56}{9}\right] + (1+x)\left[5{\rm H}_0 - 2{\rm H}_{0,0}\right]\right)$$

$$P^{(1)}_{\rm qg}(x) = 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x){\rm H}_{-1,0} - 2p_{\rm qg}(x){\rm H}_{1,1} + x^2\left[\frac{44}{3}{\rm H}_0 - \frac{218}{9}\right] + 4(1-x)\left[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1\right] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0\right) + 4\, C_F n_f \left(2p_{\rm qg}(x)\left[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \zeta_2\right] + 4x^2\left[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4}\right] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0\right)$$

$$P^{(1)}_{\rm gq}(x) = 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1\right] - x^2\left[\frac{8}{3}{\rm H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2 - 7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x)\left[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x){\rm H}_{-1,0}\right) - 4\, C_F n_f \left(\frac{2}{3}x - p_{\rm gq}(x)\left[\frac{2}{3}{\rm H}_1 - \frac{10}{9}\right]\right) + 4\, {C_F}^2 \left(p_{\rm gq}(x)\left[3{\rm H}_1 - 2{\rm H}_{1,1}\right] + (1+x)\left[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0\right] - 3{\rm H}_{0,0} + 1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x\right)$$

$$P^{(1)}_{\rm gg}(x) = 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\, {C_A}^2 \left(27 + (1+x)\left[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12{\rm H}_0 - \frac{44}{3}x^2{\rm H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\, C_F n_f \left(2{\rm H}_0 + \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5{\rm H}_0 - 2{\rm H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right).$$

NNLO DGLAP

$$P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ab}^{(2)}$$

NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

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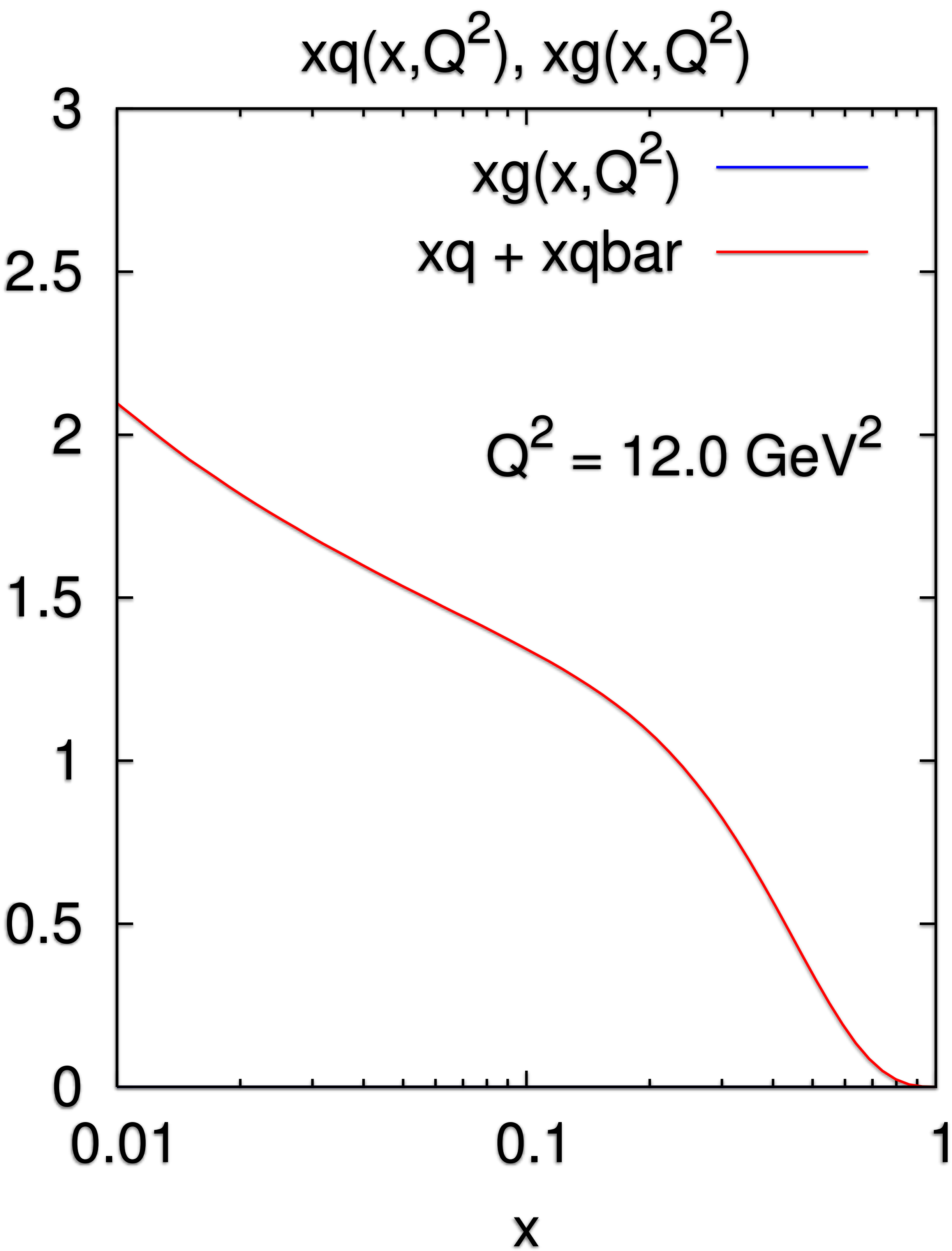
Four-Loop Non-Singlet Splitting Functions in the Planar Limit and Beyond

S. Moch[a], B. Ruijl[b,c], T. Ueda[b], J.A.M. Vermaseren[b] and A. Vogt[d]

arXiv:1707.08315v2 [hep-ph] 5 Oct 2017

DGLAP evolution (initial quarks only)



Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

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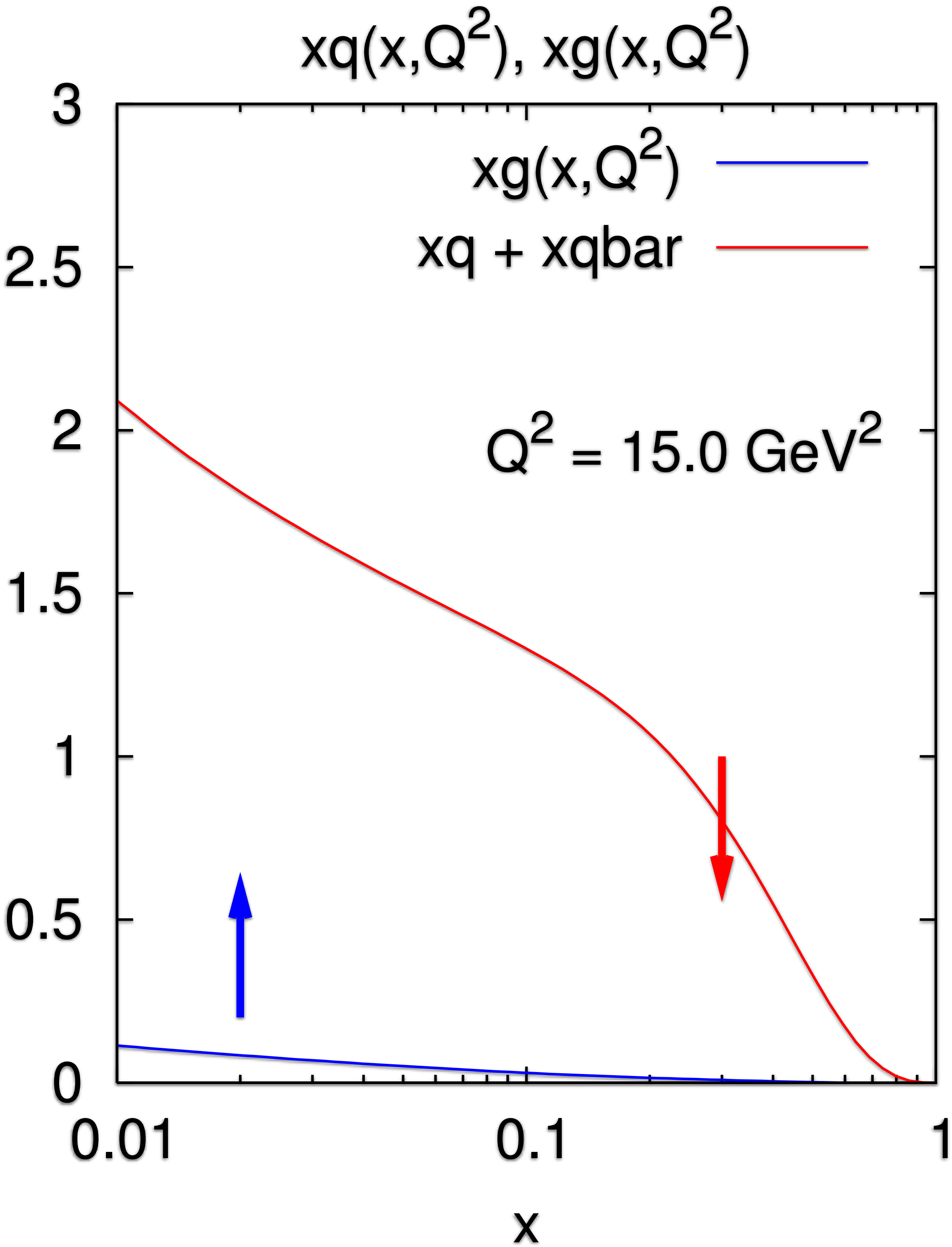

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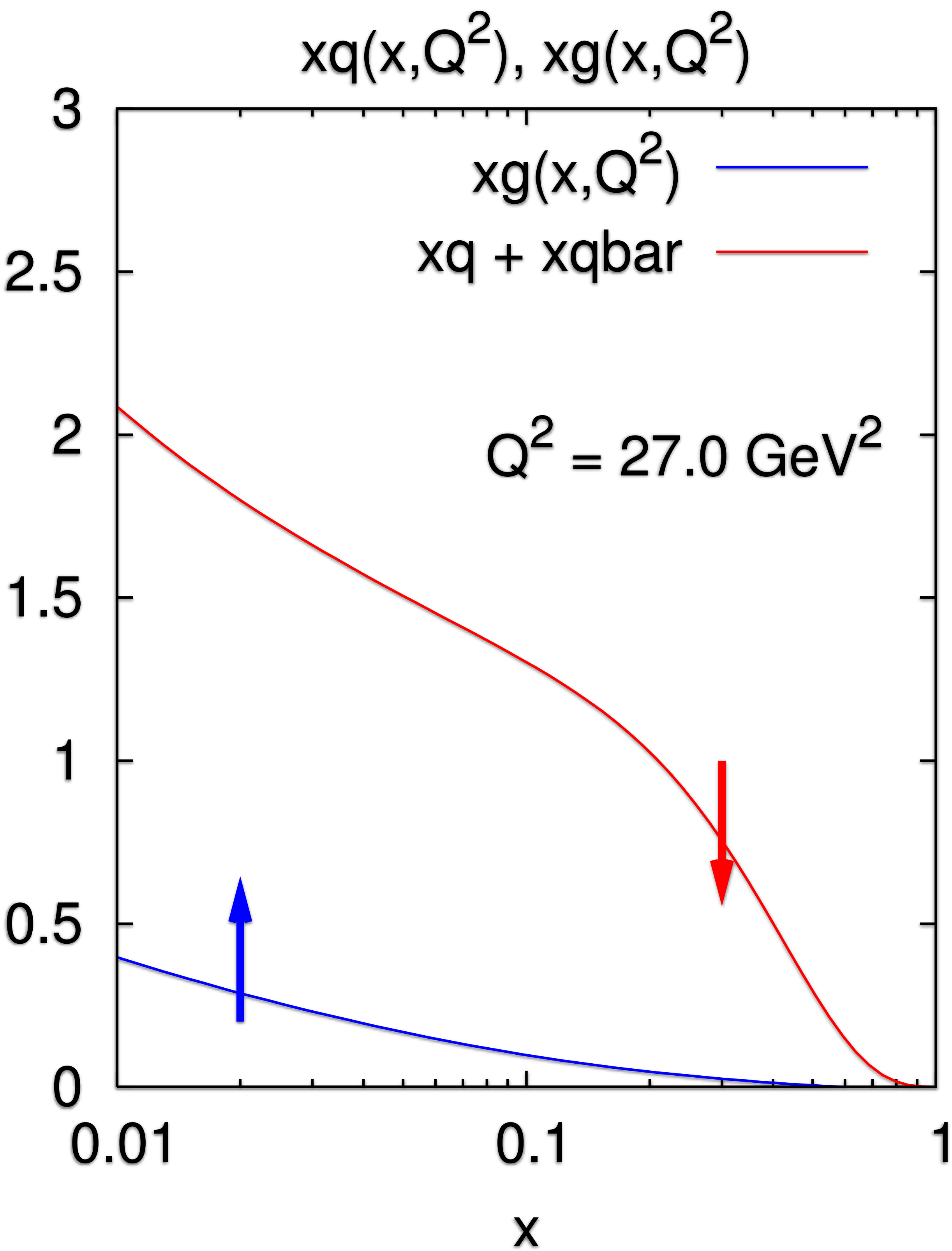

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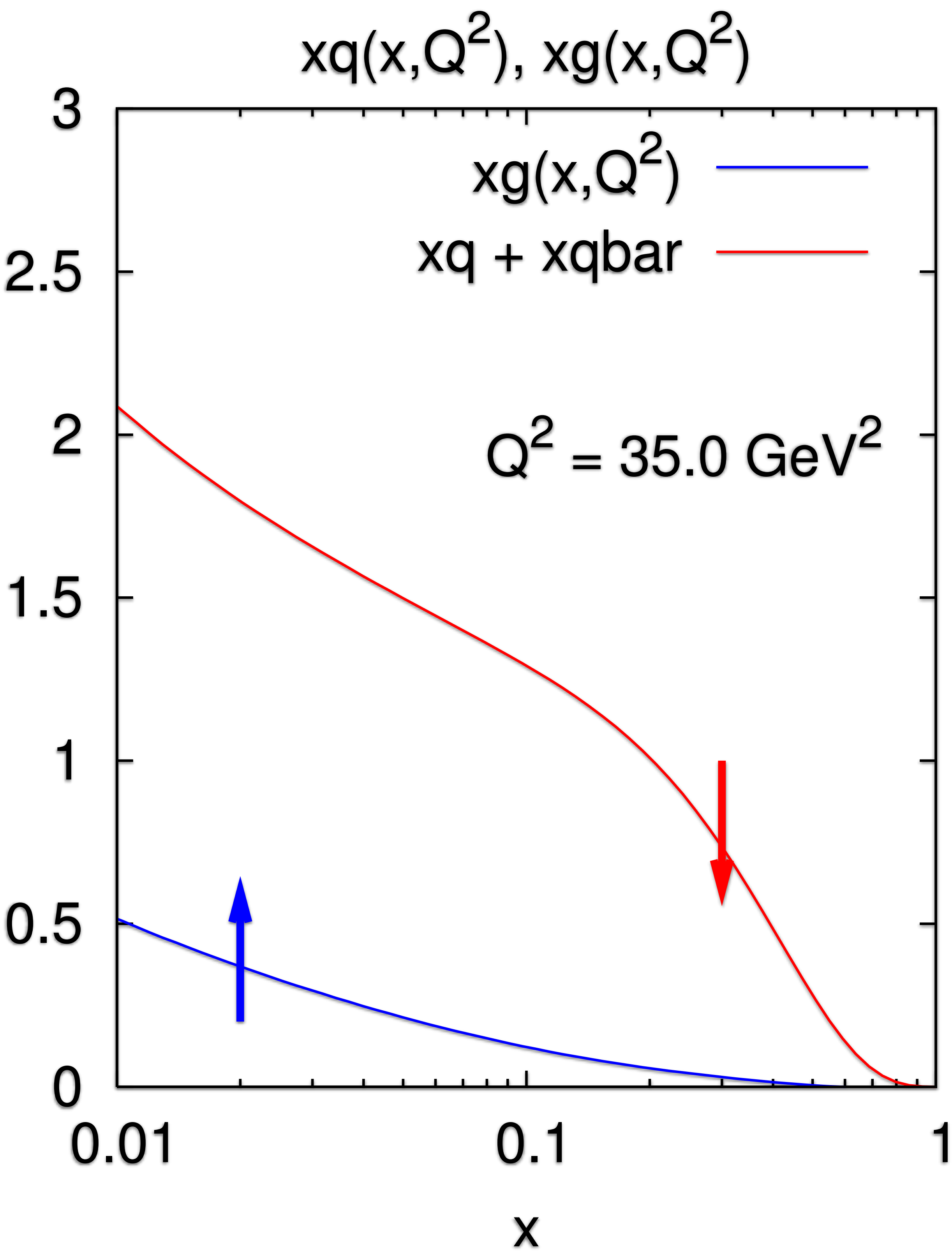

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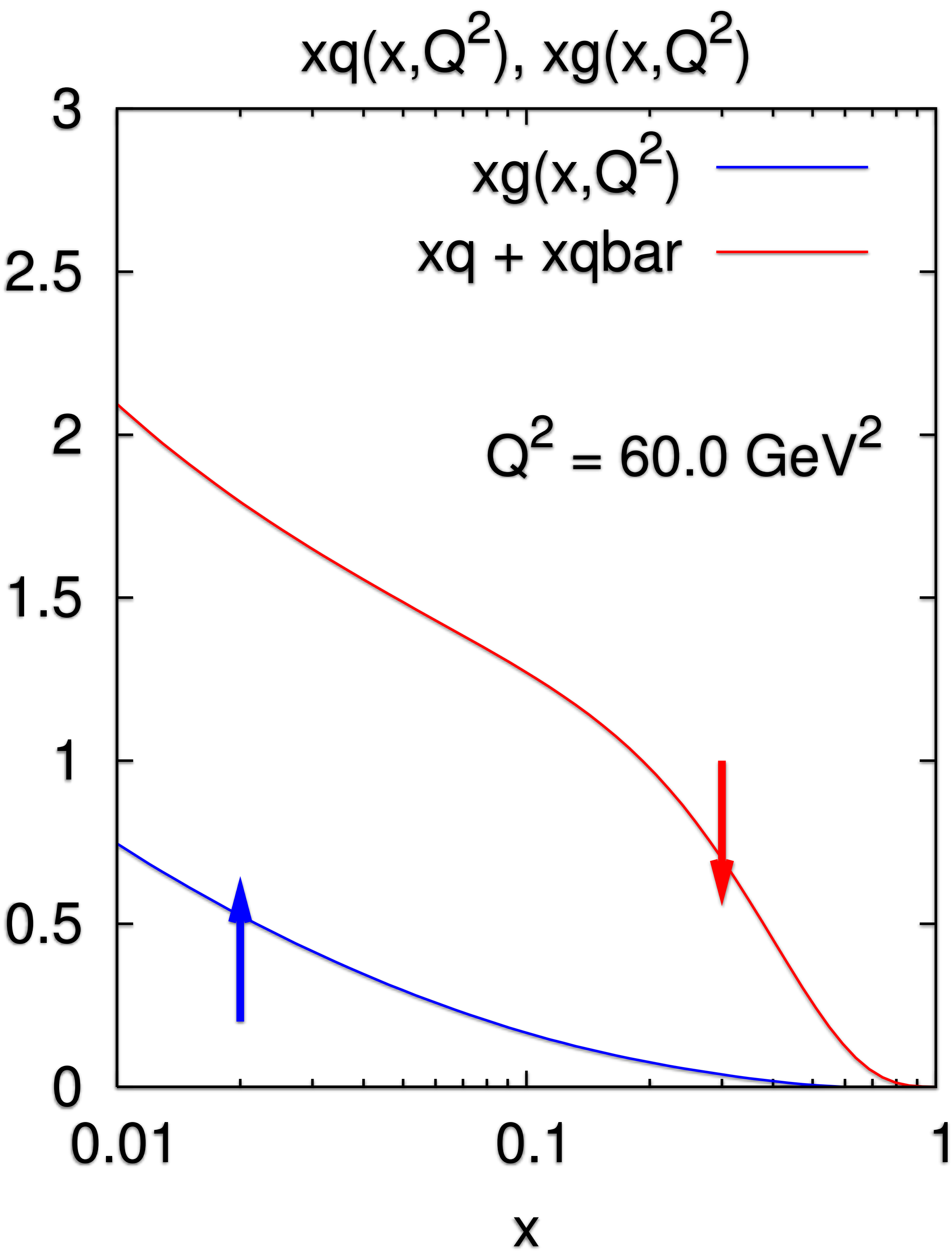
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

DGLAP evolution (initial quarks only)



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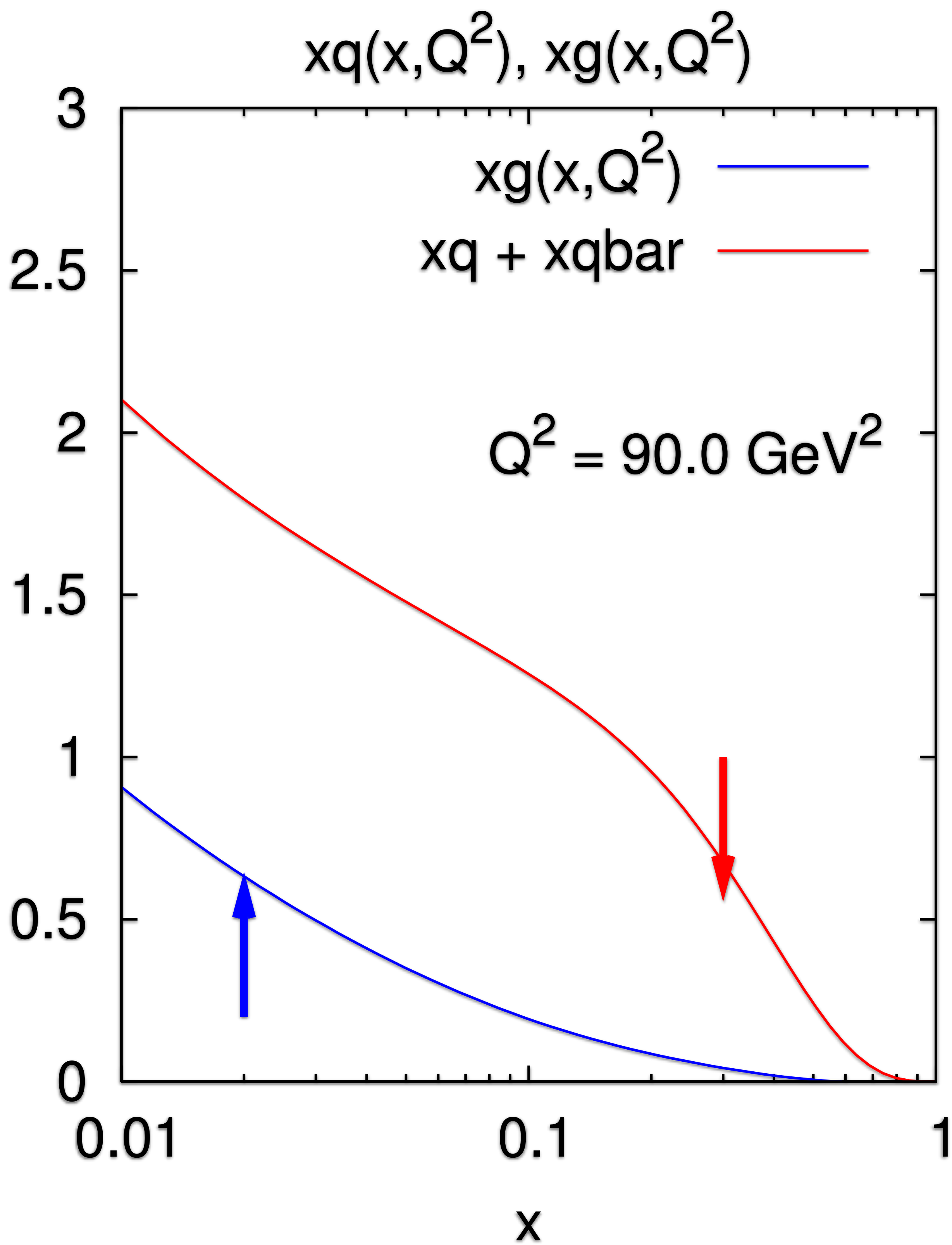

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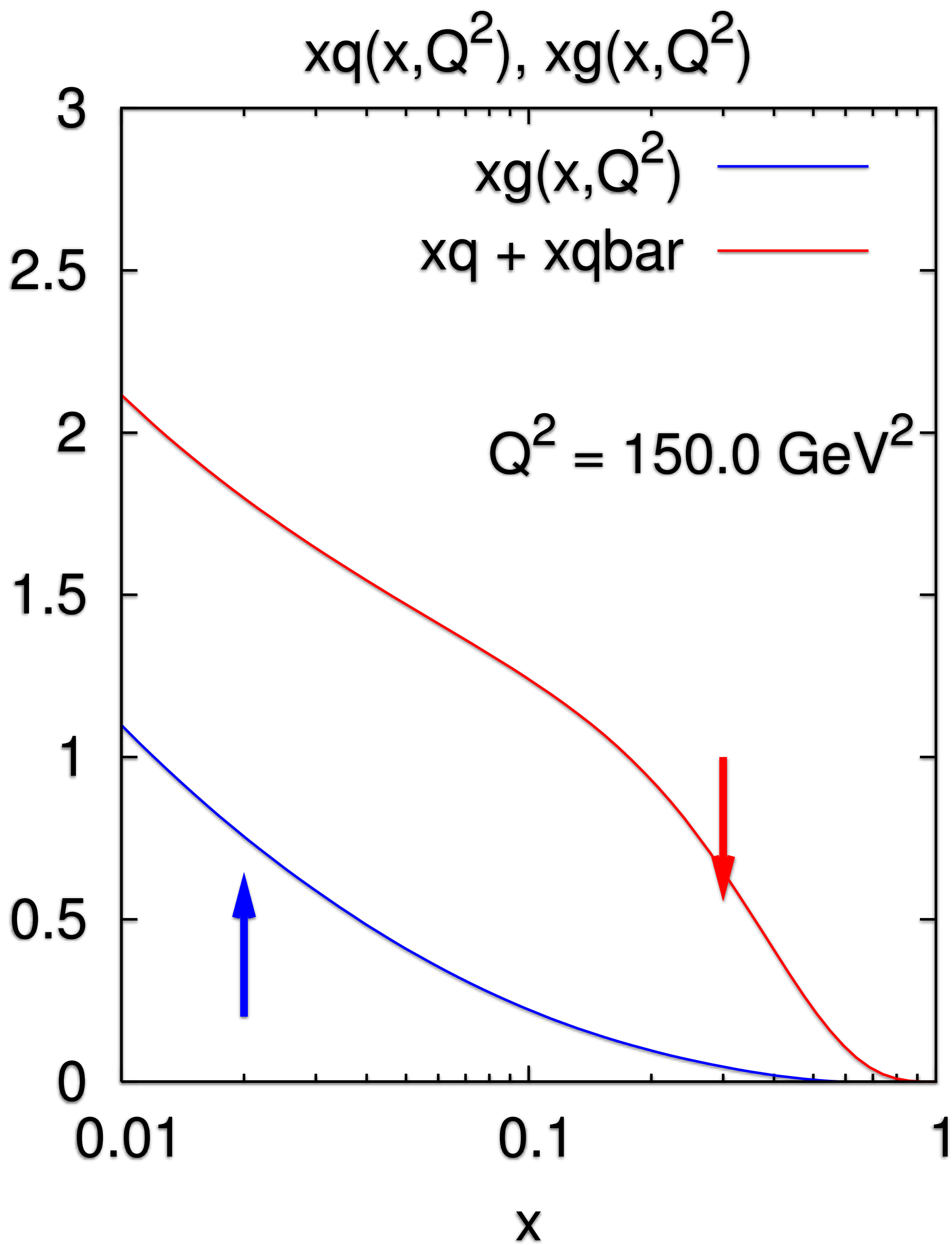

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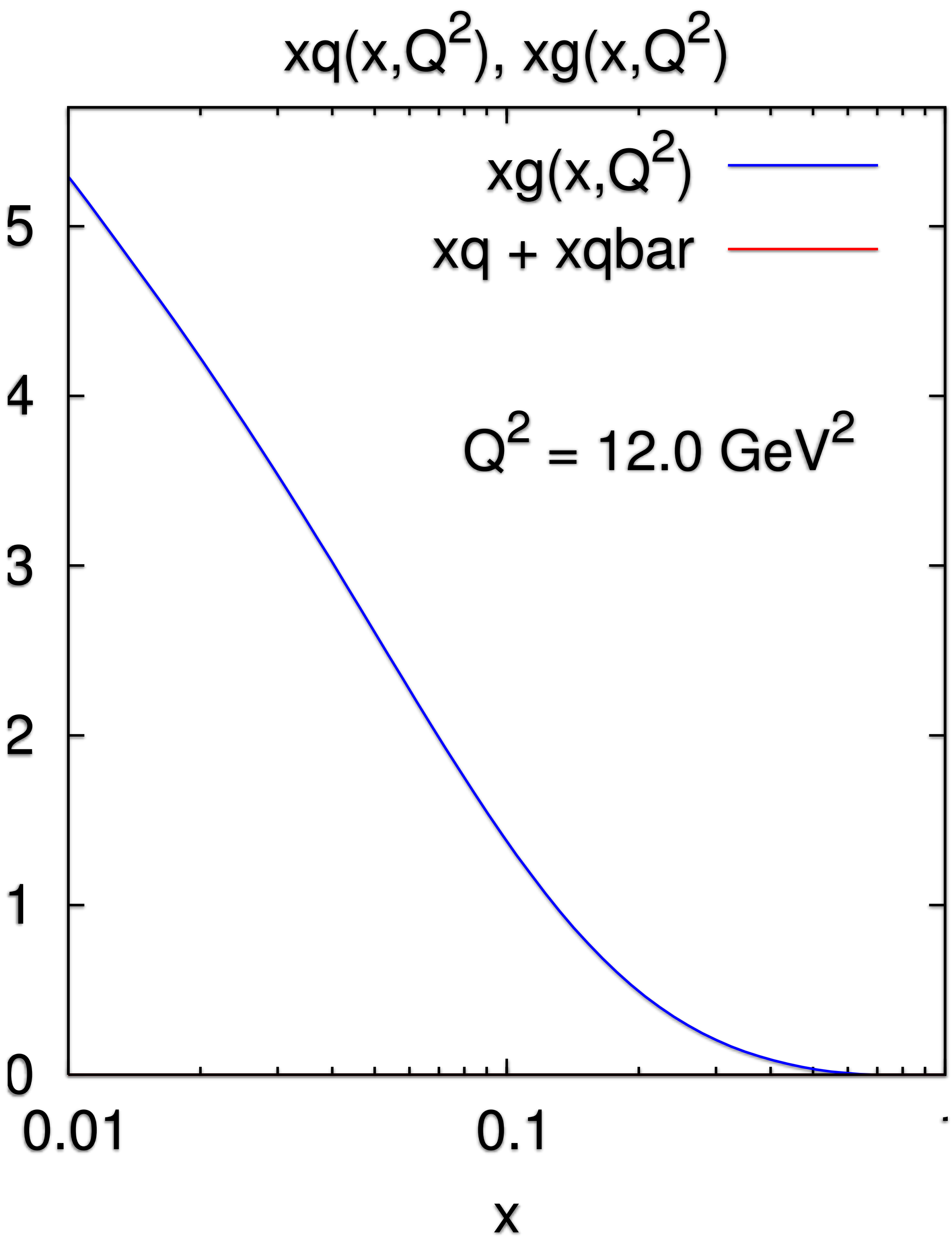

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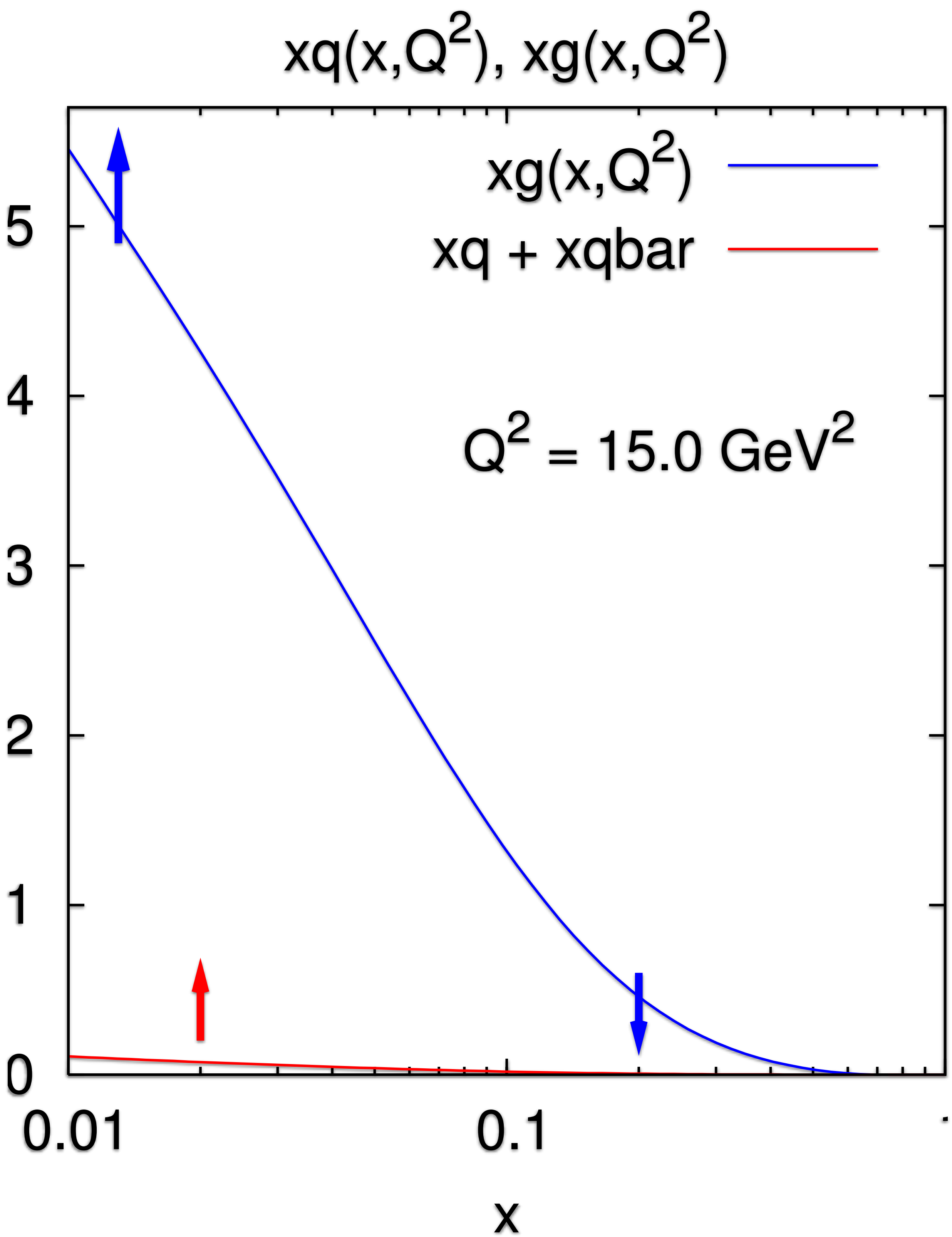

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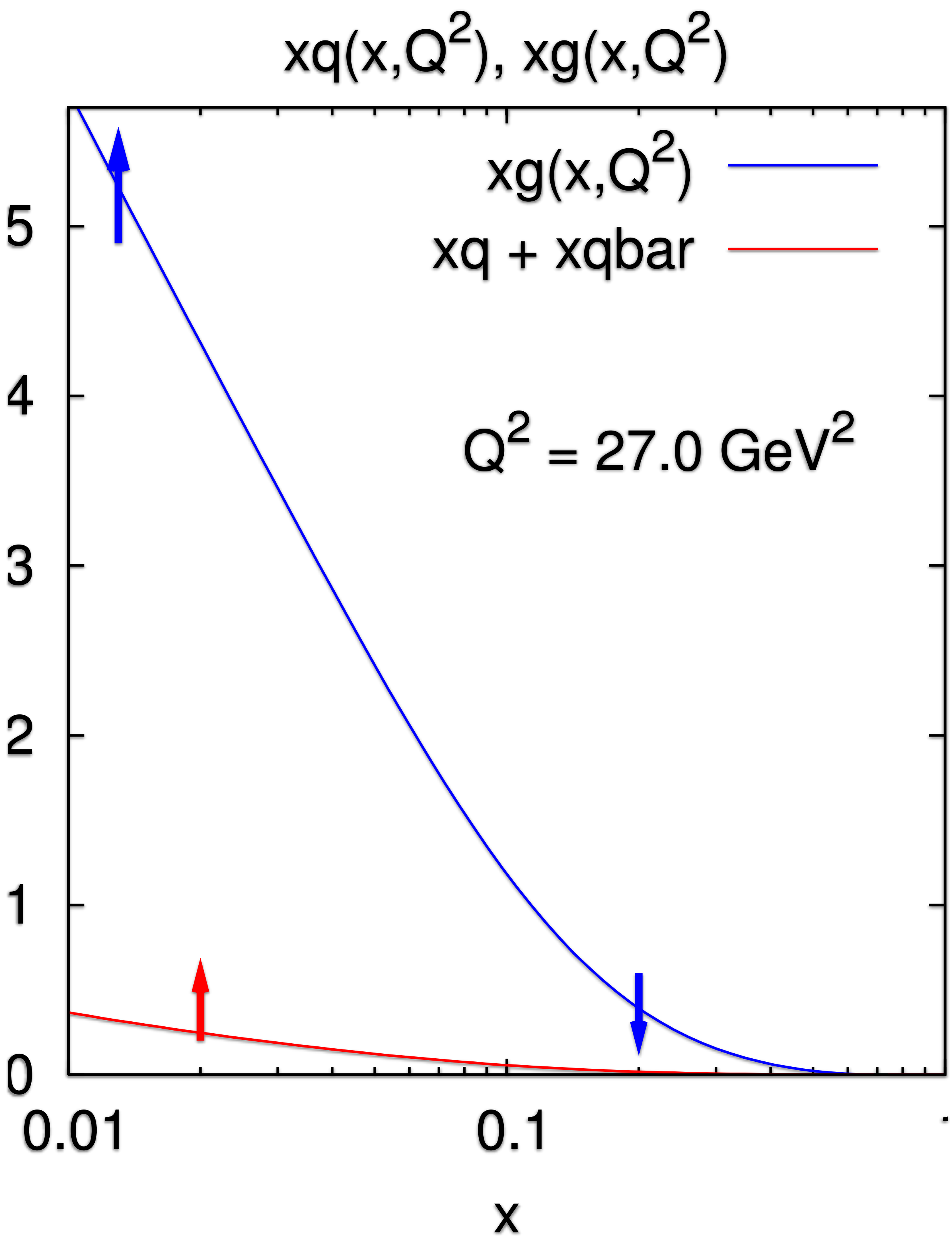

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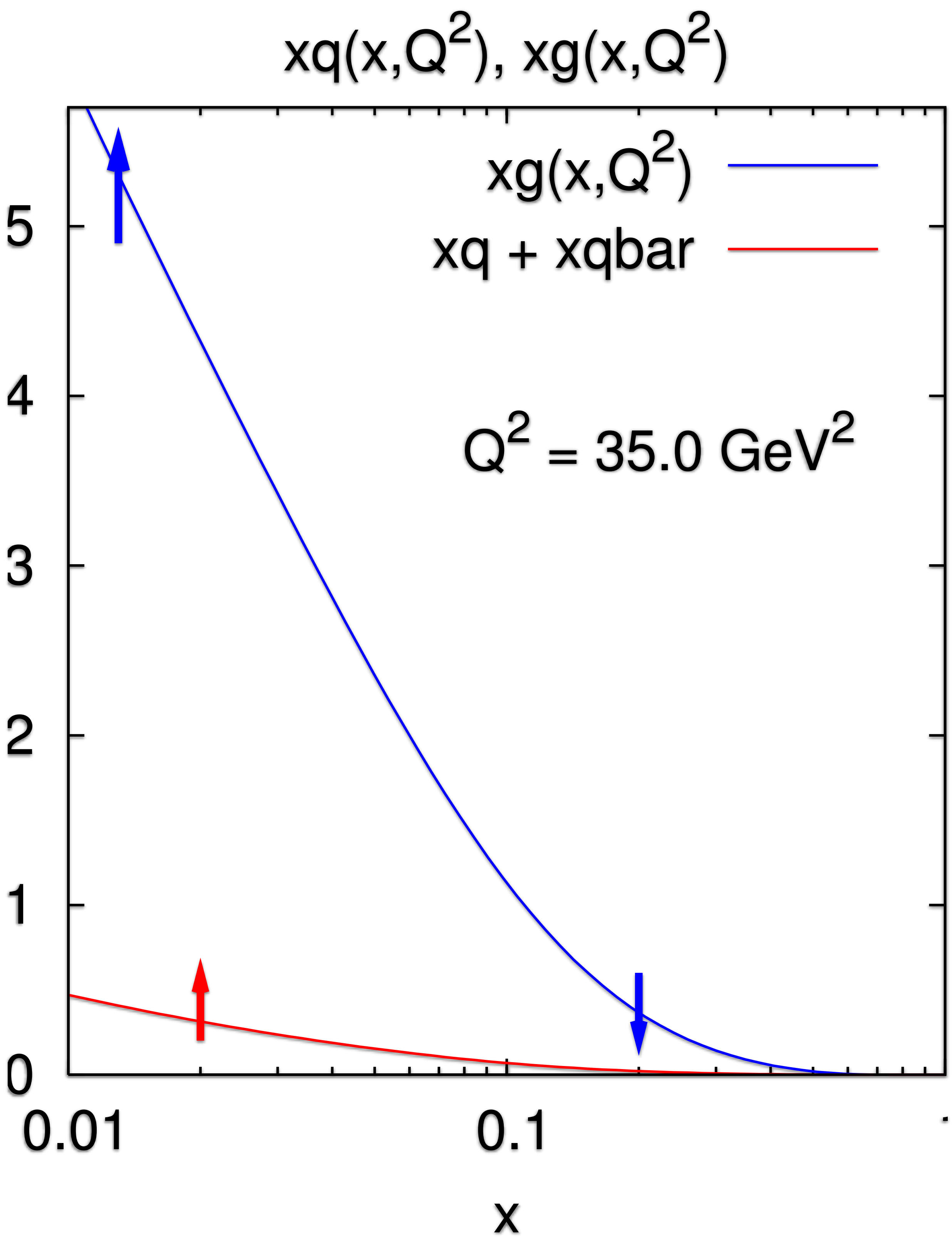

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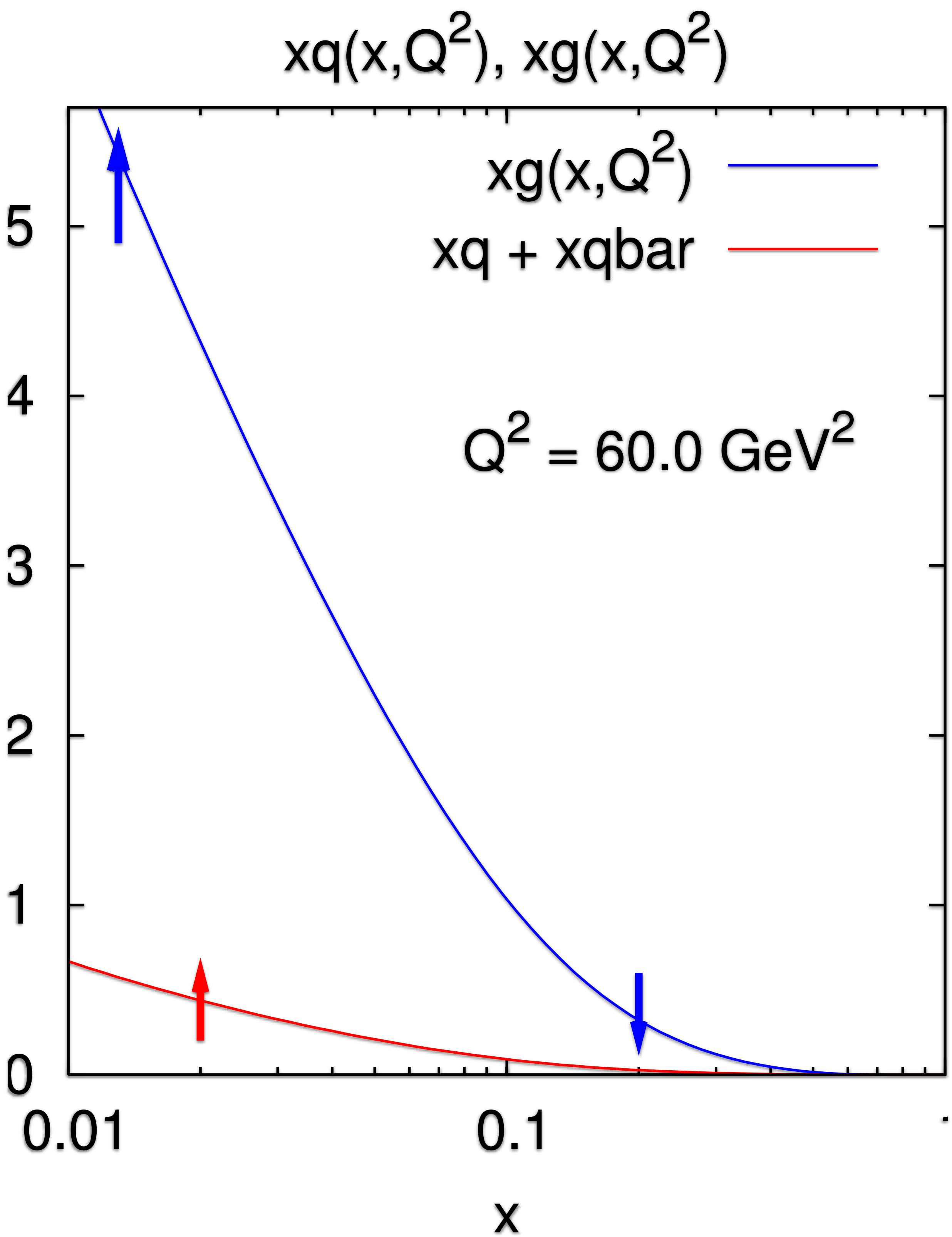

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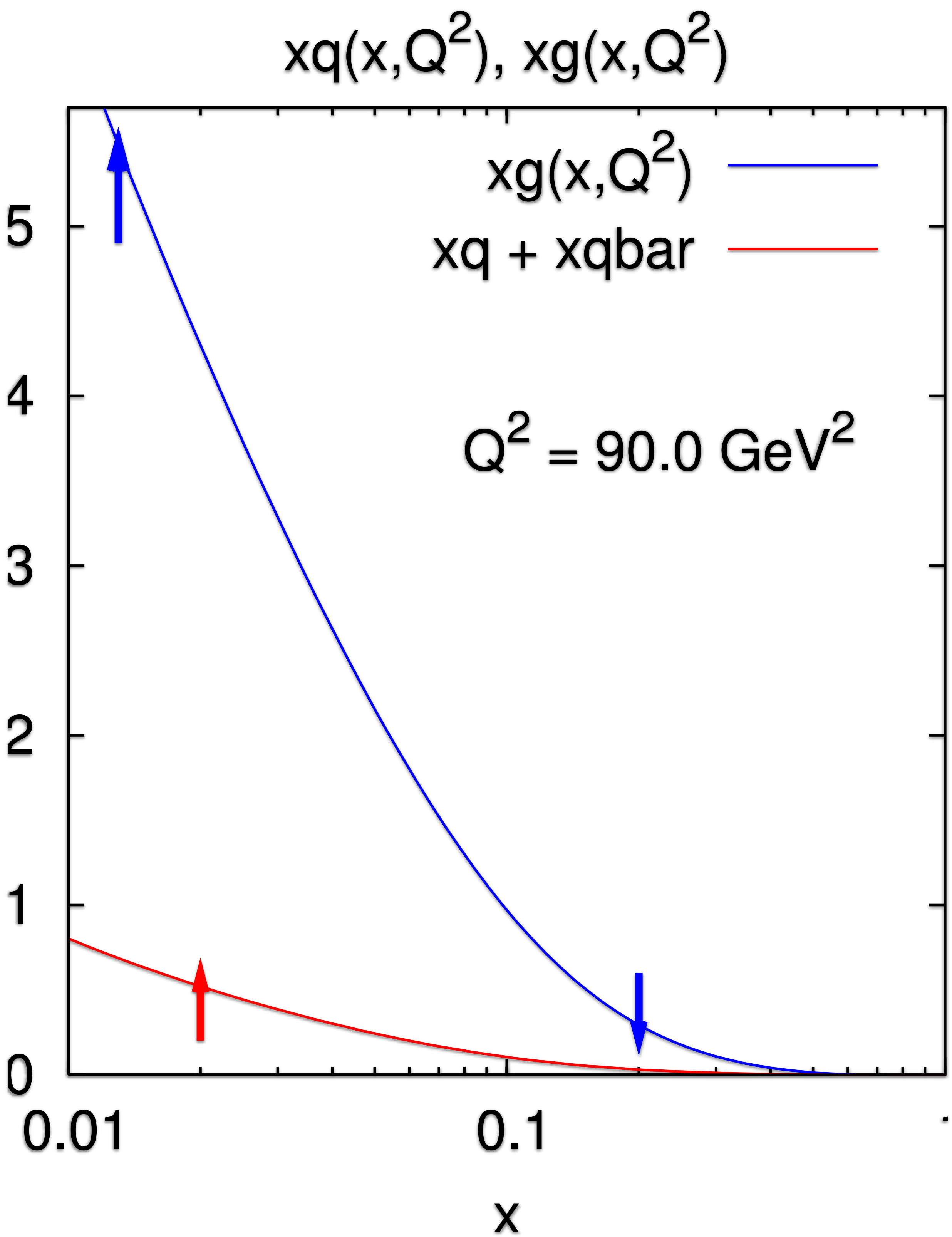

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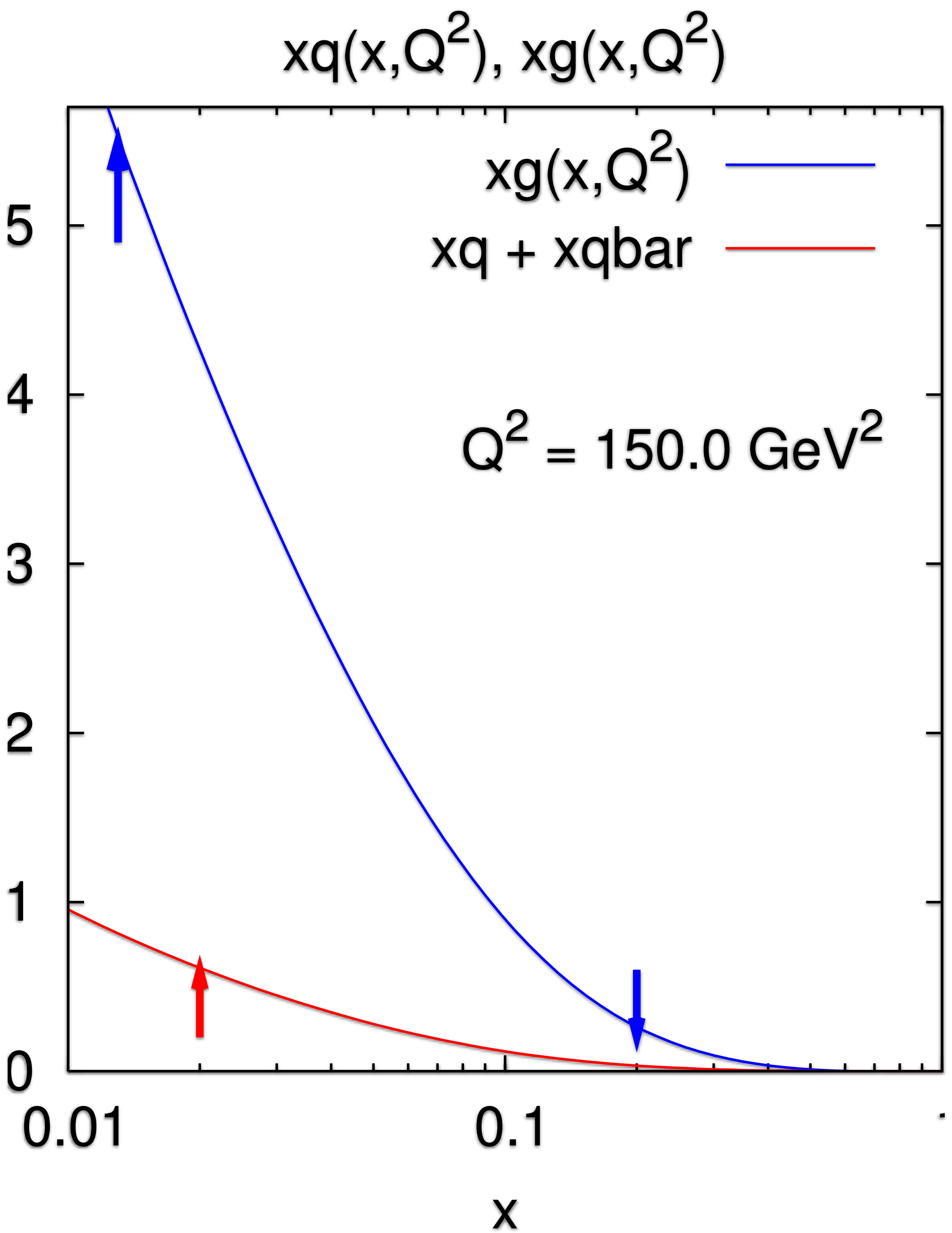

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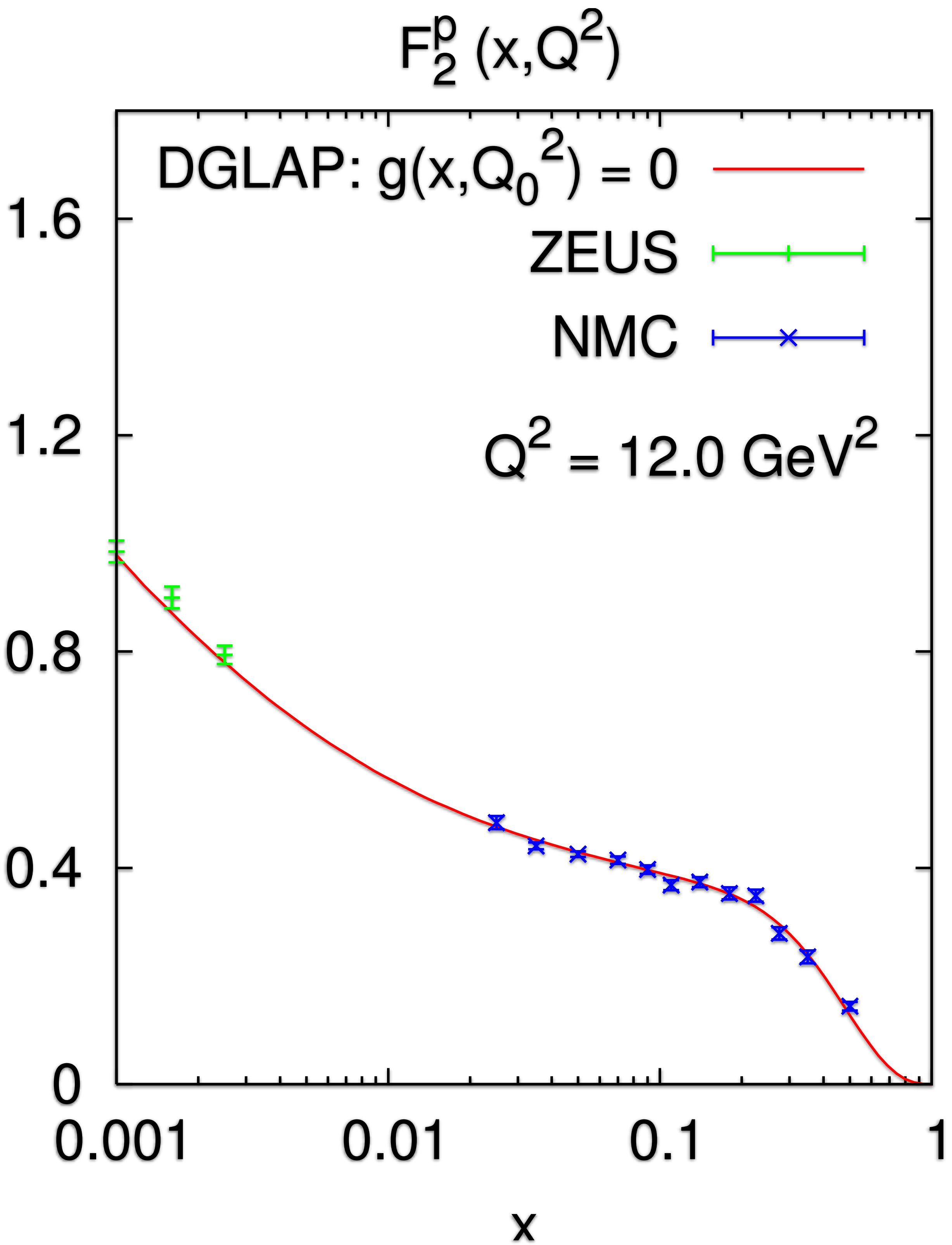
DGLAP evolution:

- partons lose momentum and shift towards smaller x
- high-x partons drive growth of low-x gluon

determining the gluon

which is critical at hadron colliders (e.g. ttbar, Higgs dominantly produced by gluon-gluon fusion), but not directly probed in Deep-Inelastic-Scattering

Consider DIS data — $F_2(x,Q^2)$ — in a world where the proton just had quarks



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

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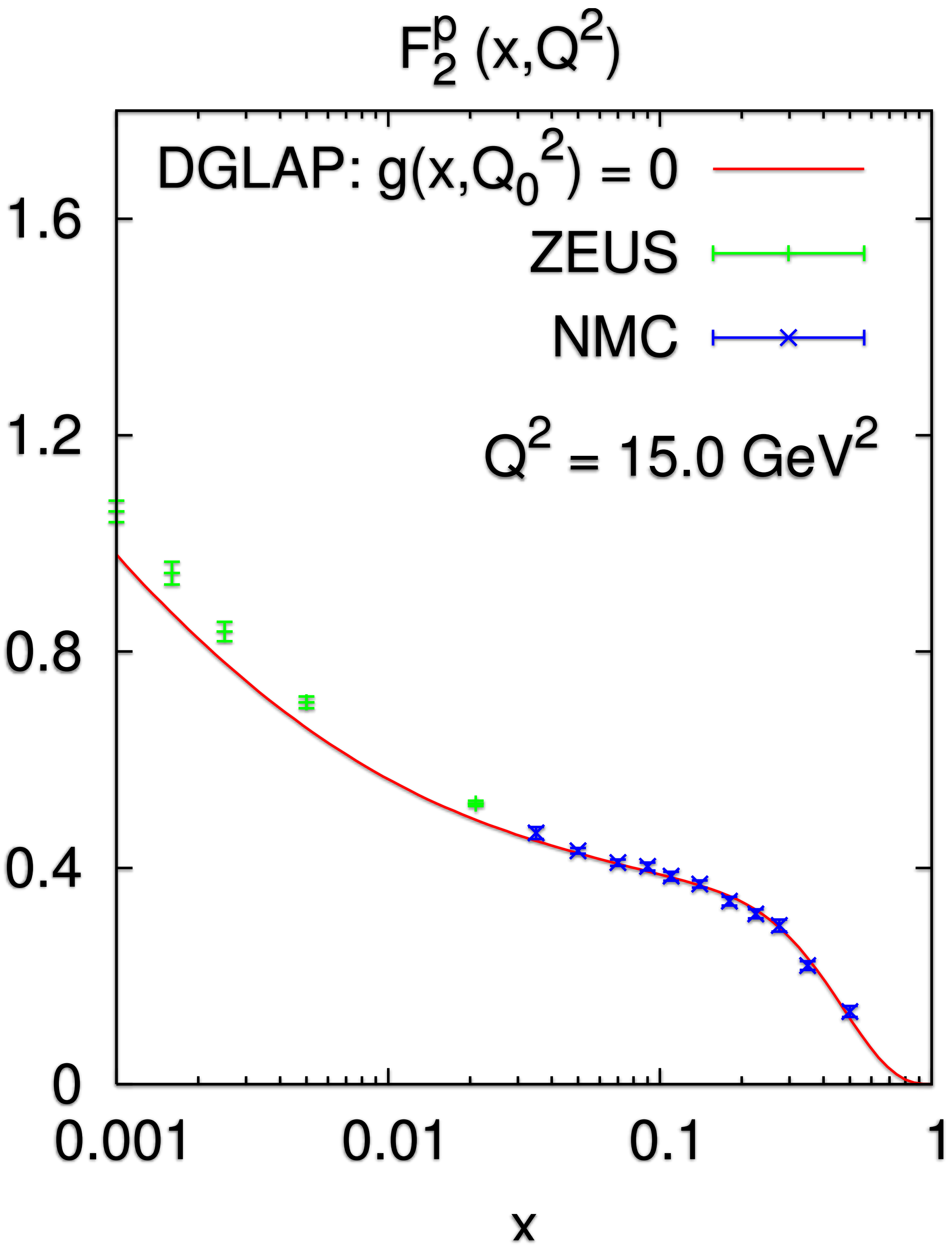

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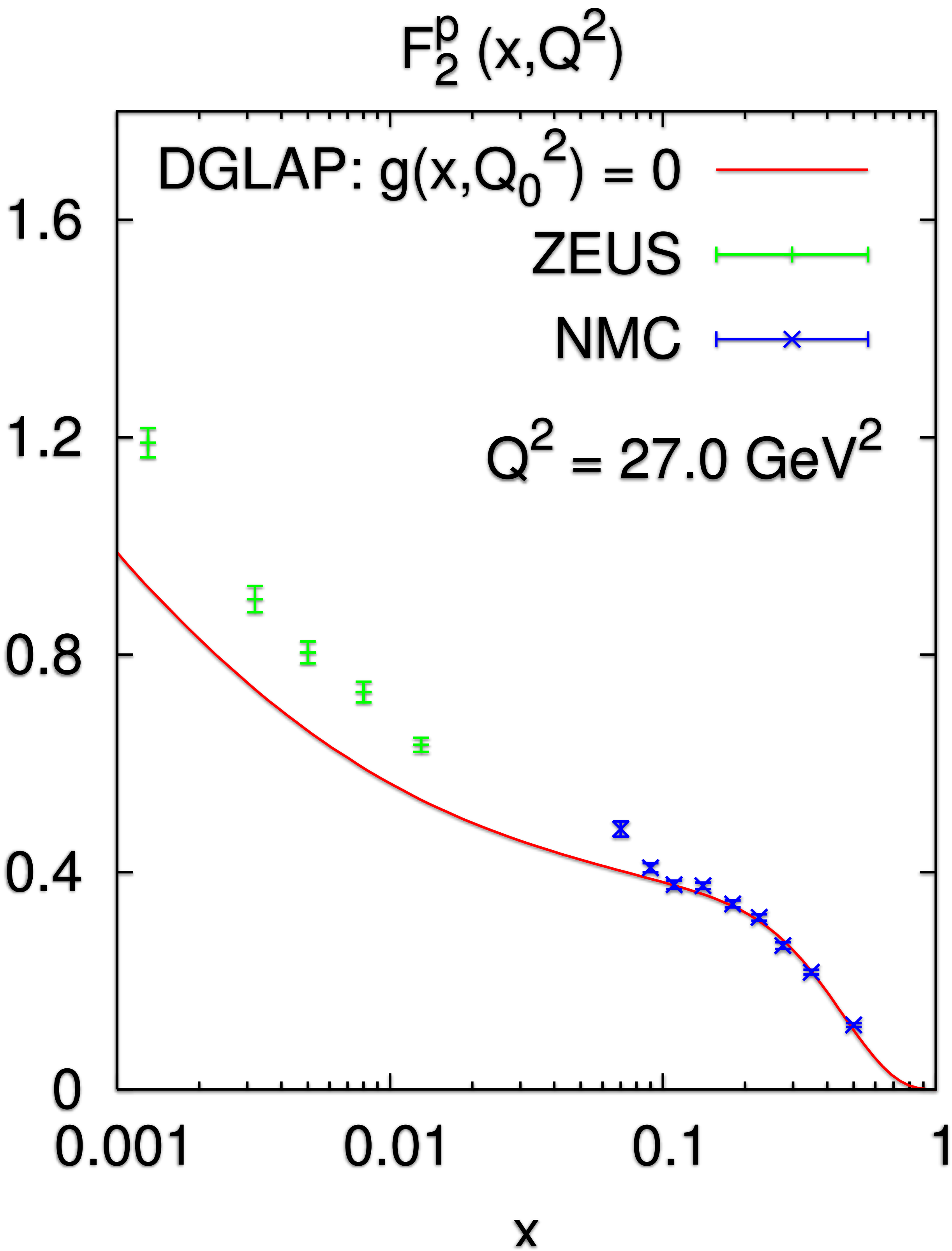

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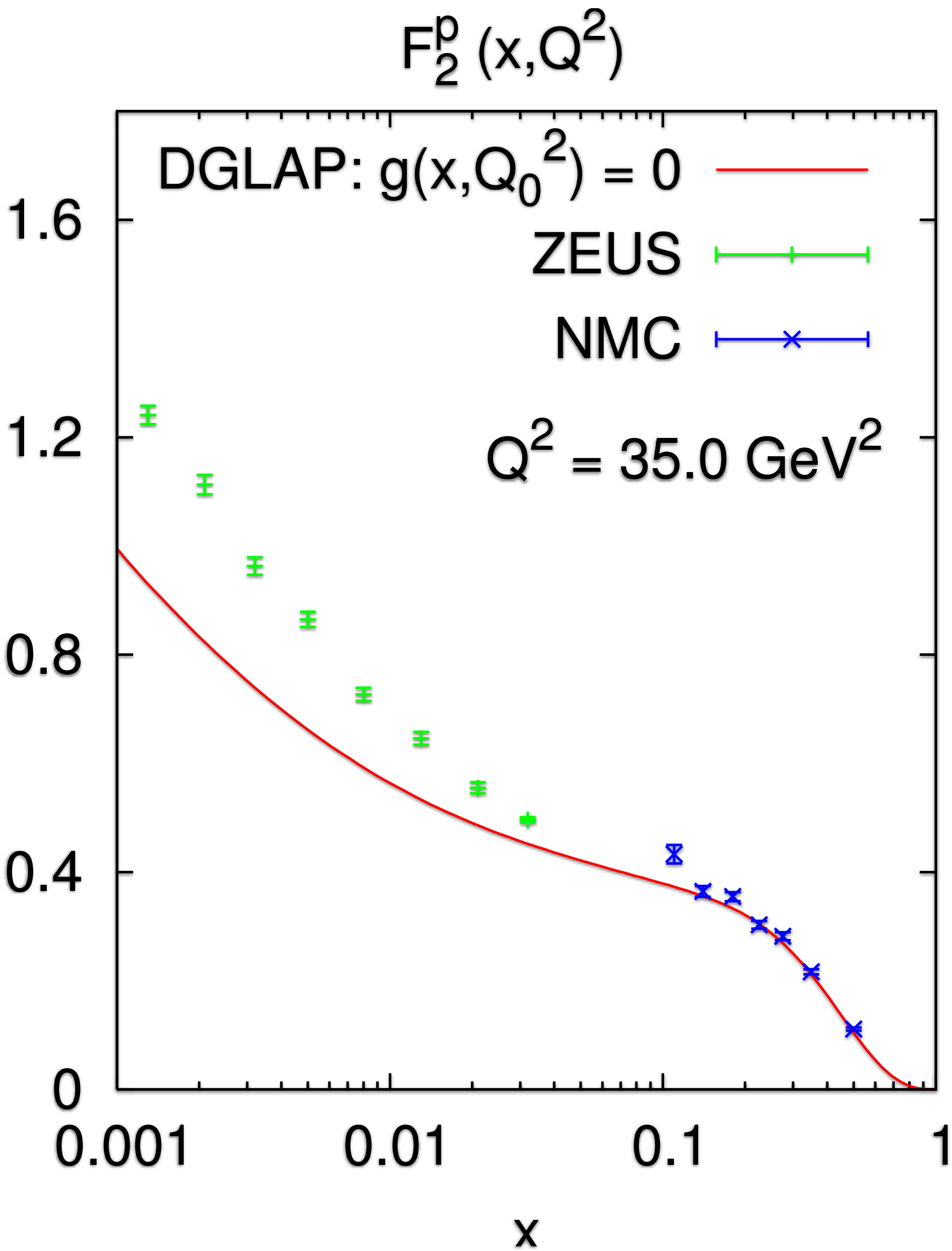

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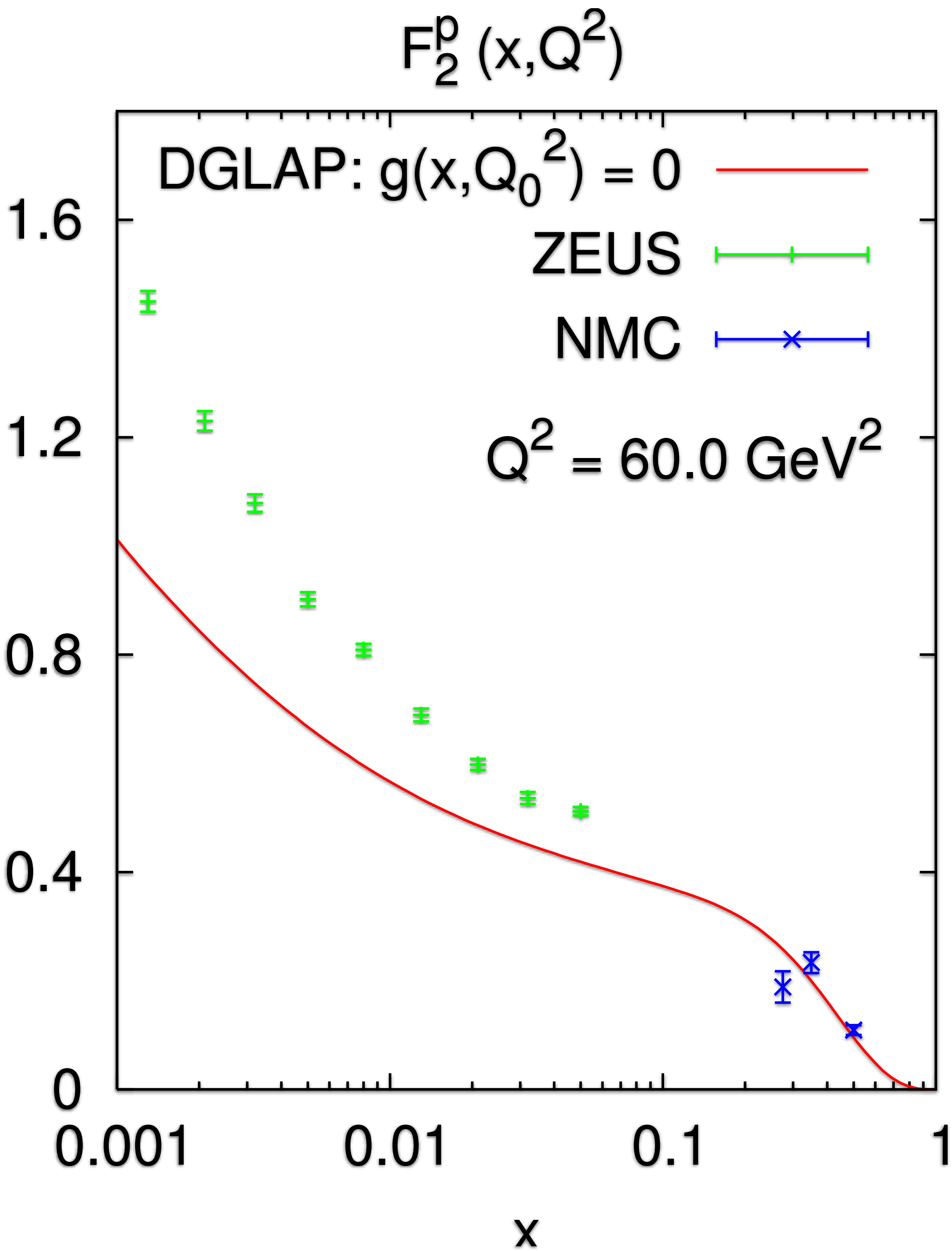

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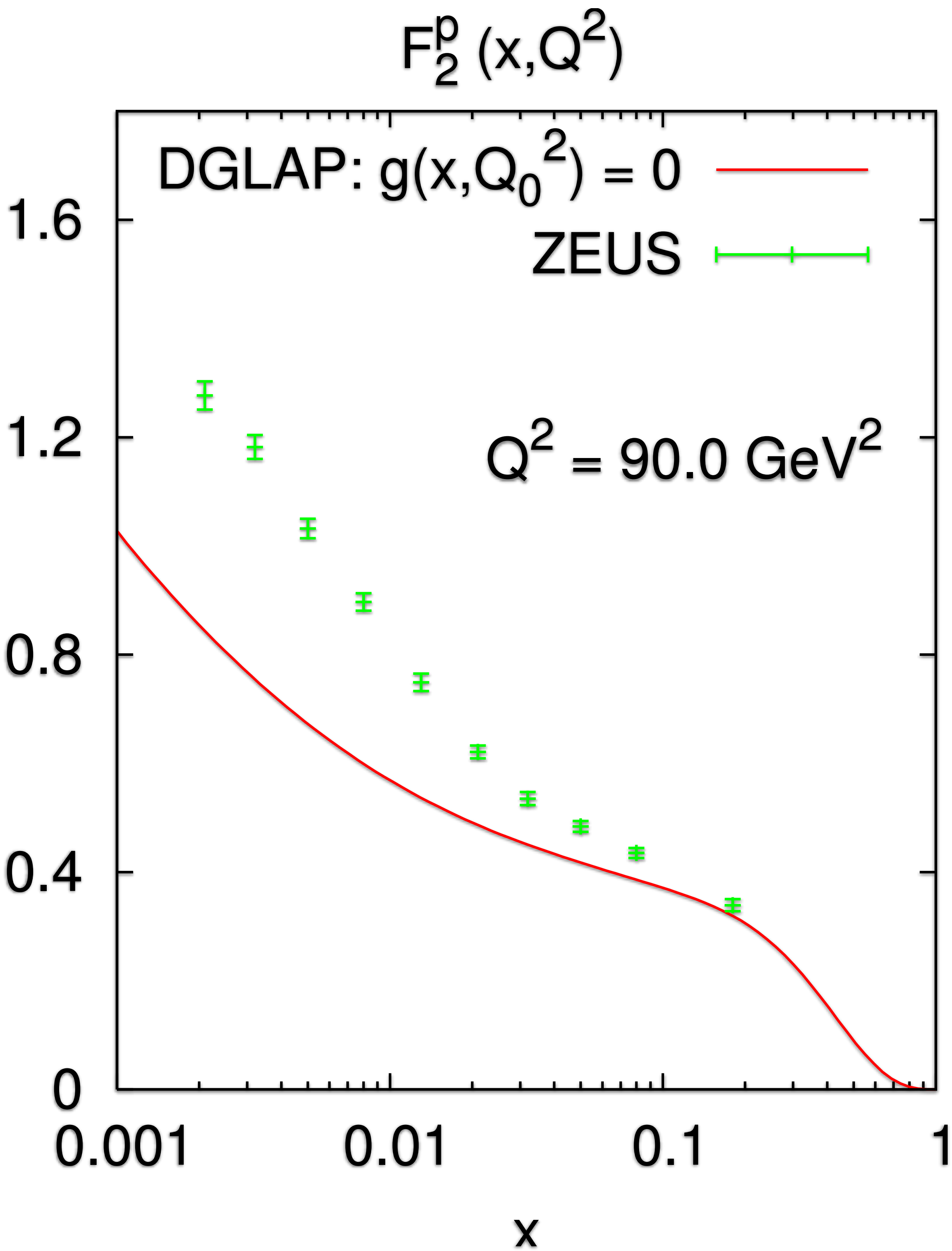

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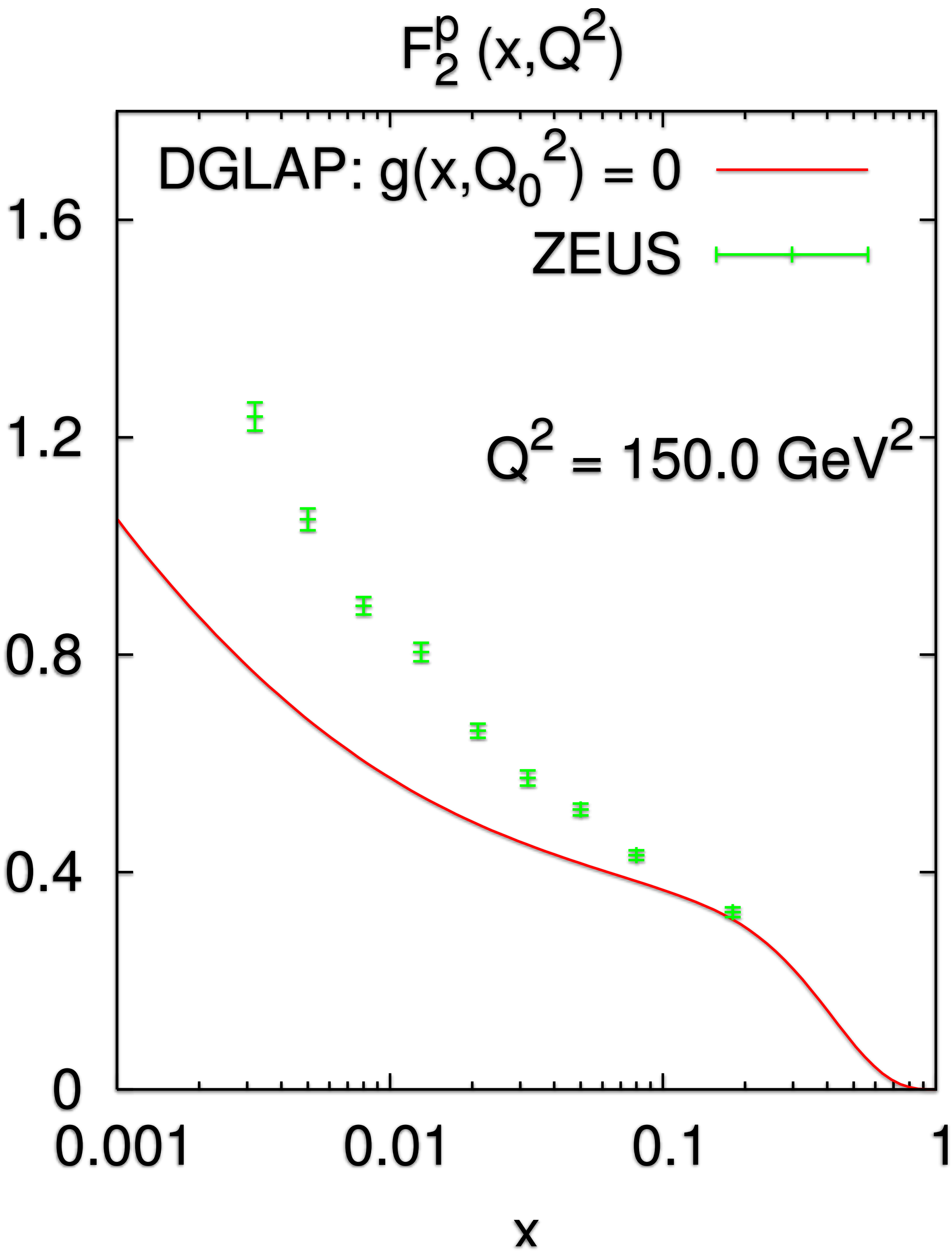

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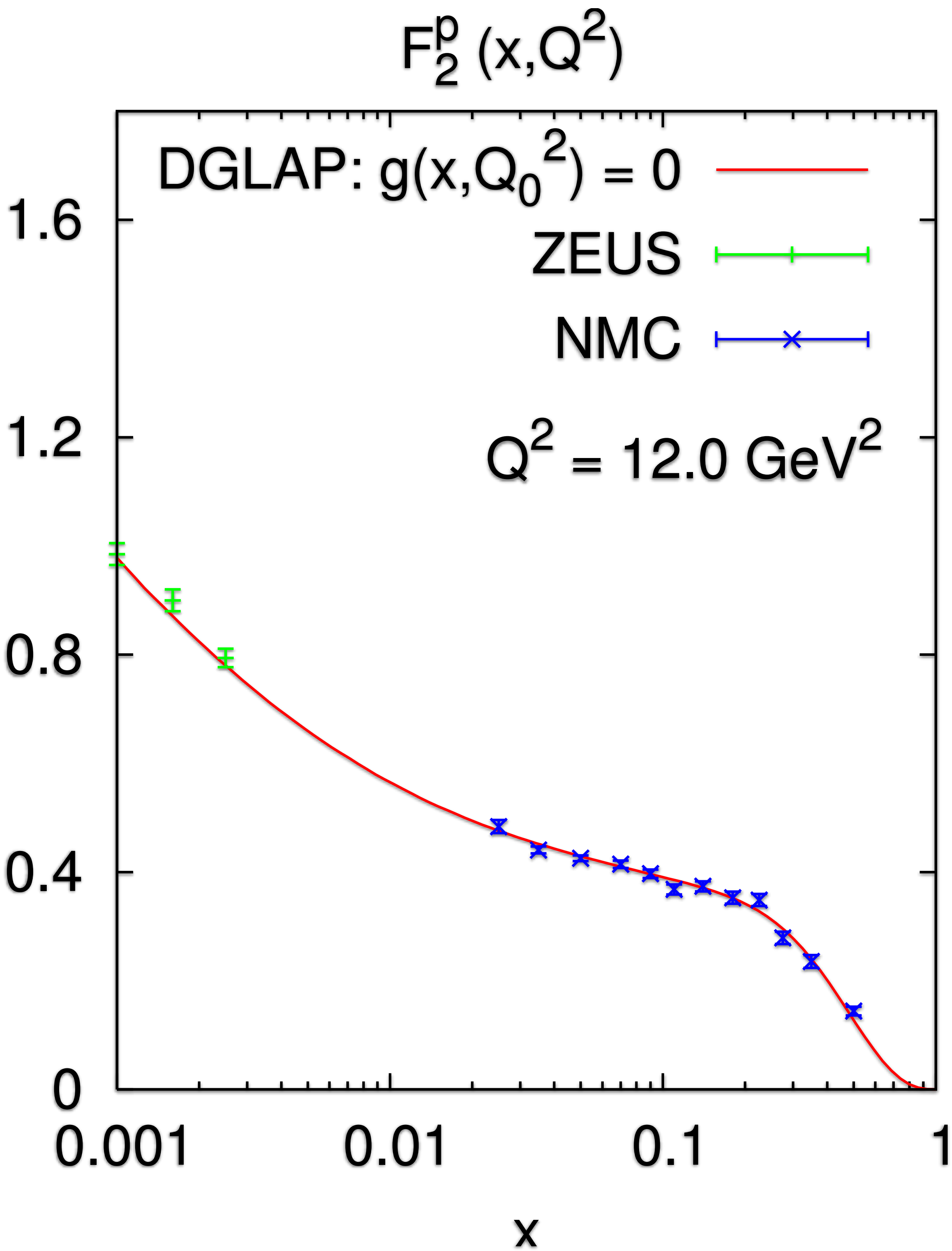
COMPLETE FAILURE
to reproduce data evolution

Consider DIS data — $F_2(x,Q^2)$ — with specially tuned gluon



If gluon ≠ 0, splitting

$$g \to q\bar{q}$$

generates extra quarks at large Q2 ➠ faster rise of F2

Global PDF fits (CT, MMHT, NNPDF, etc.) choose gluon distribution that leads to the correct Q2 evolution.

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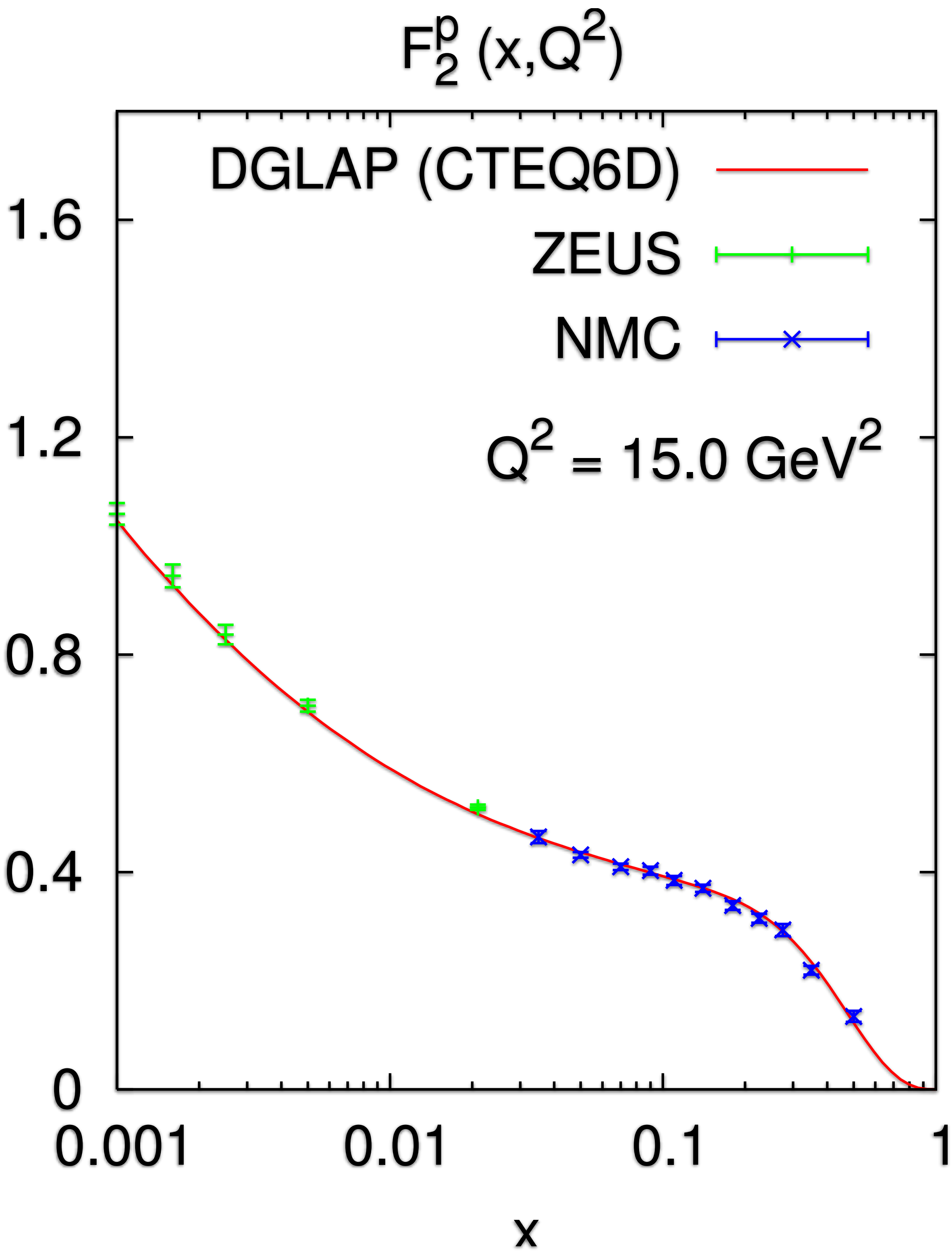

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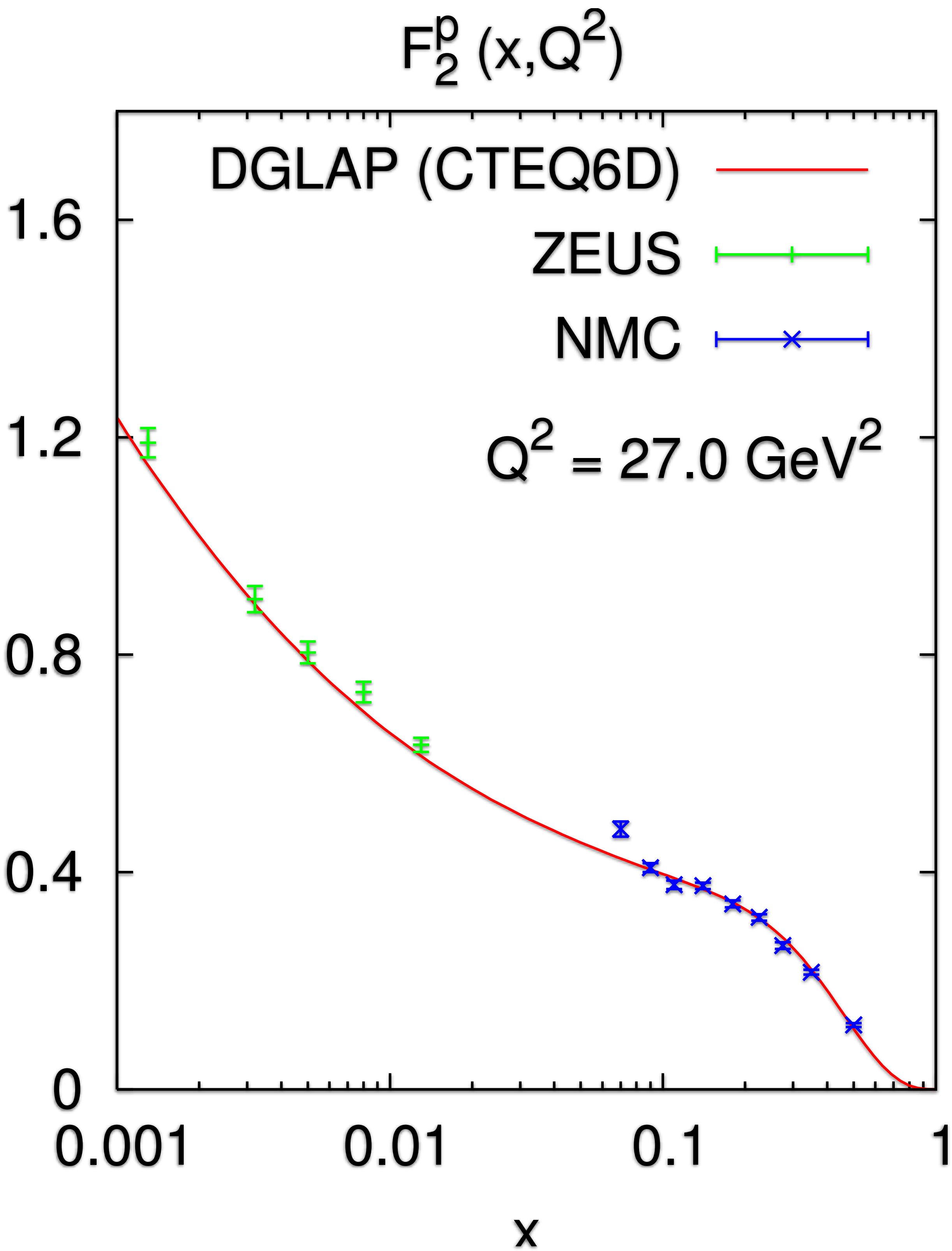

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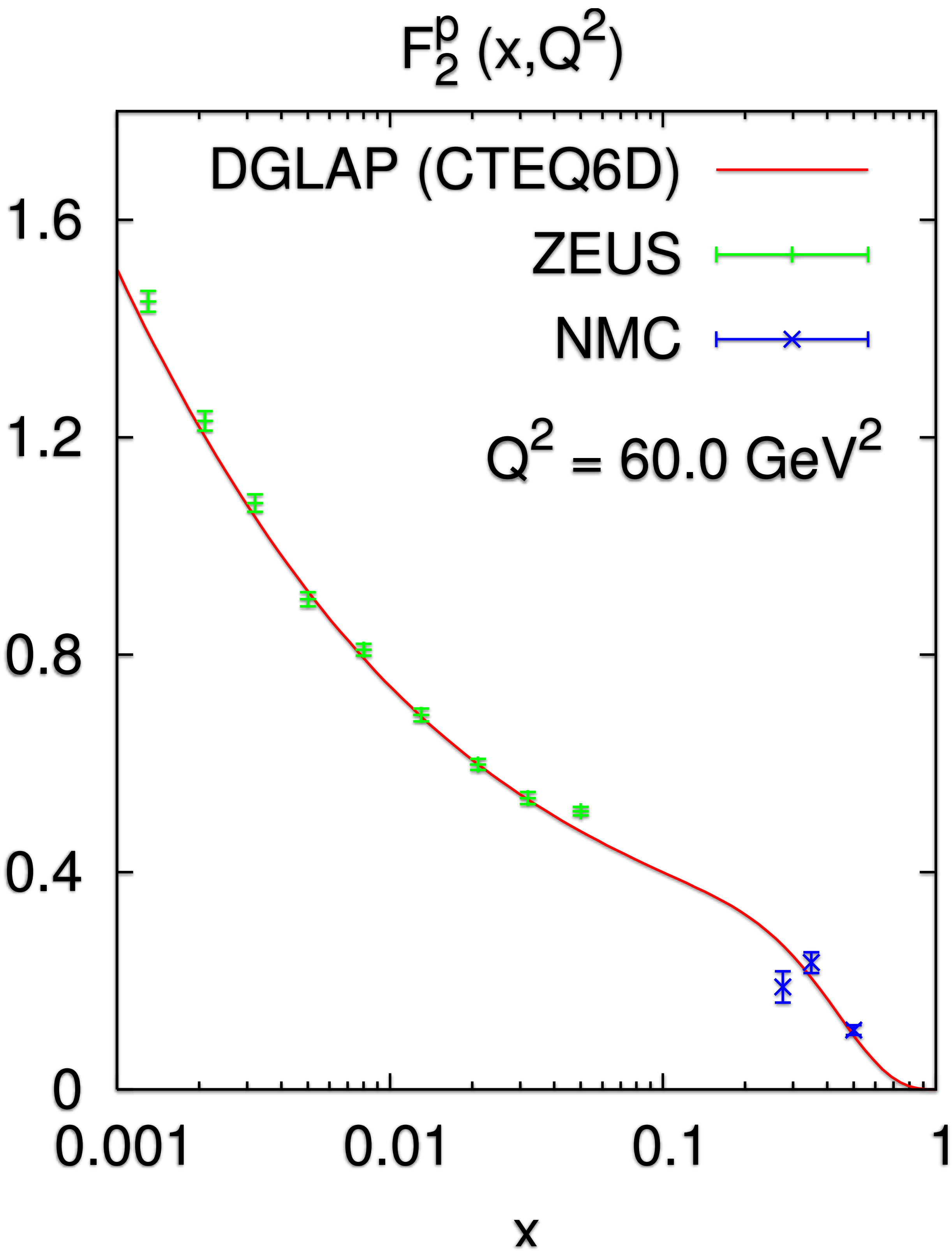

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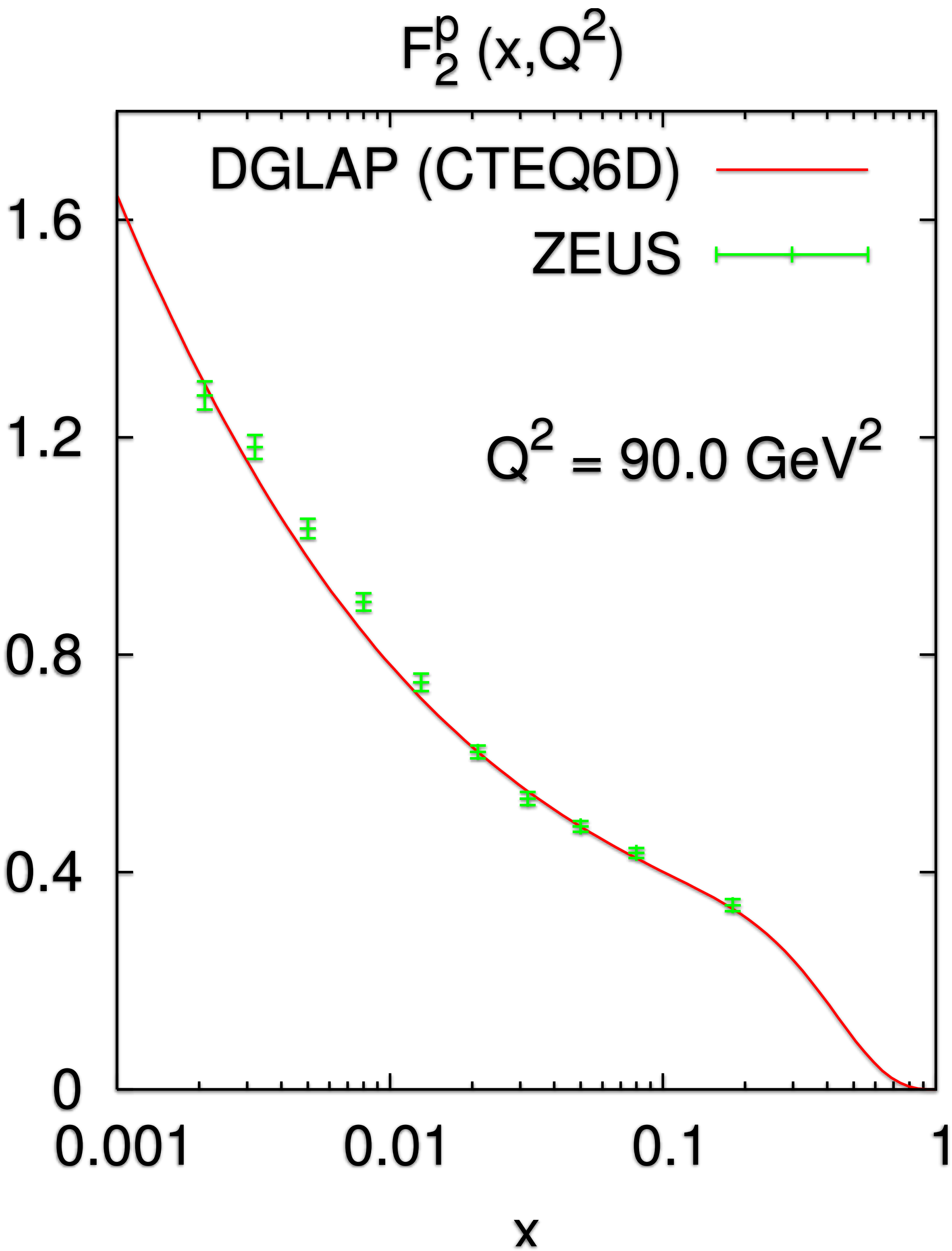

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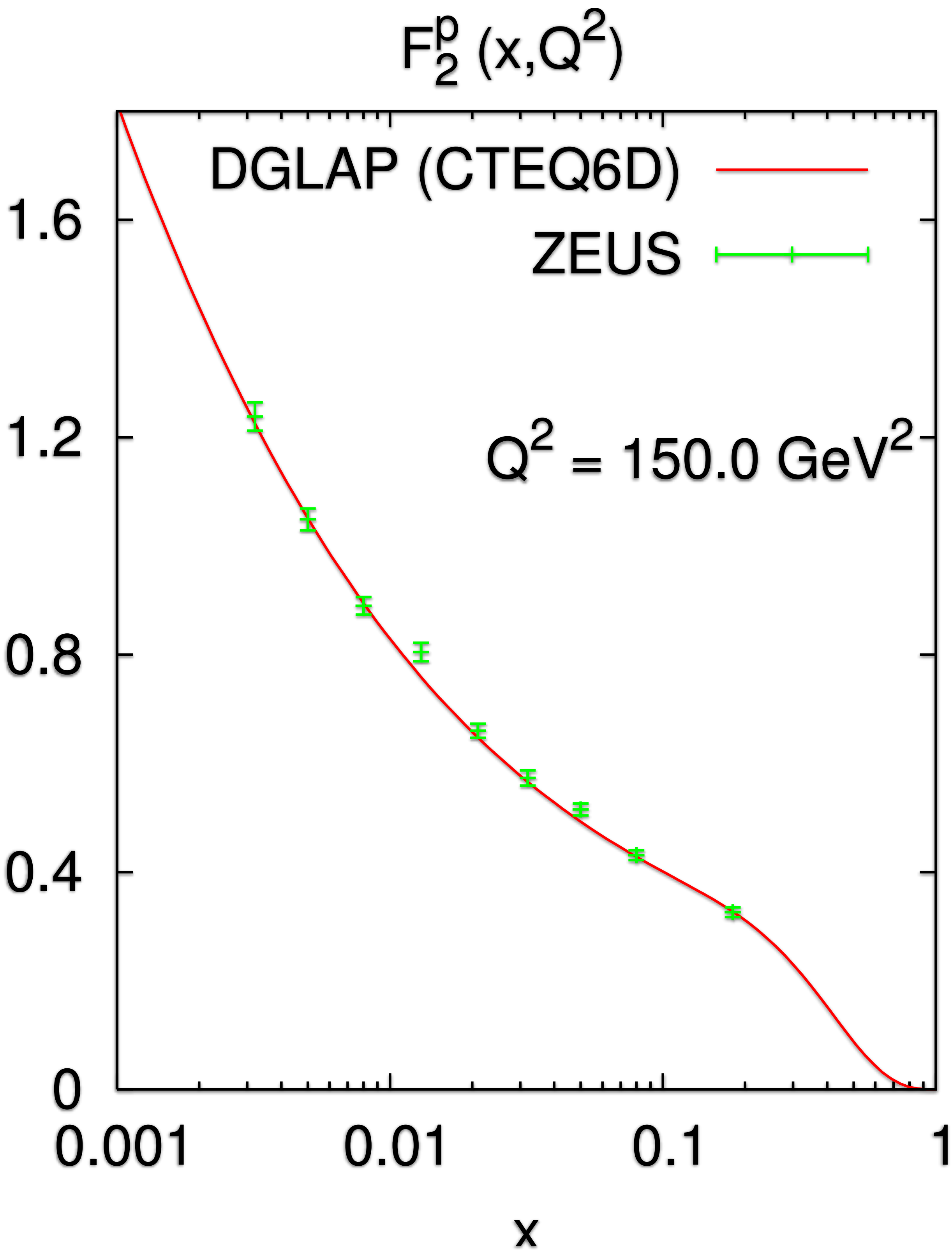

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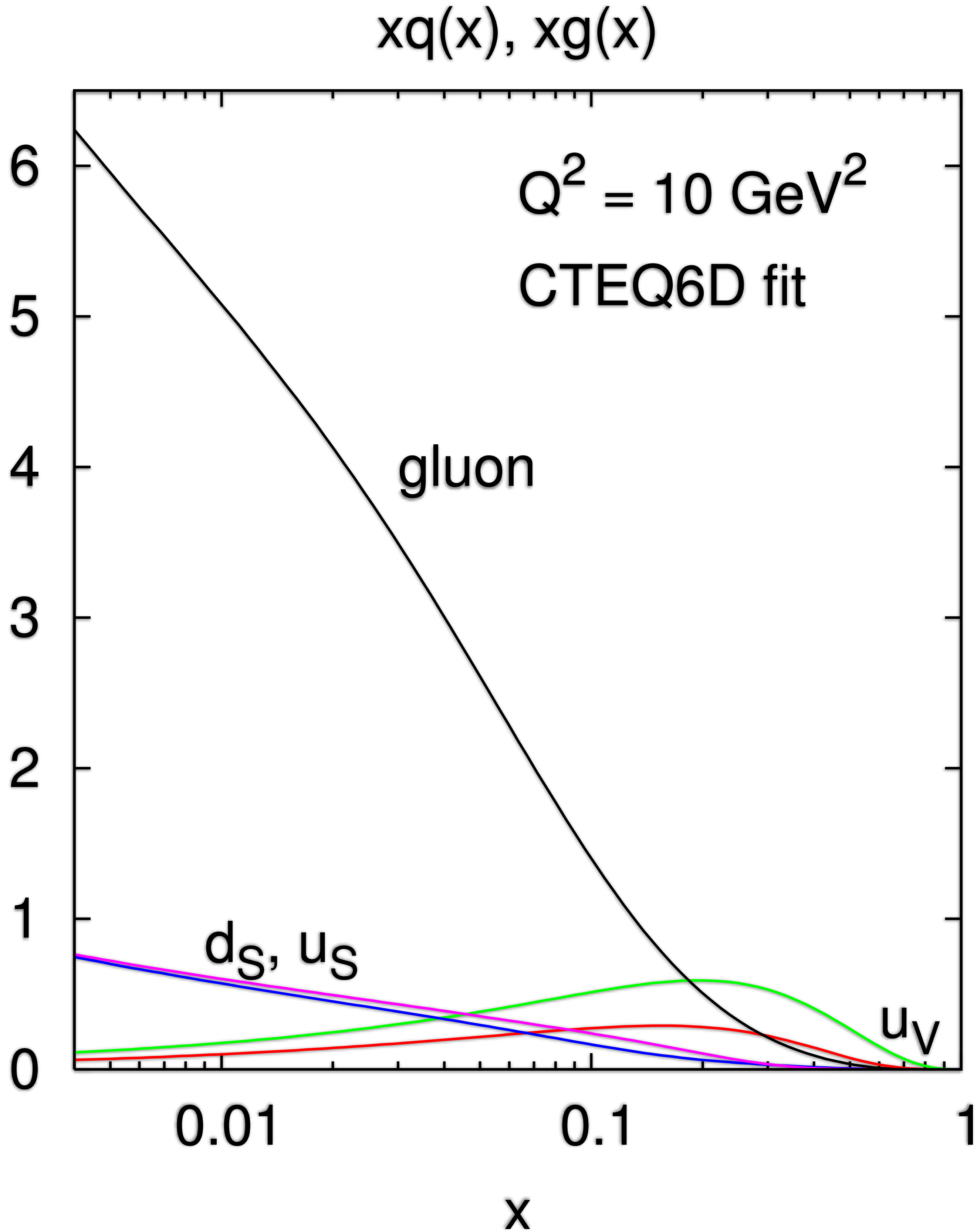
SUCCESS

Resulting gluon distribution, compared to quarks



Resulting gluon distribution is **HUGE!**

Carries 47% of proton's momentum
(at scale of 100 GeV)

Crucial in order to satisfy momentum sum rule.

Large value of gluon has big impact on phenomenology

NNPDF3.1 dataset

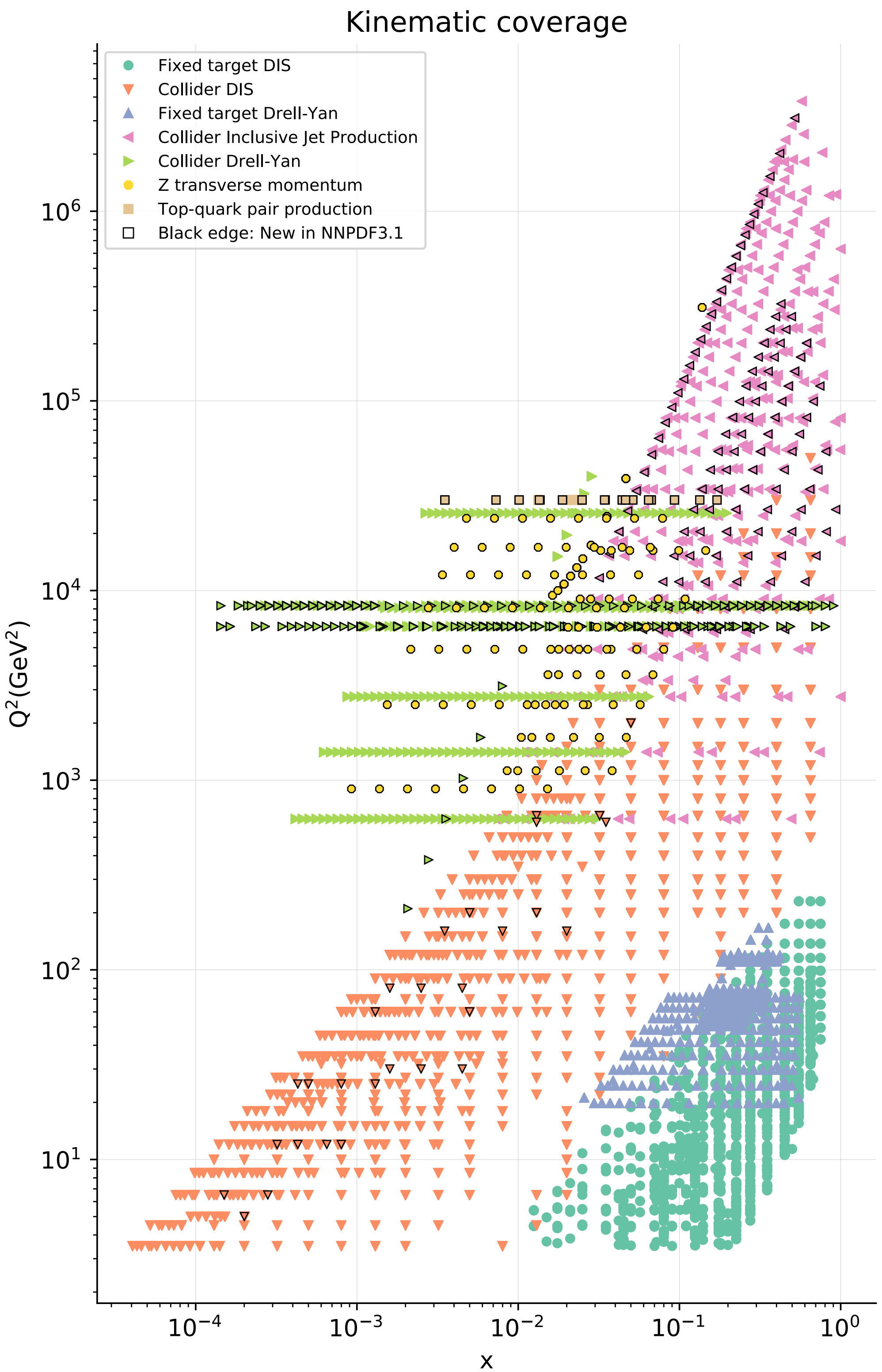
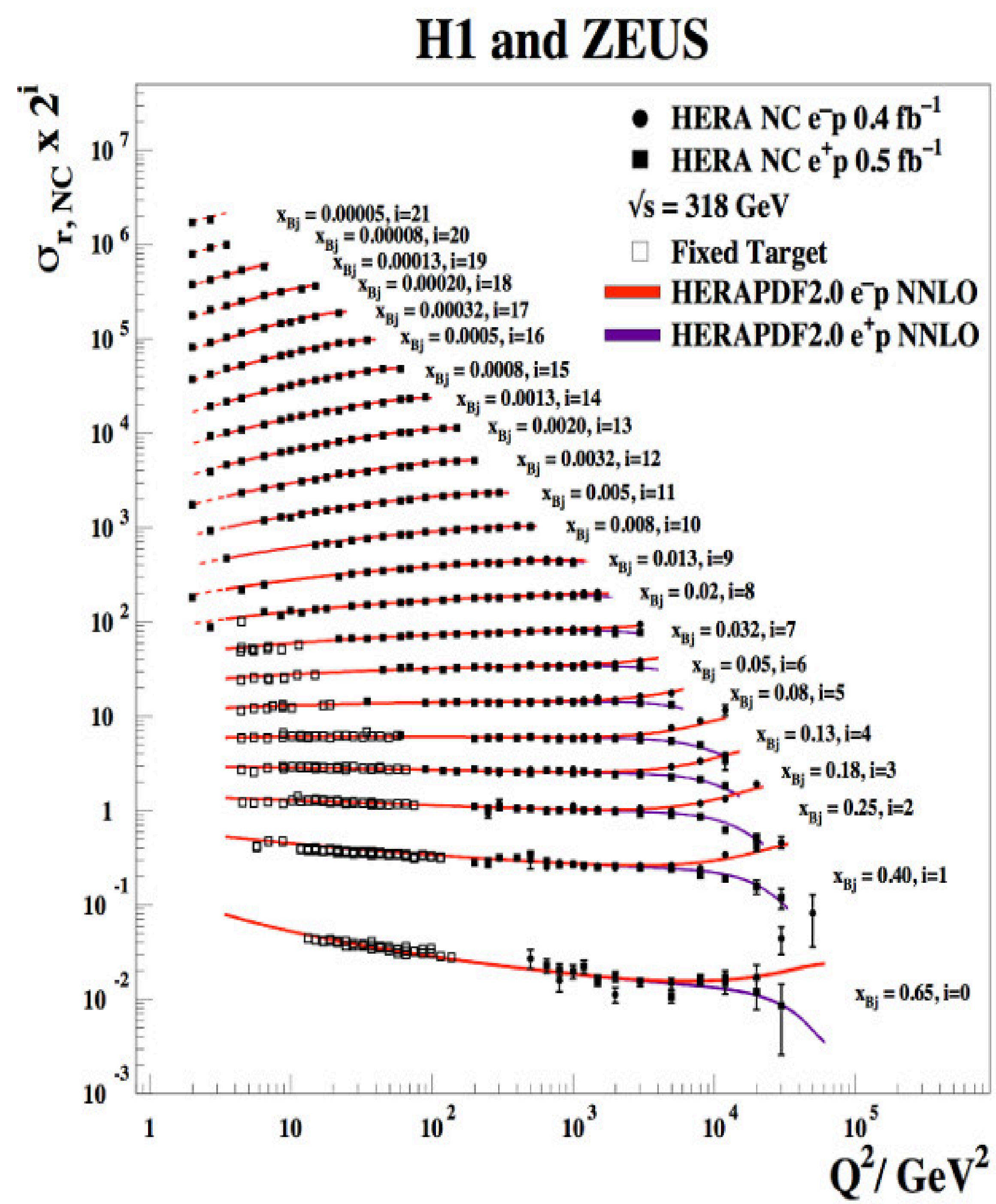
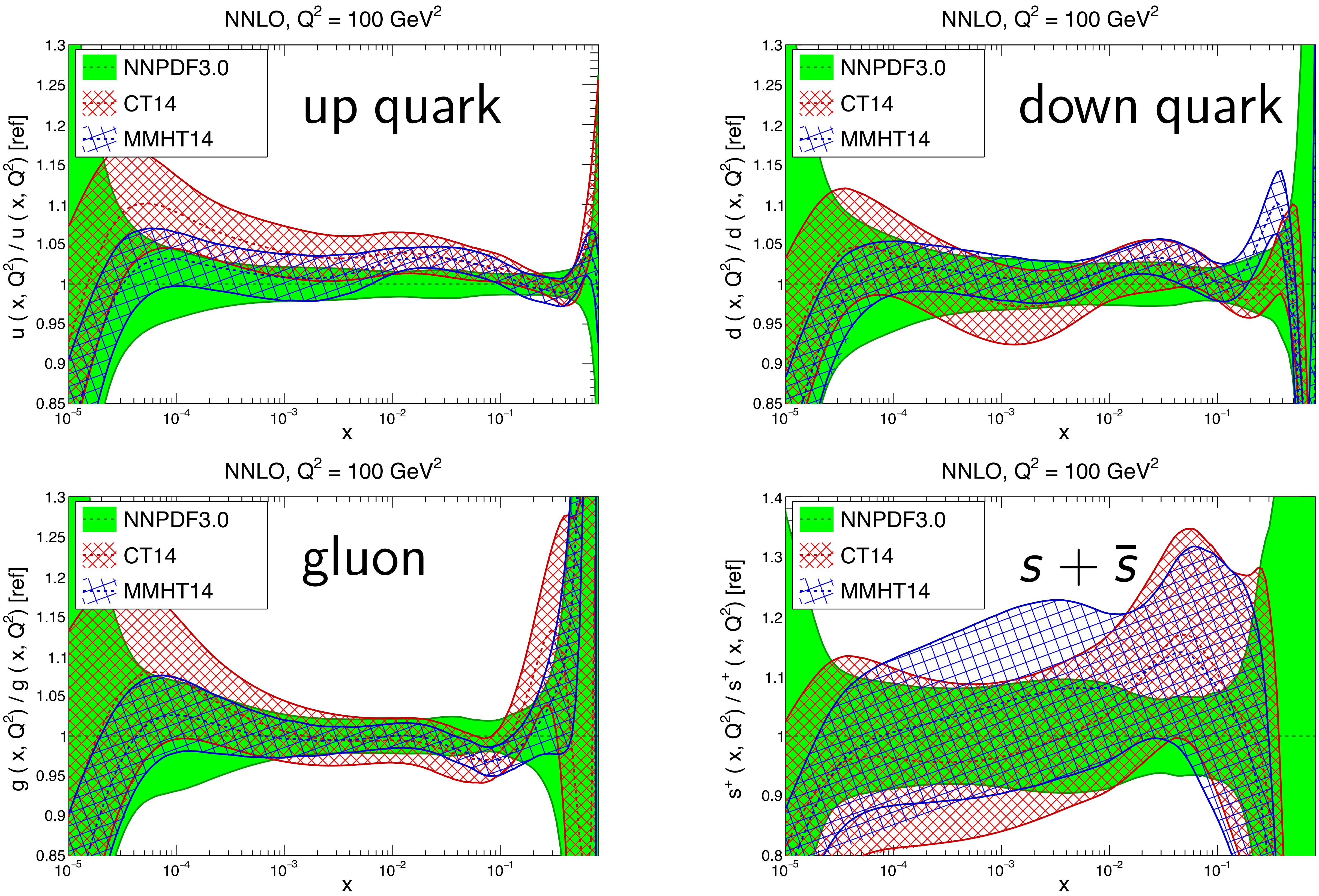


Figure 2.1: The kinematic coverage of the NNPDF3.1 dataset in the (x, Q^2) plane.

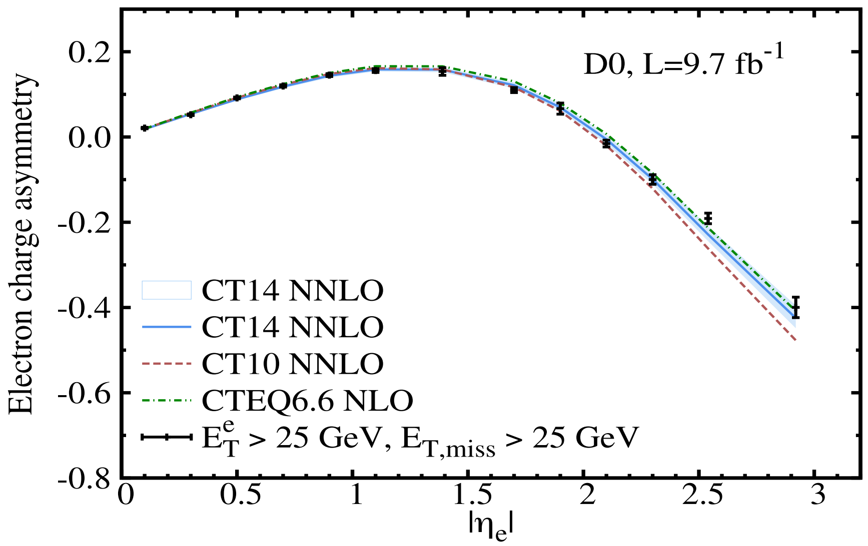


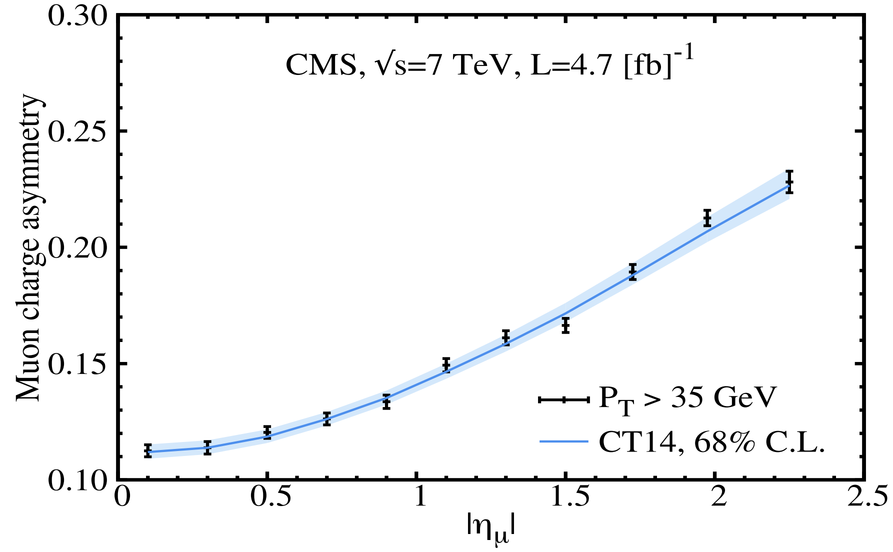
THREE GLOBAL PDF FITS: CT14, MMHT2014, NNPDF30/31

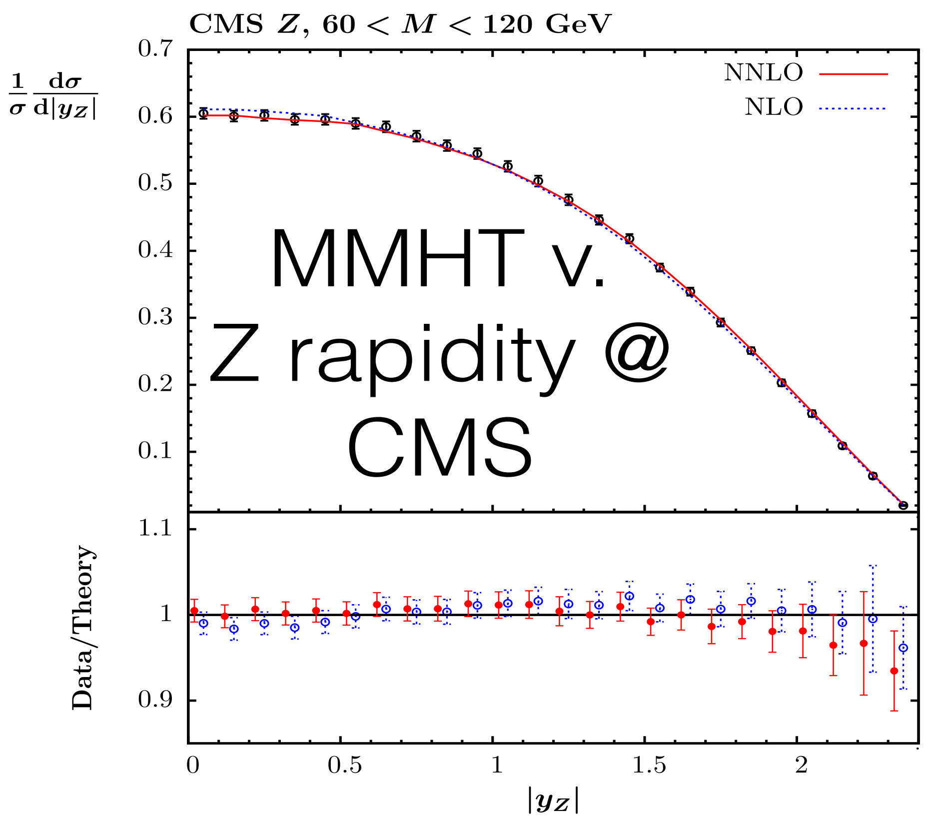


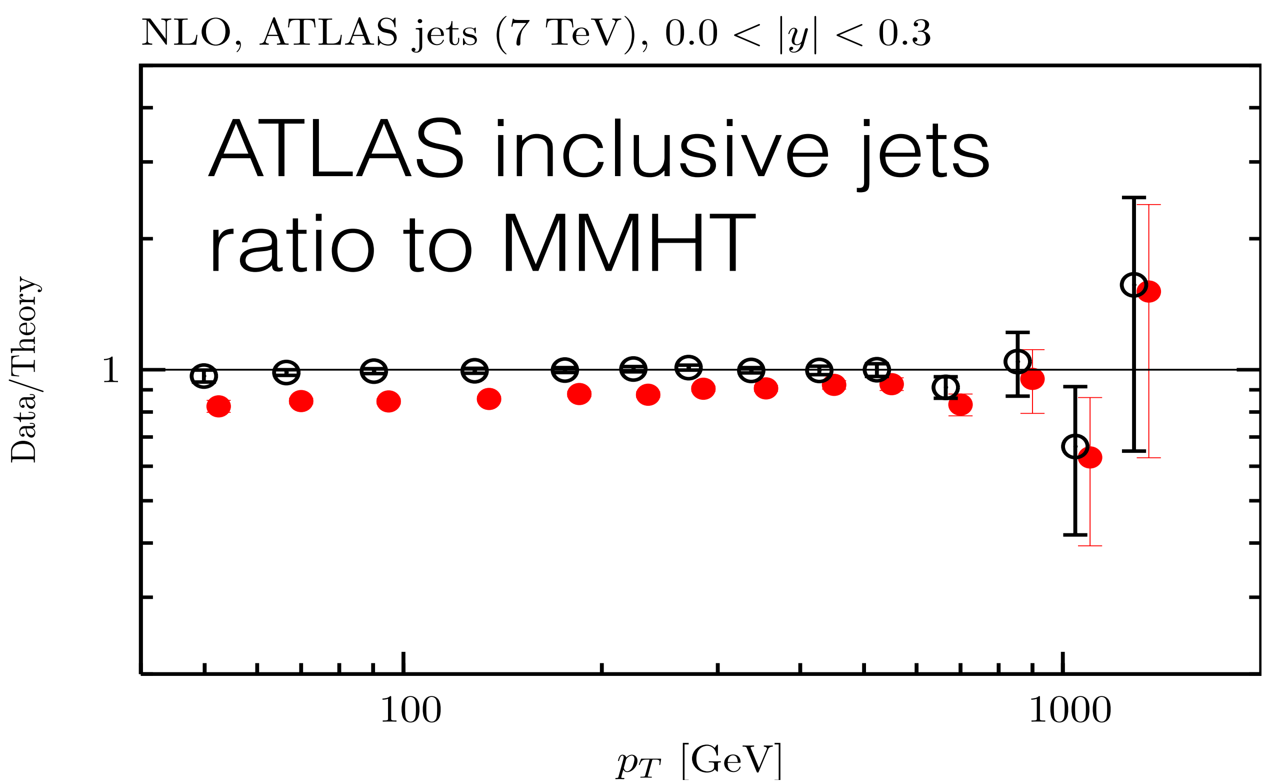
TODAY'S PDF FITS

Lepton charge asym. v. CT14 @ D0 & CMS

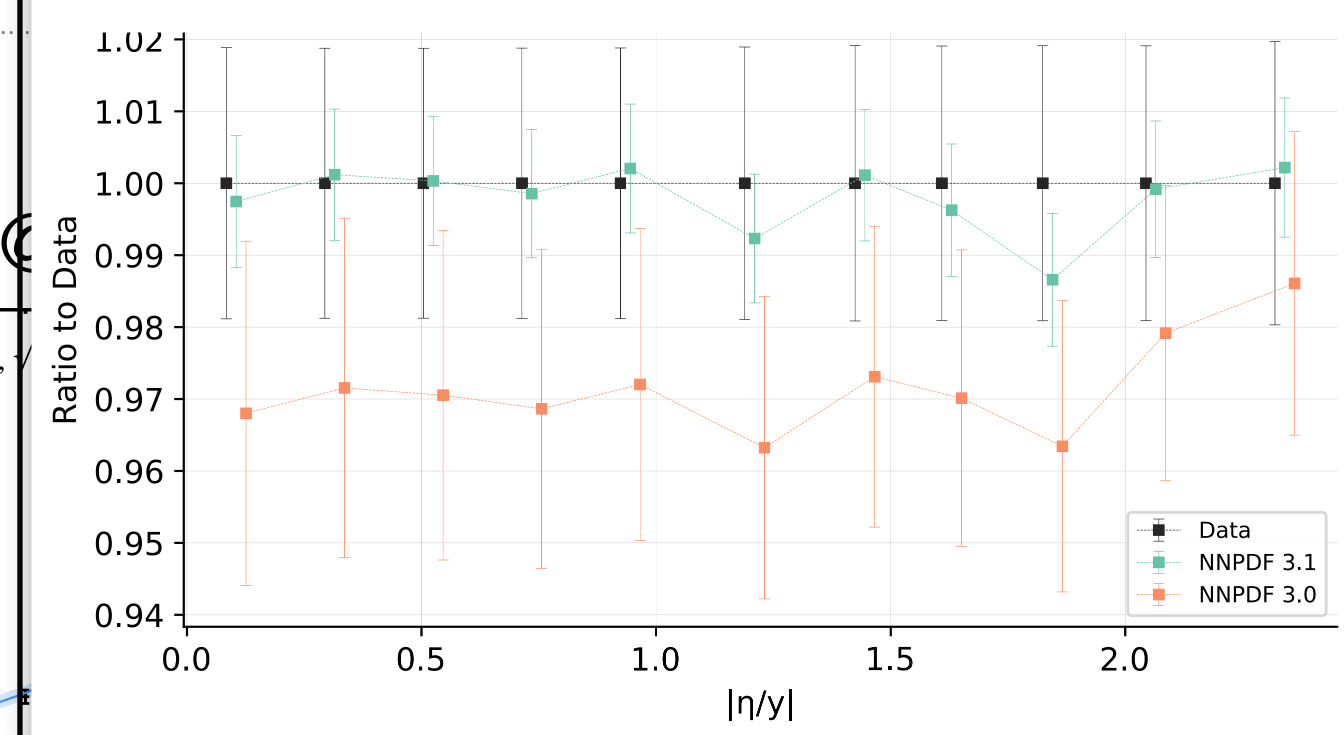




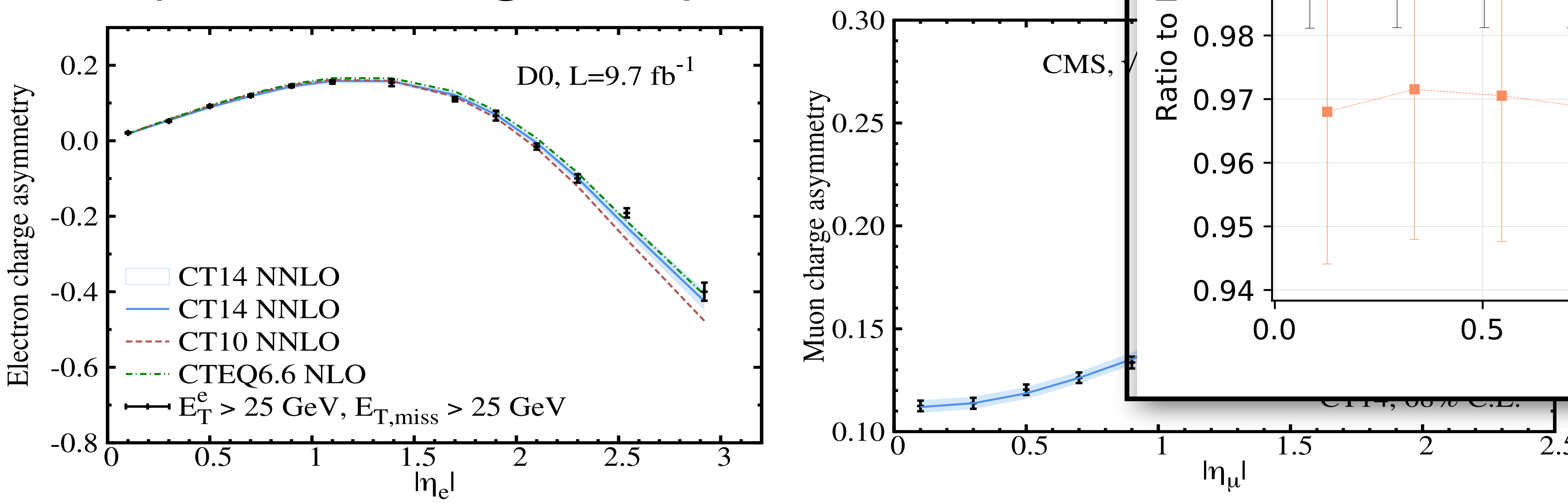


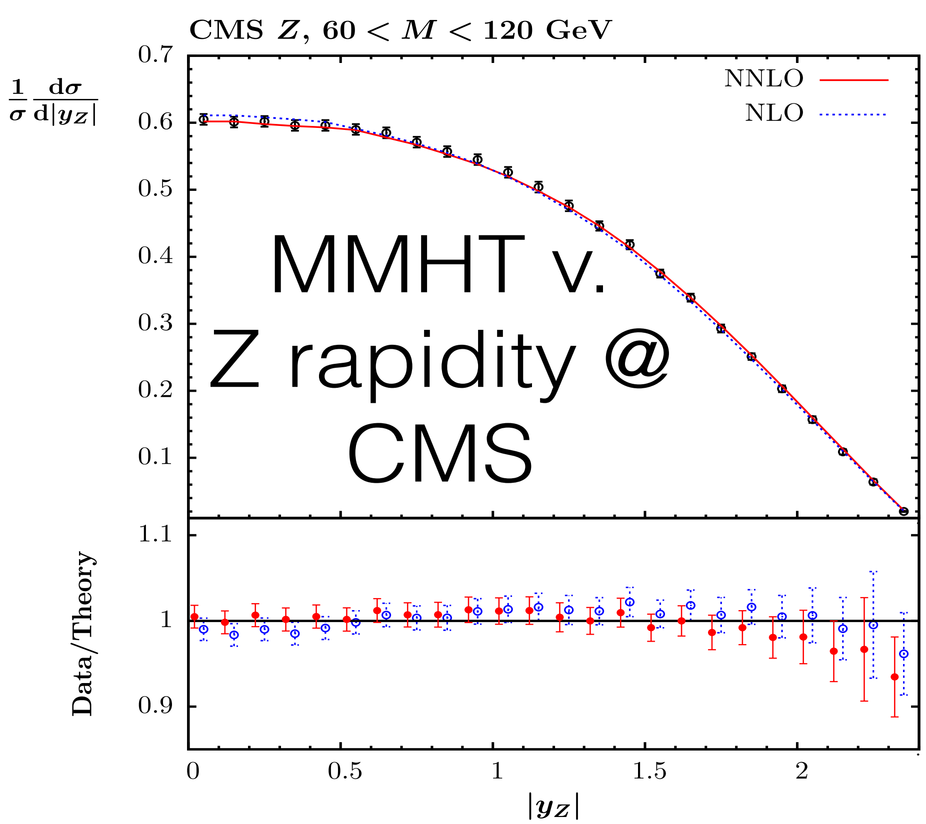


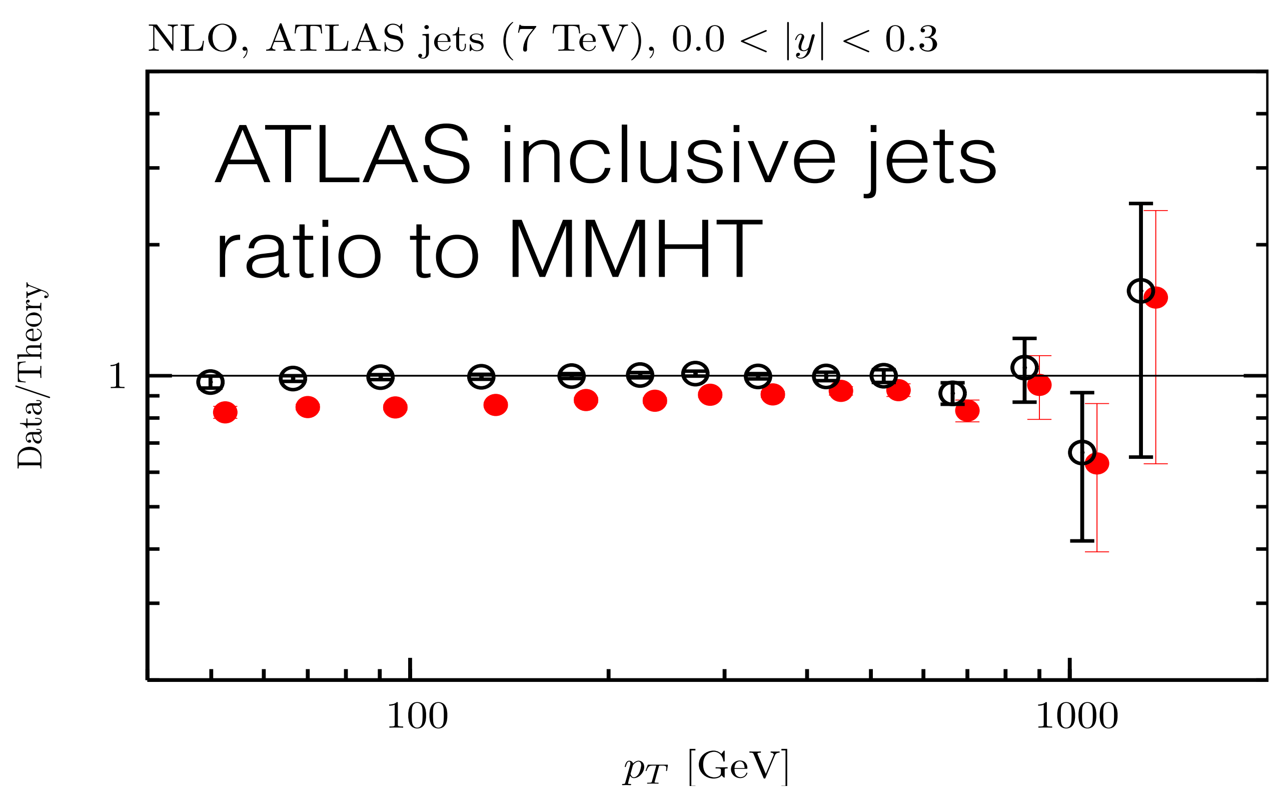
ATLAS W− v. NNPDF31



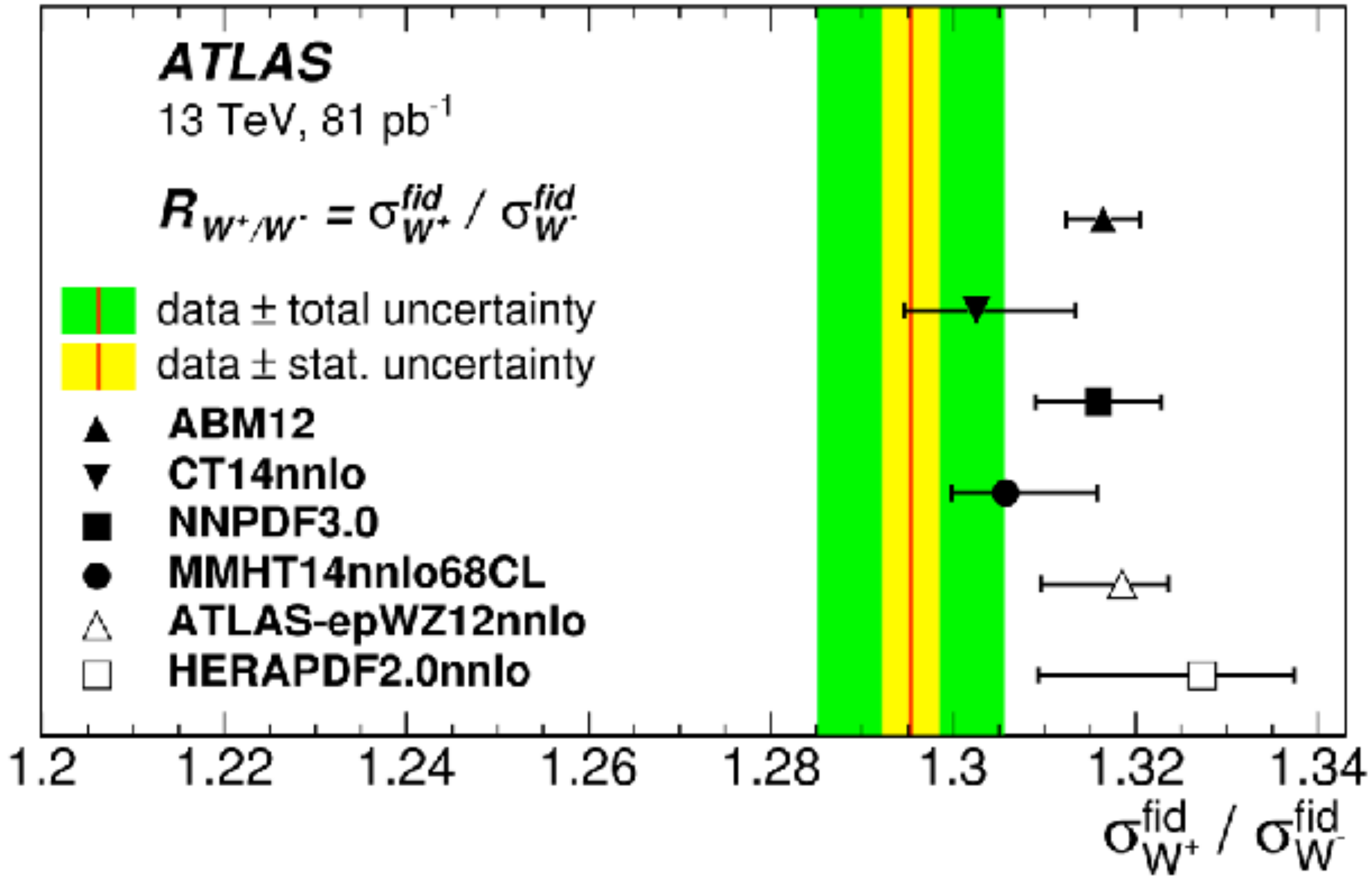
Lepton charge asym. v. CT14 @





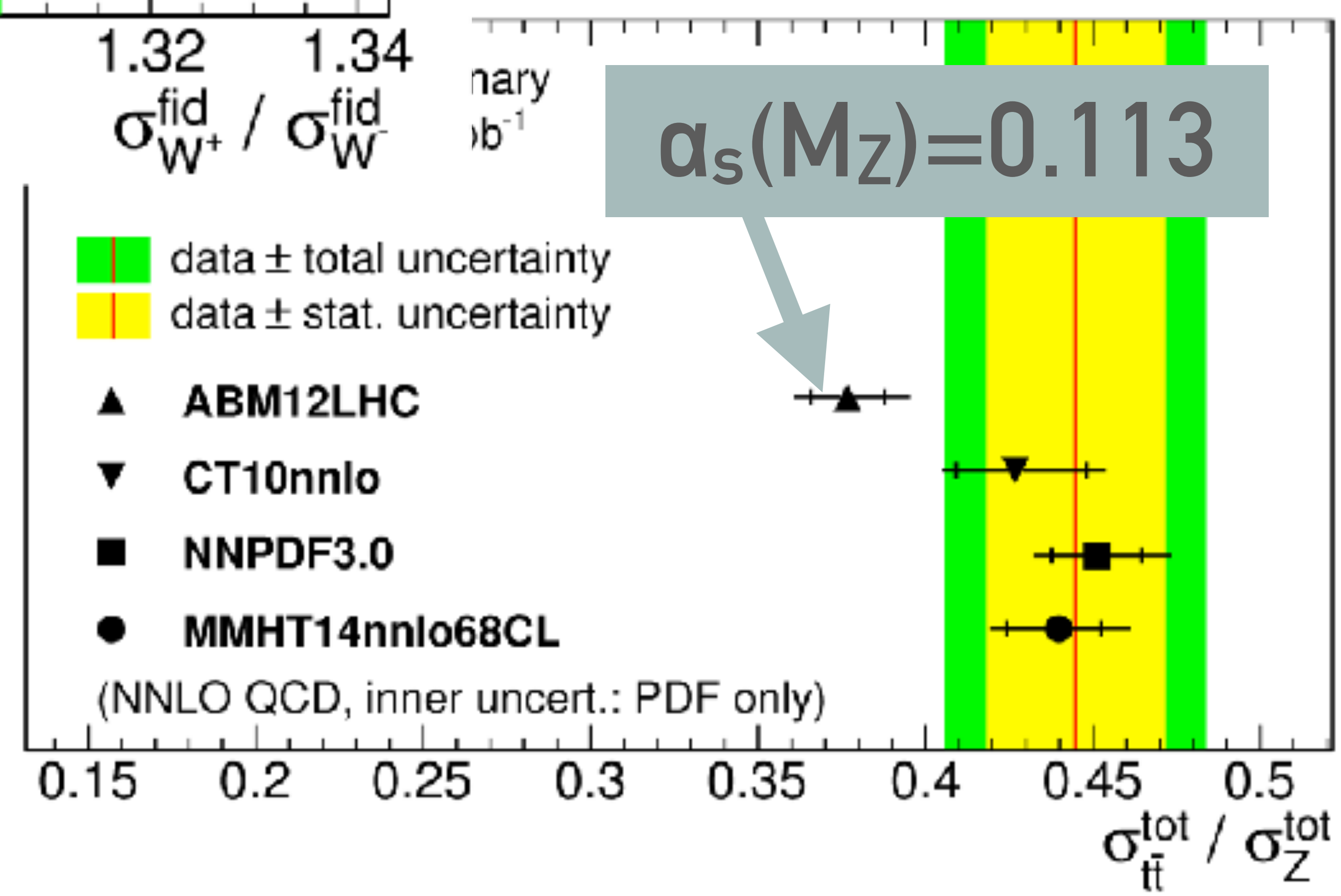


TODAY'S PDF FITS



cross-section ratios (W^+/W^-, ttbar/Z) show tensions with some PDFs

ATLAS-CONF-2015-049



NB: top-quark mass choice affects this plot

FINAL REMARKS ON PDFS

- In range $10^{-3} < x < 0.1$, core PDFs (up, down, gluon) known to ~ few % accuracy
- For many LHC applications, you can use PDF4LHC15 set, which merges CT14, MMHT2014, NNPDF30
- Situation is not full consensus: e.g. ABMP group claims substantially different gluon distribution

For visualisations of PDFs and related quantities, a good place to start is

http://apfel.mi.infn.it/ (ApfelWeb)

SO FAR

- We discussed the "Master" formula

$$\sigma\left(h_1 h_2 \to W+X\right) = \sum_{n=0}^{\infty} \alpha_s^n\left(\mu_R^2\right) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right) \times \hat{\sigma}_{ij\to W+X}^{(n)}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{M_W^4}\right),$$

- and its main inputs
  - the strong coupling α_s
  - Parton Distribution Functions (PDFs)
- **Next:** we discuss the actual scattering cross section

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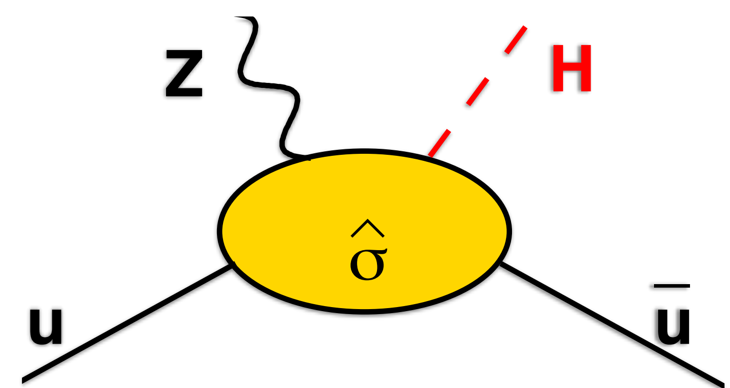

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the hard cross section

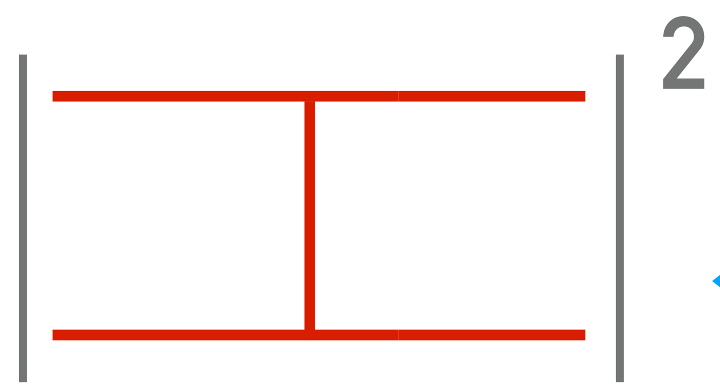
$$\sigma \sim \sigma_2 \alpha_s^2 + \sigma_3 \alpha_s^3 + \sigma_4 \alpha_s^4 + \sigma_5 \alpha_s^5 + \cdots$$

LO **NLO** **NNLO** **N3LO**

INGREDIENTS FOR A CALCULATION (generic 2→2 process)

LO

Tree
2→2



to illustrate the concepts, we don't care what the particles are — just draw lines

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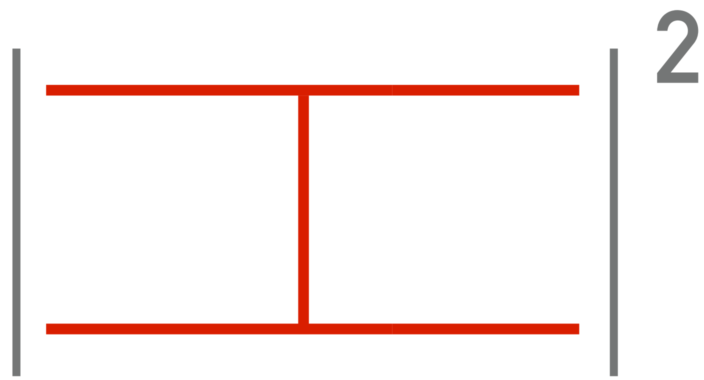

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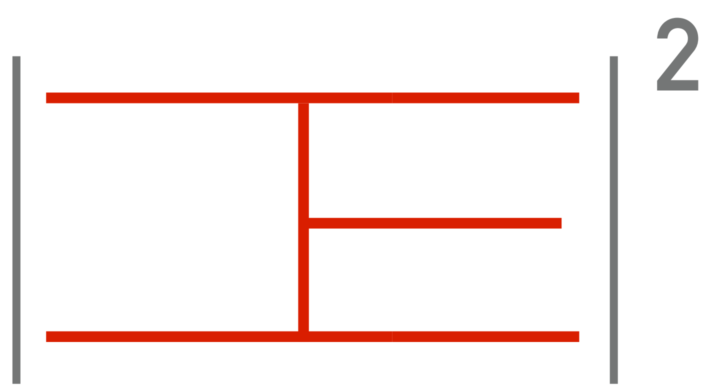
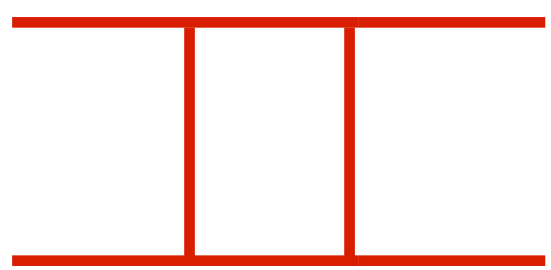
NLO

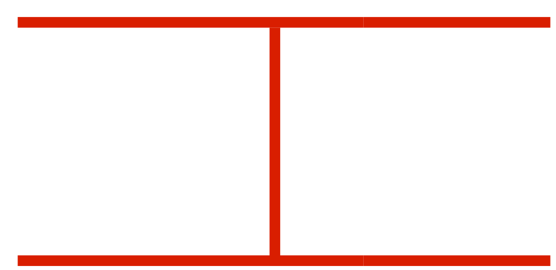
Tree
2→3



1-loop
2→2



×

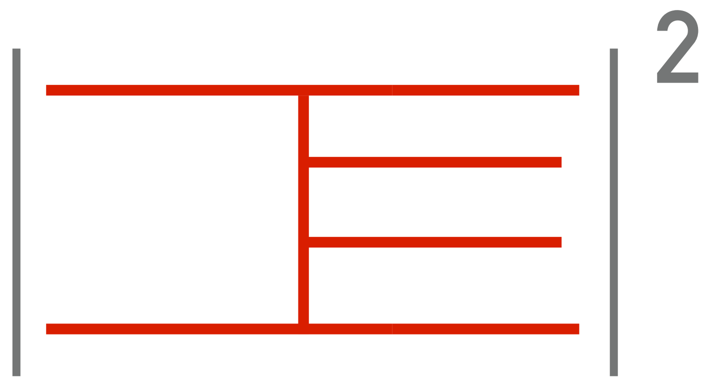


$+$ *complex conj.*

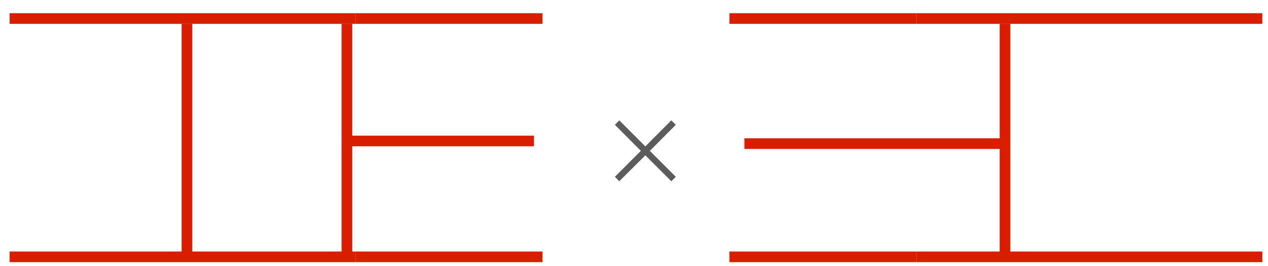
INGREDIENTS FOR A CALCULATION (generic 2→2 process)

NNLO

Tree
2→4

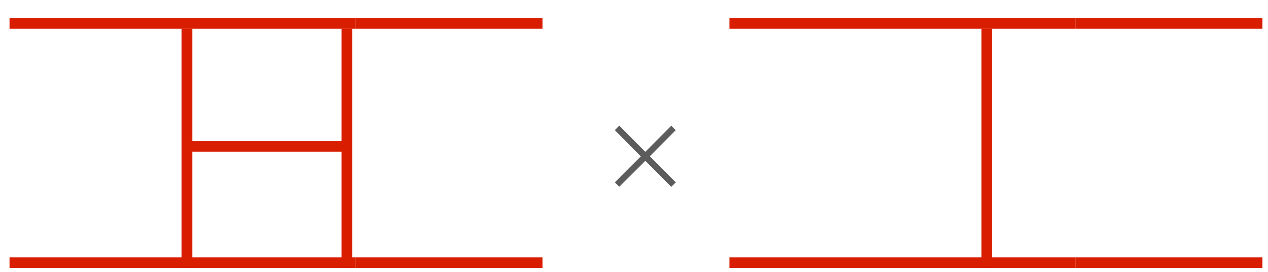


1-loop
2→3



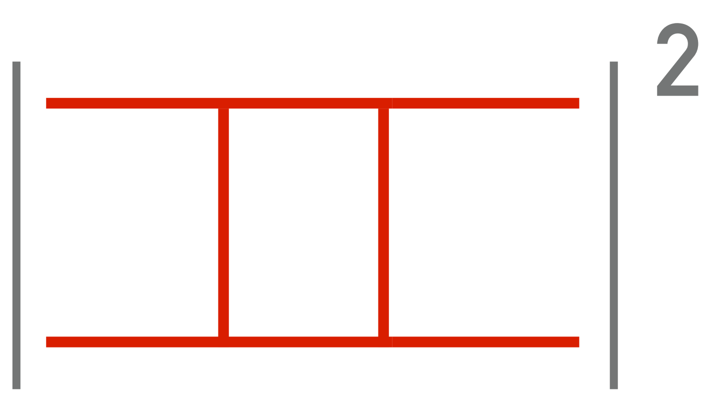
+ *complex conj.*

2-loop
2→2



+ *complex conj.*

1-loop
2→2



EXAMPLE SERIES #1

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \qquad [\alpha_s \equiv \alpha_s(\sqrt{s_{e^+e^-}})]$$

$$= R_0 \left(1 + 0.32\alpha_s + 0.14\alpha_s^2 - 0.47\alpha_s^3 - 0.59316\alpha_s^4 + \cdots\right)$$

Baikov et al., 1206.1288
(numbers for γ-exchange only)

This is one of the few quantities calculated to N4LO

Good convergence of the series at every order
(at least for $\alpha_s(M_Z) = 0.118$)

EXAMPLE SERIES #2

$$\sigma(pp \to H) = (961\,\text{pb}) \times (\alpha_s^2 + 10.4\alpha_s^3 + 38\alpha_s^4 + 48\alpha_s^5 + \cdots)$$

$$\alpha_s \equiv \alpha_s(M_H/2)$$

$$\sqrt{s_{pp}} = 13\,\text{TeV}$$

Anastasiou et al., 1602.00695 (ggF, hEFT)

pp→H (via gluon fusion) is one of only two hadron-collider processes known at N3LO

(the other is pp→H via weak-boson fusion)

The series does not converge well

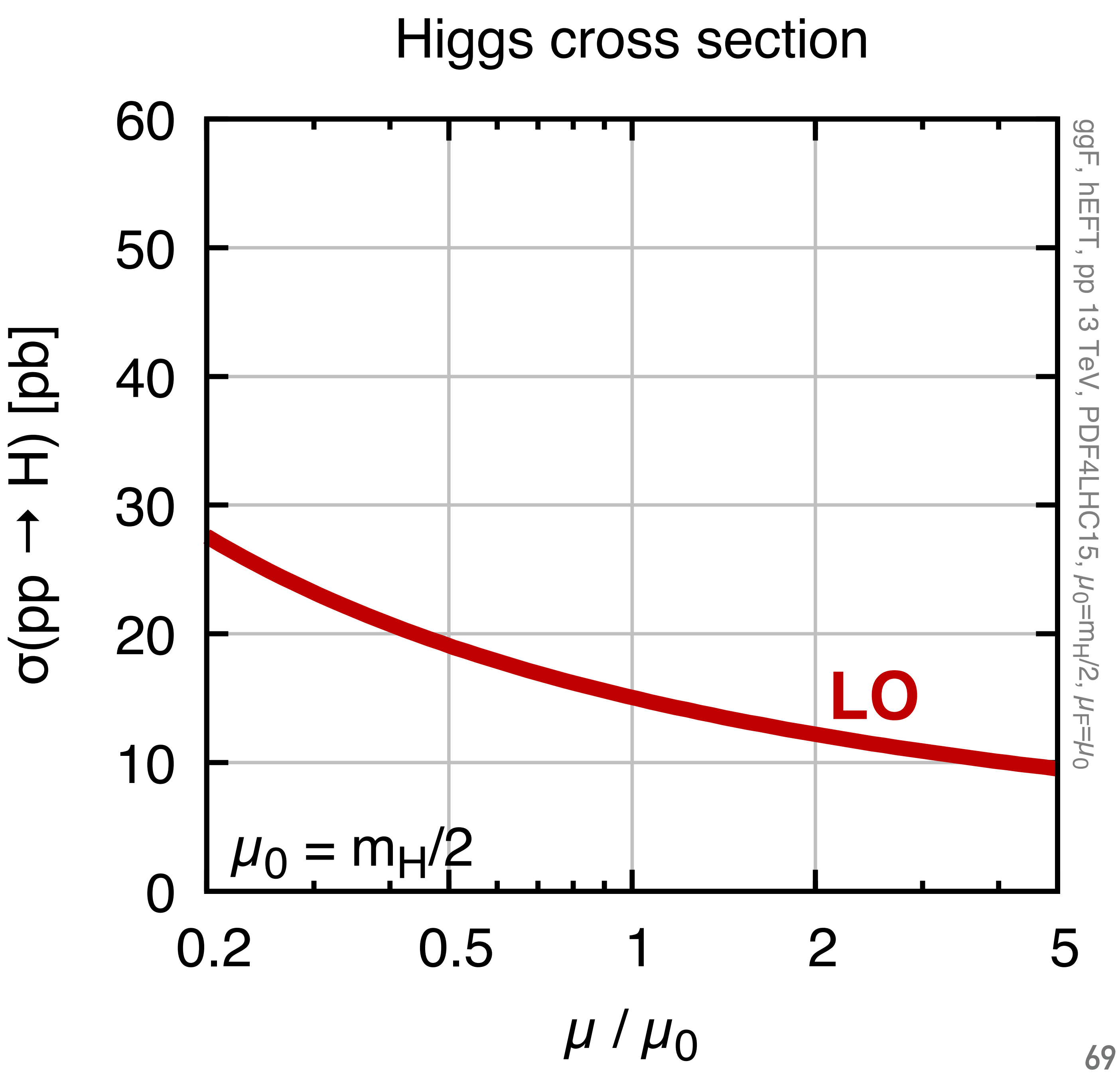
(explanations for why are only moderately convincing)

SCALE DEPENDENCE

- On previous page, we wrote the series in terms of powers of $\alpha_s(M_H/2)$
- But we are free to rewrite it in terms of $\alpha_s(\mu)$ for any choice of "renormalisation scale" μ.

LO

$$\sigma(pp \to H) = \sigma_0 \times \alpha_s^2(\mu)$$



SCALE DEPENDENCE

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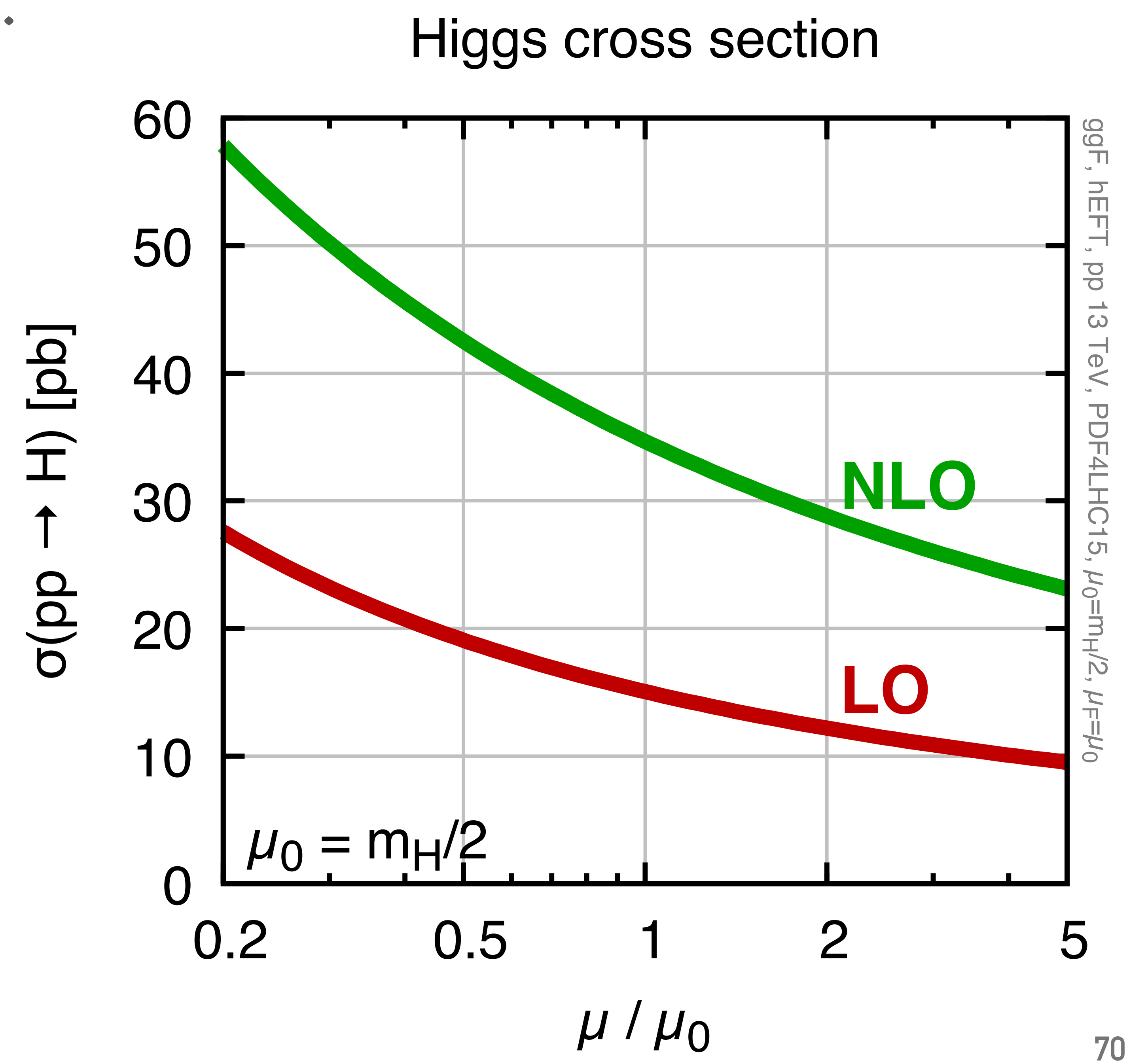
NLO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu)\Big)$$



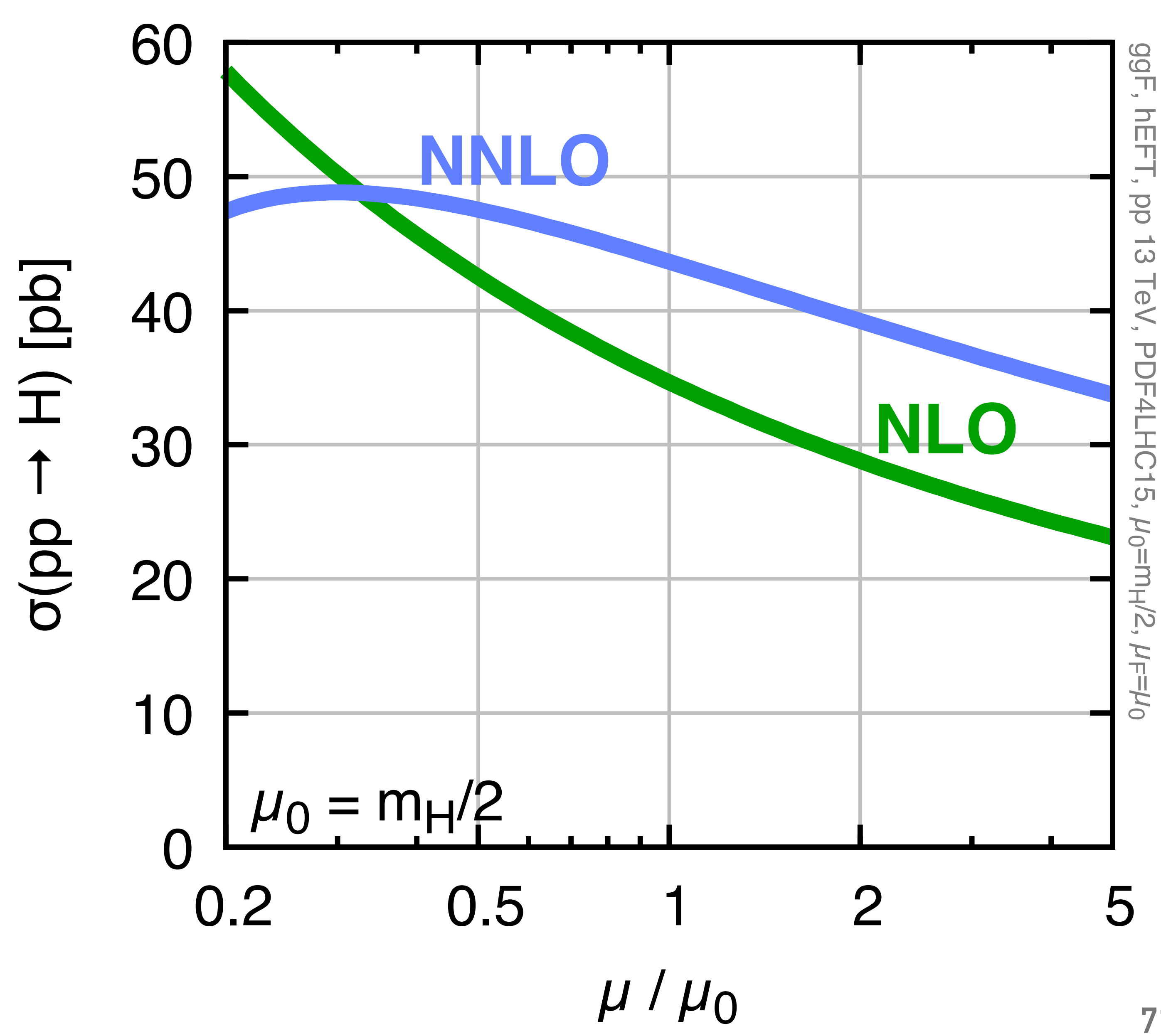
SCALE DEPENDENCE

- On previous page, we wrote the series in terms of powers of $\alpha_s(M_H/2)$
- But we are free to rewrite it in terms of $\alpha_s(\mu)$ for any choice of "renormalisation scale" μ.

NNLO

$$\sigma(pp \to H) = \sigma_0 \times \left(\alpha_s^2(\mu) + (10.4 + 2b_0 \ln\frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu) + c_4(\mu)\alpha_s^4(\mu)\right)$$

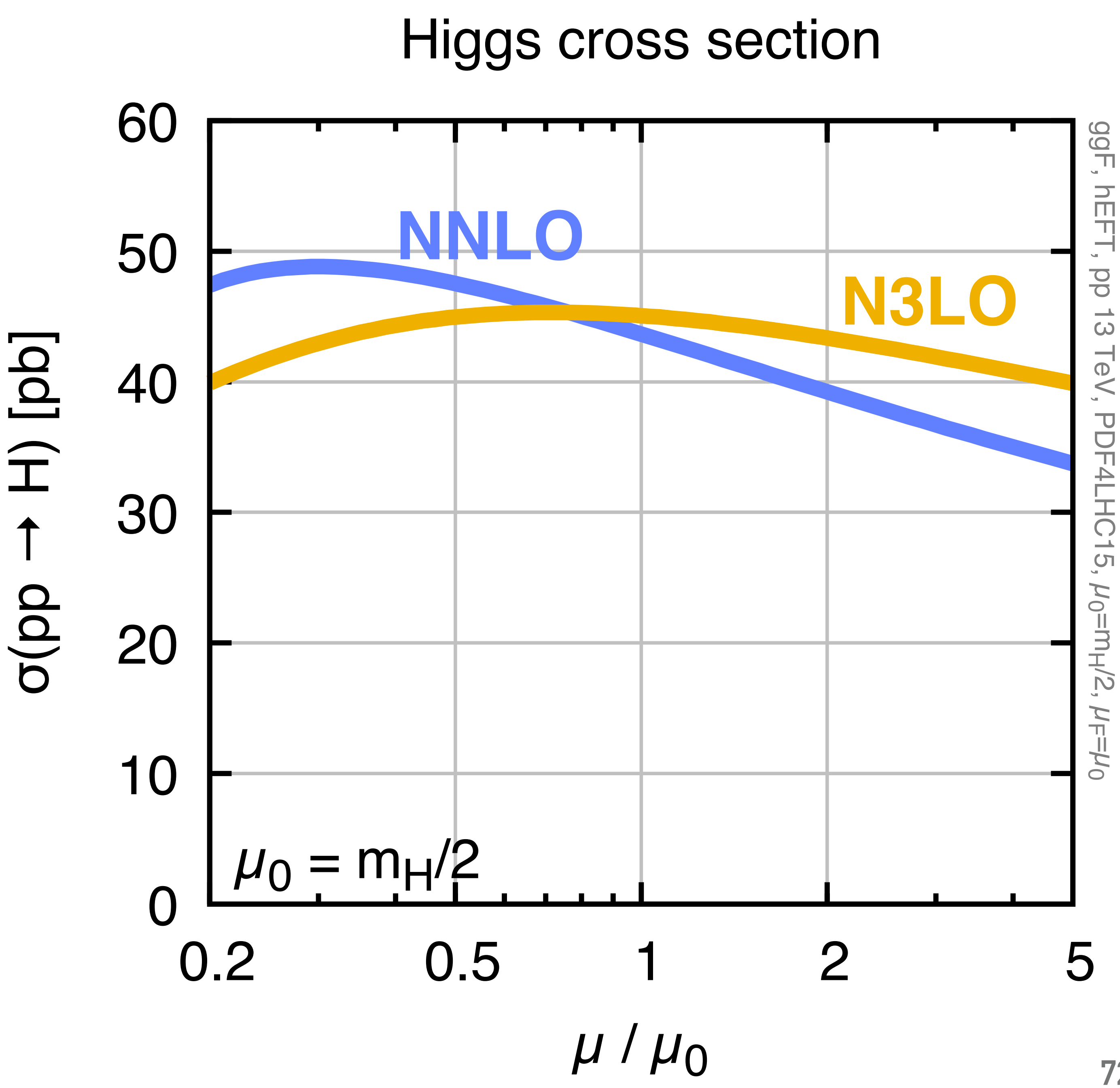
Higgs cross section



- On previous page, we wrote the series in terms of powers of $\alpha_s(M_H/2)$
- But we are free to rewrite it in terms of $\alpha_s(\mu)$ for any choice of "renormalisation scale" μ.

N3LO

$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu)$$
$$+(10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu)$$
$$+c_4(\mu)\alpha_s^4(\mu) + c_5(\mu)\alpha_s^5(\mu)\Big)$$



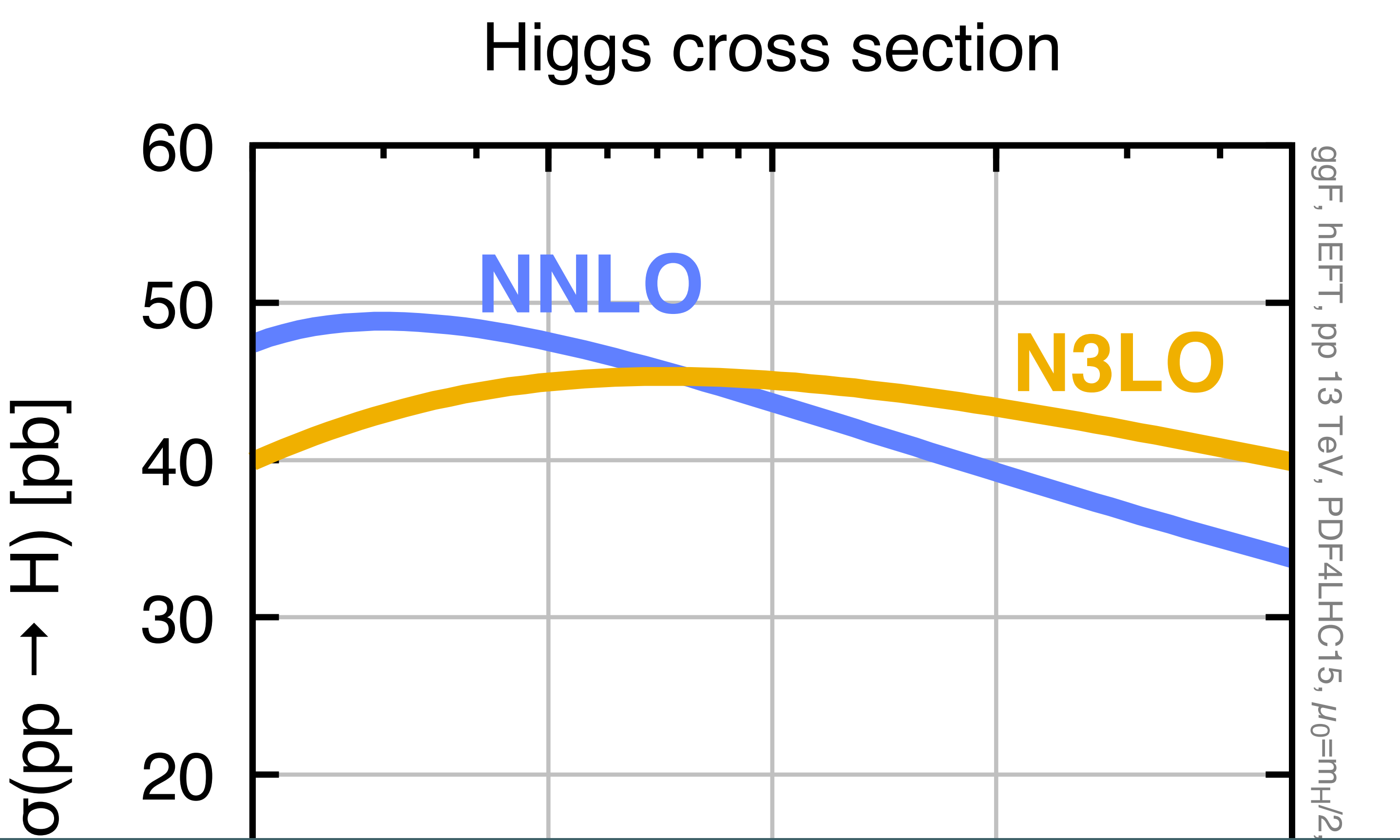
SCALE DEPENDENCE

- On previous page, we wrote the series in terms of powers of $\alpha_s(M_H/2)$
- But we are free to rewrite it in terms of $\alpha_s(\mu)$ for any choice of "renormalisation scale" μ.

N3LO

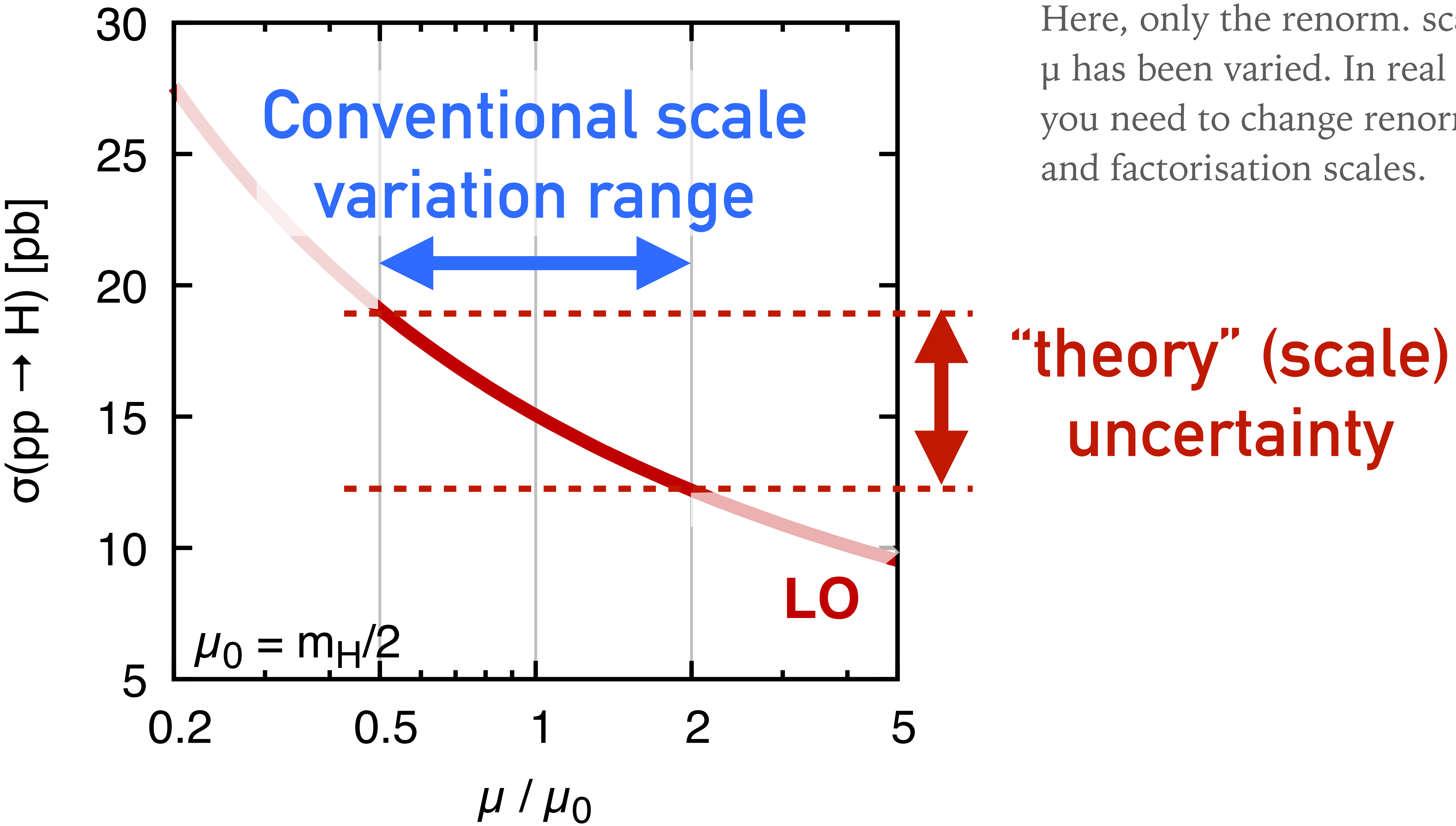
$$\sigma(pp \to H) = \sigma_0 \times \Big(\alpha_s^2(\mu)$$

$$+(10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2})\alpha_s^3(\mu)$$



scale dependence (an intrinsic uncertainty) gets reduced as you go to higher order

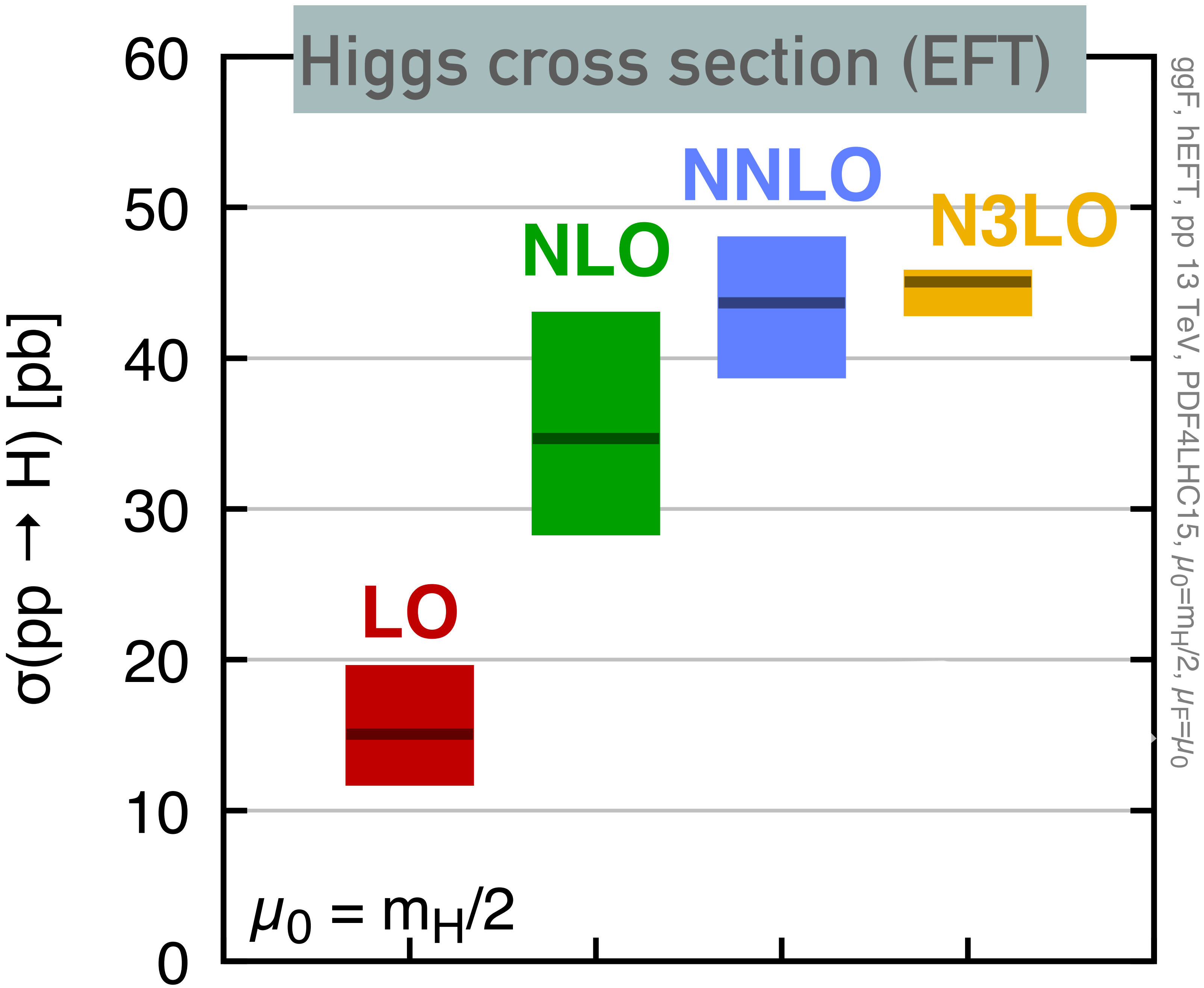
Scale dependence as the "THEORY UNCERTAINTY"



Here, only the renorm. scale μ has been varied. In real life you need to change renorm. and factorisation scales.

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying μ in range 1/2 → 2 around central value

Scale dependence as the "THEORY UNCERTAINTY"



Here, only the renorm. scale $\mu(\equiv\mu_R)$ has been varied. In real life you need to change renorm. and factorisation (μ_F) scales.

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying μ in range 1/2 → 2 around central value

WHAT DO WE KNOW?

- LO: almost any process *(with MadGraph, ALPGEN, etc.)*
- NLO: most processes *(with MCFM, NLOJet++, MG5_aMC@NLO, POWHEG, OpenLoops/Blackhat/NJet/Gosam/etc.+Sherpa)*
- NNLO: all 2→1 and most 2→
  (top++, DY/HNNLO, FEWZ, MATRIX, MCFM, NNLOJet, etc.)
- N3LO: pp → Higgs via gluon fusion and weak-boson fusion
  both in approximations (EFT, $QCD_1 \times QCD_2$)
- NLO EW corrections, i.e. relative α_{EW} rather than α_s:
  most 2→1, 2→2 and 2→3

EXTRA SLIDES

Higgs cross sections

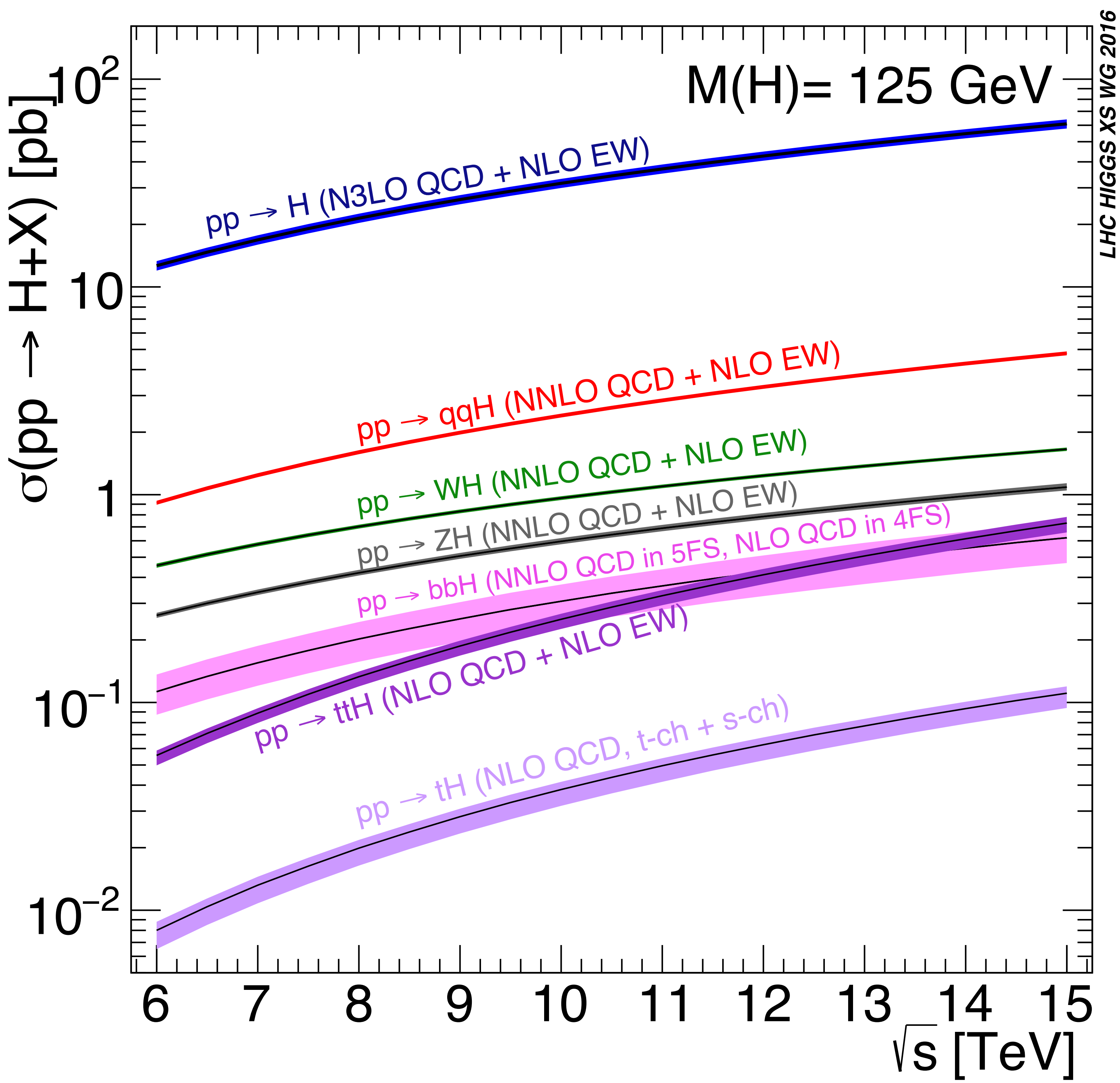


Figure 178: The SM Higgs boson production cross sections as a function of the LHC centre of mass energy.

how close are scale variations to being 1σ uncertainty?

Bagnaschi, Cacciari, Guffanti, Jenniches (1409.5036)

Observable	Leading order in α_s	Highest known order in α_s	Reference
$pp \to H$	2	4	HIGLU [27, 28]
$pp \to b\bar{b} \to H$	0	2	bbh@nnlo [29]
$pp \to t\bar{t}$	2	4	top++ [30]
$pp \to Z \to e^+e^-$	0	2	DYNNLO [31]
$pp \to W^+ \to e^+\overline{\nu}_e$	0	2	DYNNLO
$pp \to W^- \to e^-\nu_e$	0	2	DYNNLO
$pp \to Z^* \to ZH$	0	2	vh@nnlo [32]
$pp \to W^{\pm *} \to W^\pm H$	0	2	vh@nnlo
$pp \to b\bar{b}$	2	3	MCFM [33, 34]
$pp \to Z + \mathrm{j}$	1	2	MCFM
$pp \to Z + 2\mathrm{j}$	2	3	MCFM
$pp \to W^\pm + \mathrm{j}$	1	2	MCFM
$pp \to W^\pm + 2\mathrm{j}$	2	3	MCFM
$pp \to ZZ$	0	1	MCFM
$pp \to WW$	0	1	MCFM

Hadronic observables

Table 2: List of hadronic observables used in the global survey.

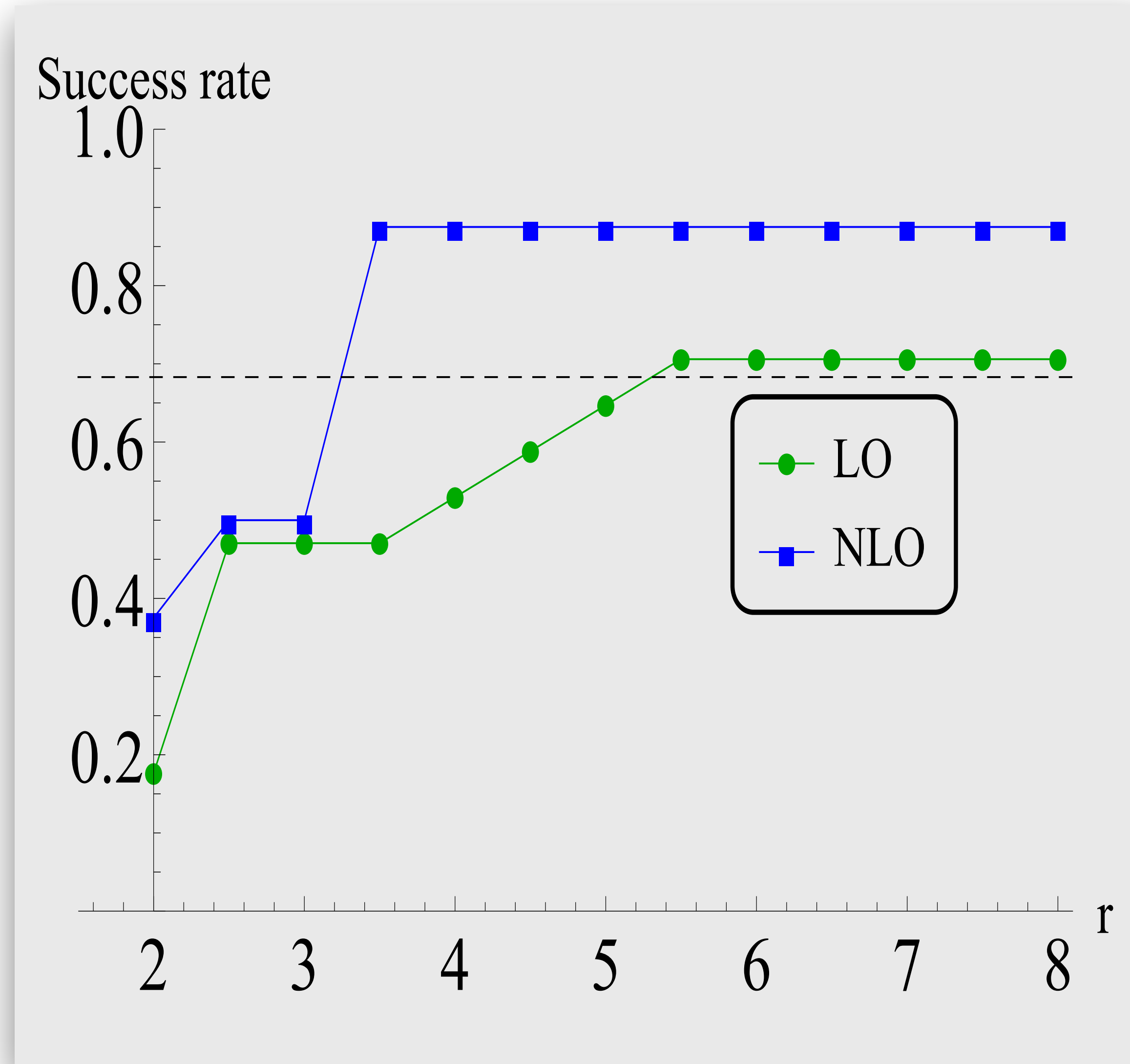
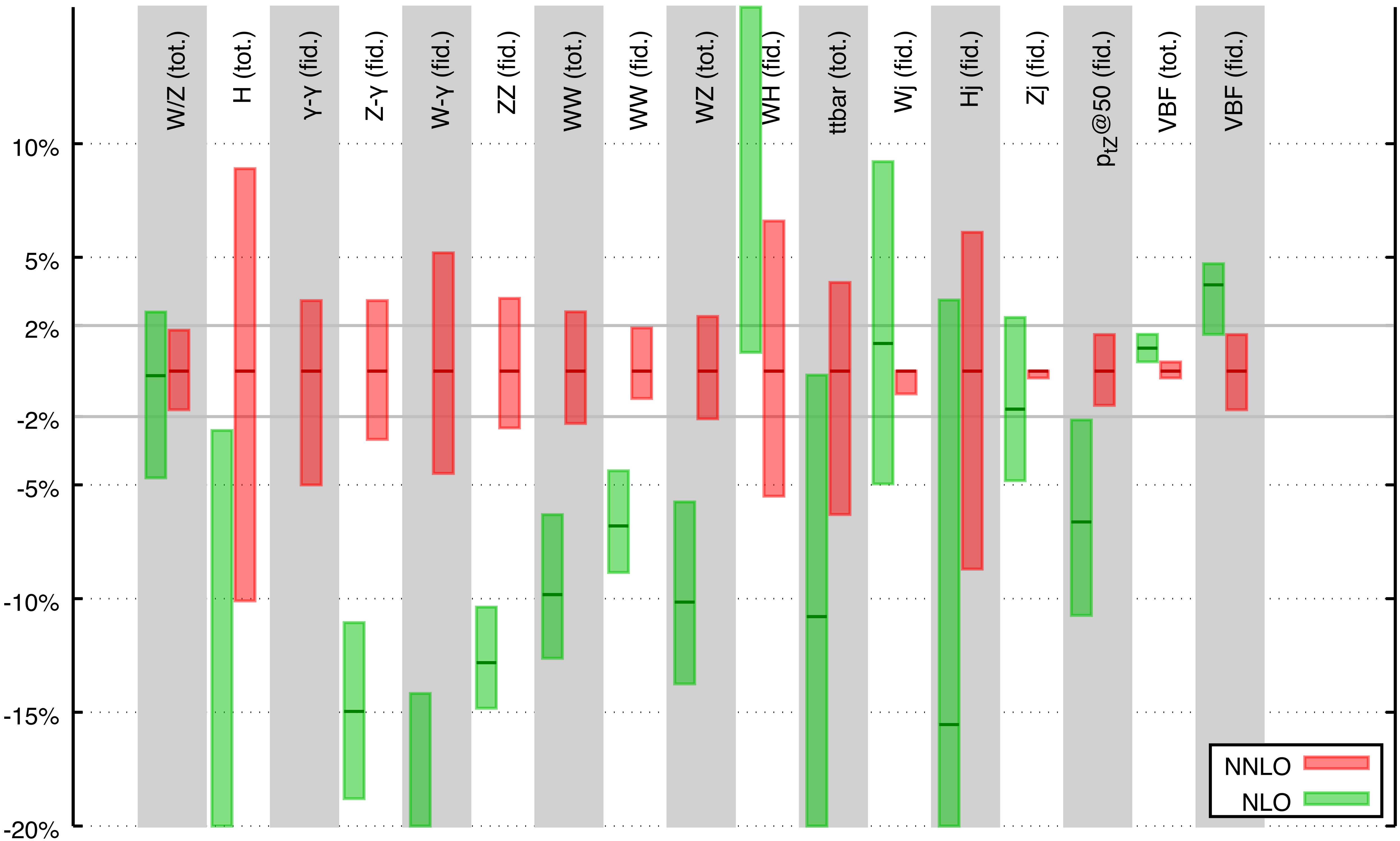


Figure 3.2: Fraction of observables whose known higher order is found to be contained within the uncertainty interval given by renormalisation and factorisation scale variation between $\mu_{r,f} = Q/r$ and $\mu_{r,f} = rQ$ with the constraint $1/r \leq \mu_r/\mu_f \leq r$. Only the seven points at the extremes and at the centre of the scale-variation interval are used. NNLO-evolved PDFs are used with all perturbative orders.

WHAT PRECISION AT NNLO?



For many processes NNLO scale band is ~±2%

But only in 3/17 cases is NNLO (central) within NLO scale band…