A non-local wave model for particles and fields (*)

W. SCHNELL
40 ch. De Valérie, CH-1292 Chambésy, Geneva, Switzerland

(ricevuto il 18 Novembre 1996; approvato il 25 Giugno 1997)

Summary. — The model universe proposed here is a Newtonian-Galilean system of extreme simplicity. Surprisingly, it can be arranged in such a way as to present itself to hypothetical model inhabitants as reality presents itself to us. It is argued, in fact, that the inhabitants of a universe formed entirely by classical waves on a macroscopically structureless medium will experience special relativity and quantum mechanics if only the medium possesses a spectrum of internal resonances, called 'masses', in addition to finite rigidity, density and very large bulk modulus. These 'masses' are then explained as compressional standing waves reflected by the spherical boundary of the universe which, thus, forms one multimode resonant cavity. Microscopic wave centres, called 'particles', are due to migrating dislocations in a simple molecular structure. The model replicates the existence of hadrons (classified in mesons and baryons) and leptons (charge carriers and neutrinos) and yields a naturally emerging explanation for electricity, including charge, the classical electron radius and the Coulomb force. It implies analytical formulae, essentially devoid of free parameters, for an approximate mass spectrum. The experimental mass ratios of many light hadrons, including all quasi-stable ones from the pion to the Ω^-, are represented with 1% rms accuracy, the charged-pion–to–muon mass ratio within 2.5×10^{-4}. The emerging electron mass is quantitatively consistent with the classical radius and the Coulomb force.

PACS 03.30 – Special relativity.
PACS 03.65 – Quantum mechanics.

1. – Introduction

The ideas presented here are quite unorthodox. They take up elementary matters which have been assumed settled for close to a century but may yet be considered very unsatisfactory.

The dissent from accepted dogma is, however, limited to a specific if fundamental point: the tentative claim that the basic elements (at least) of our reality, as described by orthodox physics, can be understood in terms of a classical, visualizable and

(*) The author of this paper has agreed to not receive the proofs for correction.

exceedingly simple model. As nothing is added to the body of accepted physics—indeed, the necessity of any addition would be an indication of the model's failure—the discussion given here ends where the formalisms of established physics begin and thus at a very elementary level.

Complete success of this admittedly unexpected model will depend on quantitative agreement with *all* known facts, including more advanced results of modern physics than those considered in this paper. It would appear, however, that the state of agreement demonstrated here is already and enough striking to merit attention.

The basic idea is this:

Instead of attempting at all costs to cope with the two self-contradictions implied in the statements that there are fields in "vacuum" and that "material" velocities are not additive, the tentative conclusions are drawn from the outset that there must be a universal medium where there are fields, but also that there cannot be any macroscopic material motion where velocities are not additive but tend towards a universal upper limit—a limit so strikingly low in fact as to be on a human-terrestrial scale.

Seen from outside, the model which will be built on these premises is governed by the Newtonian-Galilean physics which we experience in the limits of low velocities and large dimensions. The model will, however, be arranged in such a way as to present itself to hypothetical model inhabitants as reality presents itself to us. A distinction must, therefore, be made between the points of view of an external observer of the model (its constructor at least) and of its imagined inhabitants, coinciding with our reality as long as the attempt is successful. Surprisingly—and in violation of a dogma—this turns out to be the case, conceptually as well as quantitatively and far beyond the point where 19th-century physics was abandoned, albeit on a very elementary level so far.

The external observer's view will be taken unless stated or implied otherwise. Single quotation marks will designate inhabitant's terms, such as 'particle', 'energy', 'momentum'. These quotation marks will always be used when they give a different meaning to the same word, but may be omitted when no confusion is possible. Unavoidably, the model contains 'hidden' variables. It turns out to be manifestly non-local.

Although some of this model's features are rooted deeply in the first half of the 19th century, the decisive concepts—presented in sect. **3** and **6**—appear to be new and no attempt will be made to review history. Reference [1], first published in 1910, contains a quantitative account and a copious source of original references. It is clear, on the other hand, that this paper is related to an established minority dissatisfaction [2] with the orthodox interpretation of quantum physics. However, the proposed model stems from a fresh and direct look at elementary features of reality with little reference to ongoing arguments of quantum philosophy.

2. – Assumptions

The basis of this model is special relativity. According to the orthodox view, the manifest existence of a universal velocity limit is to be associated with macroscopic material transport and with ballistic particles in particular. Although impenetrable by human imagination this association is rendered acceptable by declaring it a principle—a starting point not itself amenable to a model.

There is, however, no *a priori* need for this sacrifice of intuitive logic. The existence of a universal and insurmountable velocity limit is indeed to be expected in a system in which *all* motion is restricted to the propagation of microscopic *perturbations* in a macroscopically stationary medium and, thus, to group motion of propagating waves in the most general sense.

Accordingly, the fundamental hypothesis made here is that *all* recognizable structures of our universe, including ourselves and all objects available to us, can be correctly described in terms of patterns of waves propagating in a unique, macroscopically structureless medium. The name of *ether* imposes itself although it differs from the 19th-century ether in being *the only material existing,* not an extraneous substance pervading others. Seen from outside, the ether considered here is a classical elastic medium subject to Hooke's law for small mechanical displacements, Newton's dynamics (with invariable mass) and Galilean transformations of reference frames. It will be assumed devoid of any motion other than propagating waves in every context considered in this paper, although convection on a 'cosmological' scale need not be excluded *a priori*.

The waves *forming the inhabitants' reality* tend to be composed of single-pulse perturbations possessing localized (and often permanent) centres which the inhabitants call '*particles*' and to which all dynamic interaction is confined. The existence of such wave centres requires an explanation. An apparently successful one, which establishes just as much particle behaviour as is required, will be offered in later parts of this paper. However, the definition of a wave—propagating disturbance rather than material transport—is supposed to be fulfilled at all times, each volume element of the propagating medium deviating only a very small amount from its rest position, to which it returns when the wave has passed.

Clearly, then, what the inhabitants call 'vacuum' is undisturbed ether, all their 'material' objects—such as the inhabitants themselves, their rulers, clocks, interferometers and particle detectors—are wave patterns, and *group velocity* is the only manifestation of motion.

3. – Relativistic dynamics

The inhabitants of a world formed by nothing but waves in a structureless medium find themselves in the situation of viewing a (silent black-and-white) film in a dark theatre. They have to obtain *all* information accessible to them by comparative observation of the topology and kinematics of *patterns,* the latter observation requiring references. If steady-state modes of resonant oscillation exist in the medium, these will supply references for Fourier analysis and give the inhabitants access to two—and only two—quantitative entities. Both are rational numbers, namely the numbers of consecutive extrema of a wave's central Fourier component divided by the coincident number of extrema of a reference wave, the division establishing itself by the formation of a beat pattern.

If the observation is made 'at a fixed location', to be suitably defined, the reference in the denominator forms a 'clock' and the number in the numerator is frequency $\omega/2\pi$. This defines the inhabitants' *time*. Similarly, the number of 'simultaneous' extrema along a path, divided by a coincident reference number counted along the same path, establishes a wave number $\mathbf{k}$ (2π over wavelength) and, therefore, a definition of *distance*. The meaning of 'fixed location' and 'simultaneousness' will depend on the observer's situation and may follow the usual definitions of special relativity.

The existence of reproducible reference waves is, thus, a necessary condition for the formation of recognizable structures in time and distance. In a homogeneous medium, reference waves can only be due to intrinsic modes of resonance. The inhabitants will recognize them if—and only if—there are several such modes and beating one against another gives reproducible results.

Strong support is lent to this model by the striking observation that the foundations of our dynamics appear to be mere kinematic quantities in fact. Indeed, overwhelming evidence suggests that what we call energy W (or mass W/c^2) behaves like a frequency and what we call momentum P like a wave number, the scale factor, $\hbar$, with respect to our customary units being the same in both cases. If, therefore, it is accepted that the inhabitants' 'energy' *is* a frequency ω and their 'momentum' a wave number $\mathbf{k}$, their fundamental law of 'relativistic' dynamics, relating 'energy' and 'momentum', *is* the dispersion relation (or Brillouin diagram) of the propagating medium. The question arises, therefore, whether a classical medium can be constructed whose dispersion relation equals the Lorentz-invariant law of dynamics

$$\omega^2 = \omega_0^2 + k^2 c^2 \tag{3.1}$$

actually observed.

The answer is affirmative and strikingly simple. In essence, the ether must consist of a three-dimensional array of identical coupled harmonic oscillators. There are the following perfectly plausible corollaries: they will all turn out to be part of one simple model.

– Each oscillator possesses an entire spectrum of modes with resonant frequencies ω_{0i} including zero. They may be called rest frequencies, the terms rest 'energies' $\hbar\omega_{0i}$ or (rest) 'masses' $\hbar\omega_{0i}/c^2$ being more familiar.

– The coupling constant, connecting each oscillator to its neighbour, is frequency-independent and the same for all modes ω_{0i}. The coupling changes each resonance to the cut-off of a high-pass filter. In the mode with $\omega_0 = 0$ (called 'electromagnetic') the restoring force is supplied by the coupling alone.

– Any graininess the ether may possess fails to make itself noticed in the observed dispersion diagram.

– The coupling force is perpendicular to wave propagation and expressed by $c^2 = \mu/\varrho$—the propagation velocity of shear waves—where μ is the shear modulus (rigidity) of the medium and ϱ its mass density.

The origin of the resonances ω_{0i} requires explanation (an explanation not offered by orthodox physics), but their multitude is perfectly plausible at once; a single resonance would be equivalent to the artifact of a massless spring resonating with a rigid body.

If the ether were made of spatially discrete oscillators, located a distance l_π from each other, the dispersion relation for each mode would be of the form

$$\omega^2 = \omega_0^2 + \omega_0^2 \sin^2 kl_\pi, \tag{3.2}$$

where ω_c is a measure of the coupling. At ω_0 all oscillators are in phase (0-mode). A finite oscillator-to-oscillator phase kl_π leads to $\omega > \omega_0$. At $kl_\pi = \pi$ an upper cut-off

(π-mode) would occur. As no sign of this is observed at the highest particle energies obtainable, l_π must be assumed vanishingly small. The product $\omega_c l_\pi$ then becomes a universal propagation velocity c and eq. (3.1) results, in perfect agreement with our reality if c is taken as the velocity of light.

In summary, it can be said that the actual Lorentz-invariant dynamics described by eq. (3.1) is perfectly modelled by classical mechanical waves in a dissipation-free, homogeneous, (nearly) incompressible but deformable medium of mass density ϱ and rigidity $\mu = c^2 \varrho$, provided the medium also possesses a *spectrum of internal resonances*, namely the rest frequencies $\omega_{0i} = W_{0i}/\hbar$. This presence of internal resonances and the absence of any other material in the universe are the only differences with respect to the 19th-century ether.

The resonances will later be attributed to the combination of a waveguide-like boundary condition with non-vanishing compressibility and this will furnish an immediate and quantitative explanation for the absence of observable graininess and, thus, for the exact validity of (3.1) rather than (3.2).

Familiar alternative forms of eq. (3.1)—clearly expressing relativistic dynamics as well as wave propagation in the classical medium just constructed—are

(3.3) $$\beta = \frac{ck}{\omega} ,$$

(3.4) $$\gamma^2 = \left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{1-\beta^2} ,$$

(3.5) $$v_g v_p = c^2 ,$$

where $v_p = \omega/k$ is the phase velocity of a single-frequency continuous wave and the group velocity

(3.6) $$v_g = \beta c = \frac{\partial \omega}{\partial k}$$

is seen as 'material velocity' by the inhabitants, since it is at this velocity that patterns propagate. The inhabitants' concept of 'force' is expressed by dk/dt along the path of a perturbation moving with the group velocity (3.6). In a free wave ω and k are constant. Finite derivatives will be generated by *refraction*—local variation of frequency due to interaction between wave centres.

Quantitatively, and in three dimensions, a *classical* wave packet at position x_i in a dispersive and refractive but dissipation-free medium obeys [3]

(3.7) $$\frac{dk_i}{dt} = -\frac{\partial \omega}{\partial x_i} ,$$

(3.8) $$\frac{dx_i}{dt} = +\frac{\partial \omega}{\partial k_i} .$$

The first equation [1] describes refraction which, in this model, is restricted to the vicinity of wave centres and must be made to represent one of the 'four forces of nature'. The second equation (v_g in three dimensions) describes dispersion—specifically the one of (3.1). The identity with Hamilton's equations applied to position, momentum and energy of a ballistic particle is textbook knowledge, as well as the association of particle velocity with the group velocity of de Broglie waves. Surprisingly, the inference that this might permit the construction of a Newtonian universe devoid of macroscopic motion does not seem to have been followed beyond the initial attempts by de Broglie and Schrödinger.

4. – The Lorentz transformations

It will now be argued that the inhabitants of the Newtonian-Galilean model just constructed will indeed experience the Lorentz transformations of their time and space with respect to reference frames moving through structureless ether at different group velocities.

The central point here is the existence of references for time and distance. If the ether carries no landmarks, such references can only stem from an internal resonance structure. The spectrum of rest frequencies ω_{0i} and the universal propagation constant c do form a conspicuous resonance structure and it will be assumed that there are no others. Note that this assumption, which makes the *mass spectrum the fundamental reference of time*, implies the *absence* of additional *ad hoc* constructions.

In fact, anyone of the rest frequencies forms a clock, their ratios being found invariable. The wavelength of one mode excited at the rest frequency of a higher one defines distance (for instance the wavelength of light at the electron rest frequency—the Compton wavelength). It seems noteworthy that modern standards are in fact formed by the velocity of light and an atomic transition frequency.

A straightforward demonstration of the Lorentz transformations' validity under these assumptions is presented in appendix A. Familiar ingredients of elementary special relativity are being invoked, but the essence is the observation that moving 'clocks' do in fact slow down and moving 'rulers' shrink in this Newtonian-Galilean model *because* clocks, rulers and co-moving observers are immaterial wave patterns propagating at common group velocity through a dispersive medium whose Brillouin diagram is given by eq. (3.1). And moving observers *consisting* of waves cannot know the rate at which they encounter fresh volume elements of structureless medium. They cannot, therefore, experience any "ether wind".

Accordingly, this Newtonian-Galilean model of coupled harmonic oscillators yields: relativistic mechanics (as well as classical electrodynamics, as will be shown later), the Lorentz transformations and the undistinguishable equivalence of inertial frames, in that order of deducible results not requiring any *ad hoc* principle. Contrary to orthodox dogma the model does contain a preferred reference frame but one which is fundamentally out of reach of the inhabitants' recognition.

---

(1) Explicitly written time derivatives are employed for wave motion—total derivatives when the group motion is to be followed. Dots will be reserved for microscopic material motion in an ether frame. No distinction between partial and total derivatives is required therein, since the basic assumption of an elastic solid, rather than a hydrodynamic fluid, makes convection terms negligible.

5. – Elements of quantum mechanics

The illumination of a picture cannot be deduced from a contemplation of its contents as long as these remain recognizable at all. Similarly, the wave amplitudes of this model remain fundamentally inaccessible to direct observation by the inhabitants whose 'reality', including their own identity, consists of wave patterns alone.

An amplitude will make itself noticed, however, if it becomes so small as to make the recognizable coherence of a pattern disappear in competition with others or with statistical noise. This suggests the association of a suitably chosen (scalar) amplitude with the inhabitants' 'wave function' and, hence, of this amplitude squared with the 'probability density' of finding a certain configuration. In sect. **12** the (nonrelativistic) Schrödinger equation will be derived from the model of 'mass' developed in sect. **6** and beyond. In this derivation, the 'wave function' will be identified with dynamic ether pressure.

On the other hand, it will turn out (sect. **9**) that a field of quasistatic ether displacements from rest position offers itself as a most natural explanation of the 'electric' field, accessible to only indirect recognition by enlightened inhabitants.

If it is true, then, that oscillating-wave amplitudes are inaccessible to inhabitants' direct observation, this must be even more true for intermediate values within a period of oscillation. Since, apart from the scale factor $\hbar$, the integrals of 'energy' (ω) over time and of 'momentum' ($\mathbf{k}$) over distance are identical with the phase advance of an oscillation, it follows that these integrals have to be integers, namely the numbers of (half) periods *counted* over the integration time or path. This quantization of phase—or 'action'—merely states that a universe formed by patterns is fundamentally digital. It also identifies a 'particle' that carries one 'quantum of action' as a single-pulse wave.

According to the arguments of sect. **3**, the inhabitants may describe a small 'structure'—a wave packet as seen from outside—by its rms duration Δt or length Δx_i. They may also describe it by the width $\Delta\omega$ or Δk_i of its Fourier transform and use both aspects for measurement. However, the product of rms pulse length and spectral width cannot be smaller than one-half, a mathematical fact [4] that is given the name of bandwidth theorem in communications engineering and of *uncertainty principle* in quantum physics, the minimum product being reached for Gaussian pulses. The bandwidth theorem designates a fundamental limitation for the transmission of intelligence by modulated waves. Its universal validity for conjugate variables of our world—those whose product is a phase in wave mechanics—forms a very strong suggestion that there *is* nothing but waves. Again, the identity of Heisenberg's principle with the bandwidth theorem is introductory textbook matter but the massive hint of a reality devoid of material transport is not followed in orthodox physics.

Particle-like behaviour and concomitant 'duality' must be explained by the existence of wave centres to which any interaction of waves is confined. Waves travelling with a freely moving centre extend into the surrounding space where they can be split and recombined to form an interference pattern. The wave can only manifest itself, however, by (nonlinear) intermodulation with the waves forming the 'measuring apparatus'.

If the measuring apparatus is a 'particle detector', the interaction may be assumed unstable, leading to the formation of new wave centres. It seems plausible that this avalanching nature of new-wave-centre formation makes its occurrence stochastic but subject to a probability proportional to the amplitude squared of the original wave. This seems to explain why a large number of randomly occurring localizations of the original waves into new centres—seen as impacts of 'particles'—can print out an interference pattern although self-interference pertaining to the original wave can no longer be observed once this has transformed itself into new wave centres by 'particle detection'.

While the last remarks are insufficient at this point to clear up one of the deepest mysteries of our reality, the following statements appear in order here. Orthodox contemporary physics overcomes the oxymoron of fields in 'vacuum' (or else of action at a distance) by describing all interactions in terms of mediating particles—'virtual' ones, defying eq. (3.1) under cover of uncertainty if necessary. The notion of phase cannot be dispensed with, however, and the uncertainty relation takes on an ambiguous meaning. It retains its status of an *ad hoc* postulate concerning ballistic particles while it is also identical with the deducible fact of the bandwidth theorem—which requires a medium.

In this theory the opposite view is taken. It is claimed that once a universal propagating medium is accepted, there is no difficulty with the existence of continuous fields therein, provided the parallel existence of ballistic particles is abandoned. The 19th-century problems with such a medium disappear with the notion of particles. The Lorentz transformations as well as the uncertainty relations become deducible consequences of their absence. It may also be said that this model (seen from outside) assigns reality to the inhabitants' 'abstract' notion of quantum wave mechanics while their 'reality'—consisting of ballistic particles and material structures in general—is declared an illusion.

Thus far, the model, to which 'particles' and their interaction have to be added yet, replicates nature's wave-like features alone. It is argued, however, that the evidence for the absence of anything else is so overwhelming—an insurmountable velocity limit, a simple Brillouin diagram describing apparent 'dynamics', an all-encompassing bandwidth theorem—and the traditional attempts at reconciliation of manifest wave properties with ballistic objects are so desperate that an explanation of these objects' appearance in terms of waves, rather than the inverse, seems inevitable. What remains to be accomplished, therefore, is a credible demonstration that 'particles', as distinct from diffusing wave packets, can be described in terms of propagating wave centres. The remainder of this paper—while stopping far short of the frontiers of contemporary physics—appears to go a long way in that direction.

6. – A concept for the rest masses

The presence of resonances ω_{0i} in a seemingly continuous medium asks for an explanation in terms of classical mechanics if the basic concept followed here is not to be abandoned.

A differential equation yielding the correct dispersion relation of 'free space' has the form of

$$\varrho \ddot{\mathbf{s}} = \mu \nabla^2 \mathbf{s} - d_i \mathbf{s} \,, \tag{6.1}$$

where $\mathbf{s}$ is the displacement from rest position or 'vacuum'.

With $d_i = 0$ in an incompressible medium of rigidity μ, (6.1) yields dispersionless transverse waves. Dispersion with ω and k related by eq. (3.1) is created by the spring constants d_i—per unit volume and with respect to absolute space!—which have to be made equal to $\omega_{0i}^2 \varrho$ so as to make a volume element of density ϱ resonate at ω_{0i} for $\nabla^2 \mathbf{s} = 0$. A free wave $\mathbf{s}_0 \exp[i(\omega t - \mathbf{k} \cdot \mathbf{r})]$, where $\mathbf{r}$ stands for x_i, satisfies (6.1) with $\omega(\mathbf{k})$ as given by (3.1).

The first model which comes to mind is a local one in which finite ether constituents exhibit internal resonances although they are too small to reveal their presence by an observable π-mode cut-off. Finite constituents will in fact be introduced below. It is difficult, however, to explain the presence of internal resonances at ω_{0i} in objects too small to be observable at wave velocity c and frequencies far above ω_{0i}. Moreover, the presence of a zero-frequency mode and the reference to absolute space would have to be accounted for. Short of returning to 19th-century-style contrived constructions a purely local model for the rest resonance cannot, therefore, be maintained.

This leaves no alternative, however, but an explanation in terms of macroscopic *standing* waves reflected by non-absorbing boundaries. A striking one-dimensional example is the (electromagnetic) waveguide. Standing waves, set up between the guide walls (in x, y direction say), form an intricate spectrum of 0-modes, reminiscent of a 'mass spectrum' even for simple geometric shapes—for instance the interlaced multiple roots and orders of the ordinary Bessel function for a circular guide. The infinite extension in z-direction converts each 0-mode to the cut-off for high-pass wave propagation. Higher modes are notoriously unstable via mode conversion to lower cut-off frequencies. The dispersion relation $\omega(k)$ of the uniform waveguide *is* the Lorentz-invariant law of dynamics, eq. (3.1). It may be viewed indiscriminately as resulting from the mutual coupling of an infinity of harmonic oscillators of vanishing spacing (as argued above) or from the interference of plane waves propagating at free-space velocity along zig-zag trajectories between the reflecting walls.

A straightforward extension to three dimensions—albeit an emotionally shocking one—can be made by taking the closed boundary of the entire universe as the reflecting wall for *compressional* (irrotational) waves. The ether's bulk modulus κ must then be assumed so 'enormous' as to explain the rest resonances by standing waves within this boundary, the universe being taken as one cavity-like multimode resonator. Indeed, a standing compressional wave at right angles to the propagation of a free shear wave will satisfy eq. (6.1). Pressure gradient takes the place of the restoring force $d_i\mathbf{s}$, the reflecting boundary supplying the necessary reference to 'absolute space'. This is shown by the $\operatorname{grad}(\kappa \operatorname{div} \mathbf{s})$ term in eq. (7.1)—the general equation of motion in an elastic medium—at the start of the next section, where a specific spectrum of eigenfrequencies ω_{0i} will be calculated and compared with actual particle masses.

This universe, therefore, may be viewed as a vibrating lump of clear jelly—a homogeneous substance possessing an exceedingly low but finite ratio of rigidity to bulk modulus. Compressional waves shuttle between opposite boundaries of the universe in times commensurate with $\hbar$ over rest energy. Shear wave propagation at velocity c is slower by several tens of orders of magnitude so that it finds essentially infinite space. This is also what the inhabitants find since everything they recognize consists entirely of shear waves.

Isotropy suggests a (near) spherical geometry and, therefore, radial pressure distributions governed by spherical Bessel and Neumann functions j_l and n_l given by

$$j_l = (-u)^l \left(\frac{1}{u} \frac{d}{u} \right)^l \frac{\sin u}{u} , \tag{6.2}$$

$$-n_l = (-u)^l \left(\frac{1}{u} \frac{\mathrm{d}}{\mathrm{d}u} \right)^l \frac{\cos u}{u} . \tag{6.3}$$

In a homogeneous medium $u = Kr$, where $K^2 = \omega^2 \varrho/\kappa$ and r is the radius from the centre. The two functions j_l, n_l differ by no more than π in the central phaseshift between the outgoing and incoming travelling spherical waves of which they are composed, but their behaviour near the centre is radically different. The singularity-free function $j_l(u)$ describes the response of the elastic sphere to an external excitation, while $n_l(u)$ with its central singularity describes the situation around an internal driving point.

It is customary to exclude the n_l-functions from the analysis of (empty) spherical cavities on the face of the argument that they are unphysical. In this model, their inclusion offers the unique opportunity to create particle-like wave centres. Physical reality then requires that the divergence of pressure and displacement—proportional to $u^{-(l+1)}$ and $u^{-(l+2)}$, respectively, at vanishing radius—be interrupted by a break in ether continuity. Moreover, a discrete physical oscillator is required at the wave centre which, obviously, cannot be assumed to coincide exactly with the mathematical centre of an ideal sphere.

In the absence of foreign matter, both requirements can only be fulfilled if the ether possesses *molecular structure*, however small and simple. Nothing specific has to be assumed about these molecules, but that they are nearly incompressible—so as to produce the bulk modulus κ—and held together by central forces. The existence of wave centres then finds its natural explanation in the existence of *dislocations*, non-linear disturbances of molecular order, *migrating* by rearrangement of neighbouring molecules without macroscopic transport of material.

An explicit molecular structure need not necessarily be invoked here but a most elementary one may, nevertheless, be imagined. It consists of hard frictionless spheres, pressed into nearly dense packing by a coherence pressure but kept from locking into complete 12-connection by the presence of dislocations. In spite of its simplicity, this structure appears to possess all required properties so that no further constructions are required from here on: the tendency to approach dense packing implies a (weak) rigidity μ which is unrelated to the modulus of compression κ (cf. the end of appendix B). This rigidity is subject to the possibility of reversible disruption, as required in sect. **10** and beyond. Within microscopic regions of large dynamic displacements a dislocation can jump 'instantly', by translation at compressional wave velocity, but macroscopic propagation will be free of local compression; it cannot, therefore, exceed the shear-wave velocity c.

Each propagating dislocation represents the central driving point of an n-mode configuration of standing compressional waves filling the universe. These driving points are slightly off-centre (cf. sect. 8) and propagate at group velocity βc via superimposed shear waves. The resonance conditions derived in the next two sections—eqs. (7.11) and (8.9)—are with respect to an excitation centred at a molecular dislocation and respecting the angular dependence of a surface harmonic $Y_l^m(\theta, \varphi)$ of given degree and order. From the circular-section waveguide invoked above, this model differs by the replacement of the ordinary (cylindrical) Bessel function by the two spherical ones; it differs from the ordinary spherical cavity by the inclusion of the linearly independent function n_l.

The small-argument asymptote of $n_l \propto u^{-(l+1)}$ and the fact that the displacement is proportional to the pressure *gradient* make the *ratio* of dynamic pressure to displacement vanish linearly with radius. Therefore the maximum pressure—large but finite at the finite molecular radius—can be assumed to be cushioned by central intermolecular forces. An analysis of these elastic forces remains outside the scope of this paper. Their systematic absence from the lepton model presented in sect. **10** suggests an association with the 'strong force'.

The molecular radius need not be assumed to be exceedingly small since it does *not* form a cut-off wavelength like l_π in eq. (3.2). In unperturbed propagation, the dislocation shifts gradually from one cluster of molecules to the next. The resonators, as well as the coupling rigidity μ, are formed by continua consisting of a very large number of molecules.

The combination of a 'particle' trajectory with a wave filling the universe is reminiscent of the pilot wave concept of de Broglie-Bohm—more precisely of de Broglie's *théorie de la double solution* with its singularity wave (*onde à singularité*) [2] since the propagating dislocation is a genuine part of the wave, not a ballistic particle being guided.

The ether model constructed here will not be accepted easily. It turns out, however, to be strikingly successful in representing seemingly unrelated facts and it does this without further constructions, albeit via a sequence of inferences. This will be the subject of the balance of this paper.

Without violating the fundamental assumption of there being nothing but waves, propagating molecular perturbations go as close as necessary to defining 'particles'. Interactions between them will be restricted to the neighbourhood of the n-mode wave centres with their near divergence of displacement, pressure and stress where, on the other hand, such interaction becomes very likely. In the process of interaction new dislocations may form at the expense of original ones although this is unlikely to occur outside the range of high original wave amplitude. Note that such molecular relocation will occur 'instantly' (at compressional wave velocity) and that it is bound to happen at 'particle detection'.

The propagating dislocations may be purely dynamic defects—molecules being pulled apart or pushed towards each other in small-argument n-mode motion. (At the positive peak of a vertical dipole n-mode, for instance, molecules are radially pulled away from the polar axis above the centre and pushed towards the axis below.) In addition, the near divergent motion may create *permanent dislocations* in the form of excess or deficit molecules. Creating a pair of such trapped dislocations without static pressure at large distance will create a conjugate pair of centres for permanent radial displacement and associated shear stress, as a lump of excess material leaves behind a collapsed cavity elsewhere. In sect. **9** these source points of radial displacement will emerge as perfect and perfectly natural models for the 'electric charge'. The existence of stress-free voids cannot be excluded and only such voids can support isolated monopole (breathing) modes associated with $l = 0$. Therefore, if the association of stress centres with electric charge is correct, 'particles' associated with $l = 0$ must be electrically neutral.

An exact analysis of the quasi-spherical modes postulated here will be seriously complicated by the wave centres' excentricity. Also, graded or stratified distributions of ϱ and κ may have to be taken into account and the spherical surface might be loaded with any reactive acoustic impedance—from a free surface to a rigid wall. Moreover, the finite rigidity may gain influence near the wave centre, in spite of the enormous assumed ratio of κ/μ. In the next two sections, resonant modes will nevertheless be calculated under the simplest possible assumptions of a homogeneous elastic sphere of vanishing rigidity, possessing a near central driving point and a free surface (or else be contained within a rigid wall). The result, while necessarily approximate and incomplete, lends quantitative support to this theory.

7. – An approximation to the hadron mass spectrum

In an isotropic elastic medium with constant μ, κ, ϱ everywhere the combination of Newton's and Hooke's laws (Cauchy 1828 [1]) takes the equivalent forms of

$$\varrho\ddot{\mathbf{s}} = \mu\nabla^2\mathbf{s} + \left(\kappa + \frac{1}{3}\mu\right)\nabla(\nabla\cdot\mathbf{s}), \tag{7.1}$$

$$\varrho\ddot{\mathbf{s}} = -\mu\nabla\times(\nabla\times\mathbf{s}) + \left(\kappa + \frac{4}{3}\mu\right)\nabla(\nabla\cdot\mathbf{s}). \tag{7.2}$$

The first (solenoidal) term describes the propagation of shear waves. The second (irrotational) term yields (after forming the divergence on both sides and assuming $\kappa \gg \mu$) the differential equation for pressure waves in a liquid

$$\ddot{p} = \frac{\kappa}{\varrho}\nabla^2 p, \tag{7.3}$$

where $p = -\kappa\,\mathrm{div}\,\mathbf{s}$ is the amplitude of hydrodynamic pressure. Below, eigenfrequencies for stationary solutions of (7.3) are derived. With these, (7.3) can be written as $-\varrho\omega_{0i}^2\,\mathrm{div}\,\mathbf{s} = \kappa\,\mathrm{div}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{s}$; thus $-\varrho\omega_{0i}^2\mathbf{s} = -\kappa\nabla(\nabla\cdot\mathbf{s})$. Substitution into (7.1) with $\mu \ll \kappa$ confirms (6.1) with $d_i = \varrho\omega_{0i}^2$.

Indeed, the presence of a spherical boundary and a central molecular wave centre leads to harmonic solutions of (7.3) with spatial amplitude distributions

$$p = Y_l^m(\theta,\varphi)[aj_l(Kr) + bn_l(Kr)], \tag{7.4}$$

$$\omega^2\varrho s_r = Y_l^m(\theta,\varphi)\,K[aj_l'(Kr) + bn_l'(Kr)]. \tag{7.5}$$

The radial displacement s_r equals the radial pressure gradient over $\omega^2\varrho$, the dash means differentiation with respect to the argument, r, θ, φ are spherical coordinates, Y_l^m is a surface harmonic of degree and order l, m and $K^2 \equiv \omega^2\varrho/\kappa$ as before.

At a free surface the pressure must vanish so that

$$[aj_l + bn_l]_{KR} = 0 \tag{7.6}$$

if R is the radius of the sphere. The constants a, b can be eliminated by observing that the oscillations are excited (by shear waves) at the centre of the n-mode. The j-mode is an *idler* which exchanges energy with the primary n-mode at the surface only. Therefore, the radial displacements pertaining to j_l and n_l must be equal there. This yields

$$[aj_l' - bn_l']_{KR} = 0 \tag{7.7}$$

and, therefore, the resonance condition

$$\left[\frac{j_l}{j_l'} + \frac{n_l}{n_l'}\right]_{KR} = 0 \tag{7.8}$$

stating that the acoustic impedances (pressure over radial velocity) of the two linearly independent modes, j_l and n_l, of same angular distribution Y_l^m must cancel each other at the free surface. It may be noted in passing that a rigid spherical wall would have given the same result, at least thus far. Strikingly, the almost self-evident equation (7.8) already yields most of the coincidences with experimental particle-masses listed in table I.

Barycentre motion introduces a complication for odd-order zonal modes. If the resonance were excited from outside the sphere only the pressure balance (7.6) would guarantee zero axial momentum and eq. (7.8) would remain the only resonance condition. The excitation by coupling to another vibrating system *inside* the sphere

TABLE I. – *Roots of* $[jn' + Nnj']_{KR} = 0$—*eq.* (7.11)—*compared with experimental mass values, normalized for* (1012)′ η (547.45 MeV); ν_j *and* ν_n *are the orders of neighbouring zeros of* j_l *and* n_l.

Equation (7.11)							Reality		Error
configuration	ν	N	l	ν_j	ν_n	root	mass	particle	$\frac{(7.11)}{\text{reality}} - 1$
monopole	1	1	0	1	1	2.1374	—	—	—
	2	1	0	1	2	3.7983	3.7983	η	reference
	3	1	0	2	2	5.4063	5.4253	$\omega(783)$	−0.35%
	4	1	0	2	3	6.9976	7.0728	$\phi(1020)$	−1.1%
dipole	1	1	1	1	1	3.4865	3.4250	$K^{\pm}$	−1.8%
							3.4529	K^0	+0.97%
	0	2	1	0	1	0.9735	0.9683	$\pi^{\pm}$	+0.53%
							0.9365	π_0	−3.95%
	1	2	1	1	1	3.3011	—	—	—
	2	2	1	1	2	5.3826	5.3292	$\varrho(770)$	+1.0%
	3	2	1	2	2	6.6744	6.6450	$\eta'(958)$	+0.44%
quadrupole	1	1	2	1	1	4.7292	—	—	—
	2	1	2	1	2	6.5213	6.5098	p	+0.18%
							6.5188	n	+0.04%
	3	1	2	2	2	8.2077	8.2520	Σ^+	−0.54%
							8.2740	Σ^0	−0.80%
							8.3079	Σ^-	−1.2%
	4	1	2	2	3	9.8519	9.99	N(1440)	−1.4%
	5	1	2	3	3	11.4740	11.6035	Ω^-	−1.1%
sextupole	1	1	3	1	1	5.9224	—	—	—
	2	1	3	1	2	7.7841	7.7404	Λ	+0.56%
octupole	1	1	4	1	1	7.0859	—	—	—
	2	1	4	1	2	9.0126	9.1229	Ξ^0	−1.2%
							9.1675	Ξ^-	−1.7%

permits, however, the balancing of a finite axial momentum of the resonator against the opposite momentum of the exciting system (from which 'resonance' is observed) via an exchange of surface forces. Since such forces cannot impart the necessary momentum to the n-mode directly, they will excite an intermediate j-mode—$a_b j(Kr)$, say—whose axial momentum equals that of the primary n-mode and is, thus, given by $a_b j(KR) = bn(KR)$. This must be added to the pressure balance (7.6), introducing a factor 2 in front of the second term of the resonance condition (7.8).

More precisely, it may be argued that 'resonance' as seen from inside means vanishing boundary pressure including the apparent inertial forces experienced in a frame oscillating with the primary n-mode, since it is in this frame that the compensating j-mode idler is excited. In a local frame oscillating with the local n-mode amplitude an inertial body force appears. The force component parallel to the (vertical) polar axis f_z^i (per unit normal area) is given by

$$\frac{\partial f_z^i}{\partial z} = -\varrho \ddot{s}_{zn} = \omega^2 \varrho s_{zn} = \frac{\partial}{\partial z} p_n(r, \theta), \tag{7.9}$$

where s_{zn} is the peak vertical displacement and p_n the n-mode pressure amplitude. (Note that the area density of apparent force f_z^i is a vector component; the elastic pressure $p = -\kappa \operatorname{div} \mathbf{s}$ is a scalar.) Integration parallel to the polar axis gives the total inertial force on a unit-area column intersecting the surface at θ and $\pi - \theta$ as

$$f_z^i(\theta) = p_n(R, \theta) - p_n(R, \pi - \theta). \tag{7.10}$$

For even l this force is zero; for odd-order zonal modes it is equivalent to a surface-normal force density given by $p_{\mathrm{n}} = bn_l(KR)\, P_l(\cos\theta)$ at each end of the column ($P_l(\cos\theta)$ being a Legendre polynomial) and, hence, to an apparent inertial surface pressure equal and supplementary to the elastic n-mode pressure just inside the surface ([2]). Again, a factor 2 in front of the second term of eq. (7.8) is the result.

In conclusion, a general resonance condition can be written in the form of

$$\left[\frac{j_l}{j_l'} + N\frac{n_l}{n_l'}\right]_{KR} = 0, \qquad N = 1 \text{ or } 2 \tag{7.11}$$

(or in any of numerous equivalent forms such as $(1+N)(KR)^2 nj' + 1 = 0$) with $N = 1$ for all even orders and all sectorial modes and optionally $N = 2$ for odd-order zonal modes. Independent tesseral configurations (they only exist for $l \geqslant 3$) have not been investigated.

Replacing $N = 1$ by $N = 2$ for given l creates an additional non-vanishing root (henceforth labelled $\nu = 0$) below all others. For $l = 1$ this forms the lowest of all compressional resonances. The influence of the factor N on higher roots amounts to a few per cent at most, j/j' and n/n' being steep functions of KR near these roots.

([2]) Note that in a dipole n-mode with $KR < 2.7$ the motion perpendicular to the axis makes the peak upward shift of the spherical envelope coincide with peak negative pressure in the upper hemisphere, thus with peaks of upward acceleration and downward apparent inertial force felt in a barycentre frame.

An objection to this model will be its limitation to inhabitants living near the centre of their universe and the concomitant reminiscence of pre-Copernican geocentricity. The objection is attenuated by the observation, based on symmetry, that deviations from eq. (7.11) cannot be of lower order than second in relative excentricity. This will be confirmed in the next section where a first-order model for an excentric wave centre will be presented.

With all due caveats, then, the roots of eq. (7.11) may be compared with experimental particle masses. This is done in table I, more extensive, albeit more tentative, associations being given in sect. 8 and fig. 1. The fourth column of table I lists short series of roots of (7.11) for $l \leqslant 4$. The first two columns give the mode configuration and the root order ν ($\nu = 0$ being reserved for the extra root appearing with $N = 2$). The third column gives the barycentre factor N (1 or 2), the Bessel-function order l and the orders ν_j, ν_n of neighbouring zeros of j_l, n_l. Henceforth the four numbers $Nl\nu_j\nu_n$ will be used to designate a mode.

The fifth column lists experimental masses [5] of light mesons and baryons, including all quasi-stable hadrons up to the Ω^-. The masses are normalized to make the η-meson (547.45 MeV, the lowest-mass singlet) coincide with (1012), this being the lowest purely radial mode possessing an internal node. As can be seen, the picture would not change significantly if the reference were changed to the proton or the charged pion (their mass ratio of 6.723 is replicated with 0.36% accuracy) or if an average had been fitted.

The agreement of the spectrum defined by the roots of eq. (7.11) with the actual mass spectrum of the lightest mesons and stable baryons is striking and hard to dismiss as fortuitous (cf. fig. 1). The errors are of the same order as typical charge-state splits which the model does not pretend to address. Moreover, beyond numerical agreement at percent level, a rudimentary but distinct order emerges:

– Consistent with the model, modes with $l = 0$ do correspond to chargeless mesons (and the extended tables of the next section will show no exception to this).

– Among the stable baryons the sum $l + \nu_j + \nu_n$ appears to take on an ordering function: it amounts to 5 for the p/n, 6 for the Σ and Λ, 7 for the Ξ and 8 for the Ω^-.

– All mesons listed correspond to radial or zonal modes, thus configurations of one or two dimensions. All baryons listed are consistent with sectorial or tesseral—thus three-dimensional—modes. It is tempting, therefore, to think of an association of quarks with one-dimensional mode components. The association remains, however, obscure; what corresponds to the simplest mode of this model, the η, is an intricate mixture of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ states in the quark model, while the simple $u\bar{d}$, $d\bar{u}$ states of the $\pi^\pm$ require the additional argument about barycentre motion here.

– With the exception of (1111), the modes with $\nu_j = \nu_n = 1$, that is those devoid of a radial node, fail systematically to correspond to real particles. An explanation may be sought in connection with barycentre stability for an excentric wave centre.

It may be objected that there are low-mass particles not covered by (7.11) and low-order roots not corresponding to any known particle. Up to about 1.3 GeV, the first defect will be essentially remedied by the inclusion of additional resonances due to finite excentricity (sect. 8) although this will aggravate the second (more acceptable) defect. It would appear, however, that the left-hand side of fig. 1 alone, containing all established particles to the $\phi(1020)$, does constitute conspicuous evidence. Moreover, even at higher energies, where the close relative spacing tends to devalidate any 1% correlation, closer inspection reveals that the clustering of roots with common $l + \nu_j + \nu_n$) —a mathematical fact—is indeed validated, albeit at increased spread *within* clusters of actual masses. A qualitative explanation for this spread is attempted in sect. **8**. Finally, strong collateral support is lent to this hadron model by the success of the concomitant lepton model presented in sect. **10**. It implies an unambiguous prescription for the charged-pion–to–muon mass ratio under the assumption that the association of the pion with the lowest root of (7.11) is indeed correct, and the prescription agrees with reality to within 0.025%.

If, therefore, and in spite of many open questions, the listed associations of hadron masses with roots of $jn' + Nnj' = 0$ are accepted, a universal hadron scale length

$$r_s = \frac{cK_i R}{\omega_{oi}} = R\sqrt{\frac{\mu}{\kappa}} \approx 1.37 \times 10^{-15}\ \text{m} \tag{7.12}$$

emerges. If R were taken as 10^{26} m the ratio $\sqrt{\kappa/\mu}$ of longitudinal to transverse propagation velocity would be of the order of 10^{41}.

8. – Finite excentricity

The apparent success of spherical symmetry—for low masses and at one-percent level—justifies pursuing the model in spite of its limitation to inhabitants living near the centre of their universe. However, propagating wave centres do entail finite, if small, excentricity and concomitant mathematical complication. Short of computational analysis, the concept proposed below permits at least a first-order consideration.

The resonant standing waves of radial dependence $aj_l(Kr) + bn_l(Kr)$ discussed in the last section are composed of travelling spherical waves whose pressure and radial displacement are given (apart from a scale factor and $e^{i\omega t}$) by

$$p(r,\theta,\varphi) = Y_l^m(\theta,\varphi)[n_l(u) \pm ij_l(u)], \tag{8.1}$$

$$s_r(r,\theta,\varphi) = Y_l^m(\theta,\varphi)\,\frac{K}{\omega^2\varrho}\,[n_l'(u) \pm ij_l'(u)], \tag{8.2}$$

where $u = Kr$. The upper signs refer to radially outgoing waves for which (8.1) is negative real and (8.2) positive real near the centre. The wave impedance $p/\dot{s}_r$ is complex in general but real at the surface at resonance for $N = 1$. A j-mode standing wave is formed if the phase between outgoing and incoming waves is π at the centre, an n-mode if it is zero.

The n-mode centre will now be assumed to have a position $\mathbf{e}\varepsilon R$ with respect to the sphere's centre where $\mathbf{e}$ is a unit vector of arbitrary direction and $\varepsilon \ll 1$. The model adopted for analysing this situation is one of bifocal reflection, apparently justified for first-order analysis since pathlength differences depend on ε^2. Spherical waves travelling outward from the molecular dislocation—called the main focus—are reflected at the surface towards a phantom focus at $-\varepsilon\mathbf{e}$ from where they travel back to the main focus via a second surface reflection. Since the phantom focus cannot be assumed to contain a molecular dislocation, it must be singularity-free and this forms another boundary condition.

The minimum solution is a superposition of two systems of travelling displacement waves in quadrature, one with scale factor one, say (thus of radial dependence $n'(u) + ij'(u)$ for an outgoing wave), the other with scale factor ia_1, both centred in the main focus. They will be named *reference wave* and *a_1-wave,* respectively, for convenience. The composite wave leaving the centre with phase $\psi_0 = \arctan a_1$ with respect to the reference has the phase $\psi_0 + 2\psi_R$ at the phantom focus if

(8.3) $$\psi_R = \arctan[j'/n']_{KR}$$

is the phase advance per radius R. The condition of freedom from singularity at the phantom focus is $\psi_0 + 2\psi_R = \pm\pi/2$ and therefore

(8.4) $$\tan - \left(2\psi_R \pm \frac{\pi}{2}\right) = a_1 ,$$

since only in this way is reflection by π, there, compatible with the symmetry required for cyclic closure of phase, namely a main-focus return-phase of zero for the reference wave and of π for the a_1-wave. Equations (8.3) and (8.4) combined are equivalent to

(8.5) $$a_1 = \frac{1}{2}\left[\frac{n'}{j'} - \frac{j'}{n'}\right]_{KR} .$$

The global result is the superposition of three systems of standing pressure waves, an n-mode and a j-mode centred at the main focus and another j-mode centred at the phantom focus, with scale factors 1, a_1 and a_2, respectively. Continuity of power flow requires that

(8.6) $$a_2^2 = a_1^2 + 1 .$$

But this, together with (8.5), is consistent with

(8.7) $$[(a_1 + a_2)j' - n']_{KR} = 0$$

and therefore with (7.7) if $a = a_1 + a_2$ is taken as the total j-mode amplitude (and $b \equiv 1$ for convenience). Together with the surface pressure balance $aj_l + Nn_l = 0$, the resonance condition (7.11) comes out unchanged for this bifocal model, as was to be expected.

It appears, however, that a second set of solutions can be generated by the addition of yet another j-mode, of scale factor a_3, say, which is centred in the sphere and, hence, linearly independent of the bifocal modes. The surface excitation of this is described by

(8.8) $$[a_3 j_l' - n_l' - (a_1 + a_2) j_l']_{KR} = 0 ,$$

while (8.7) remains valid for the bifocal waves (suffering slightly skew reflection at the surface). Together with the surface pressure balance $(a_1 + a_2 + a_3)j + n = 0$ this gives a new resonance condition, namely

(8.9) $$[3jn' + nj']_{KR} = 0 .$$

The roots, which are not far from those of (7.11), can also be characterized by the orders of neighbouring zeros of j_l and n_l. They will be designated by the symbol $\{1 l \nu_j \nu_n\}$ and identified with particles as well.

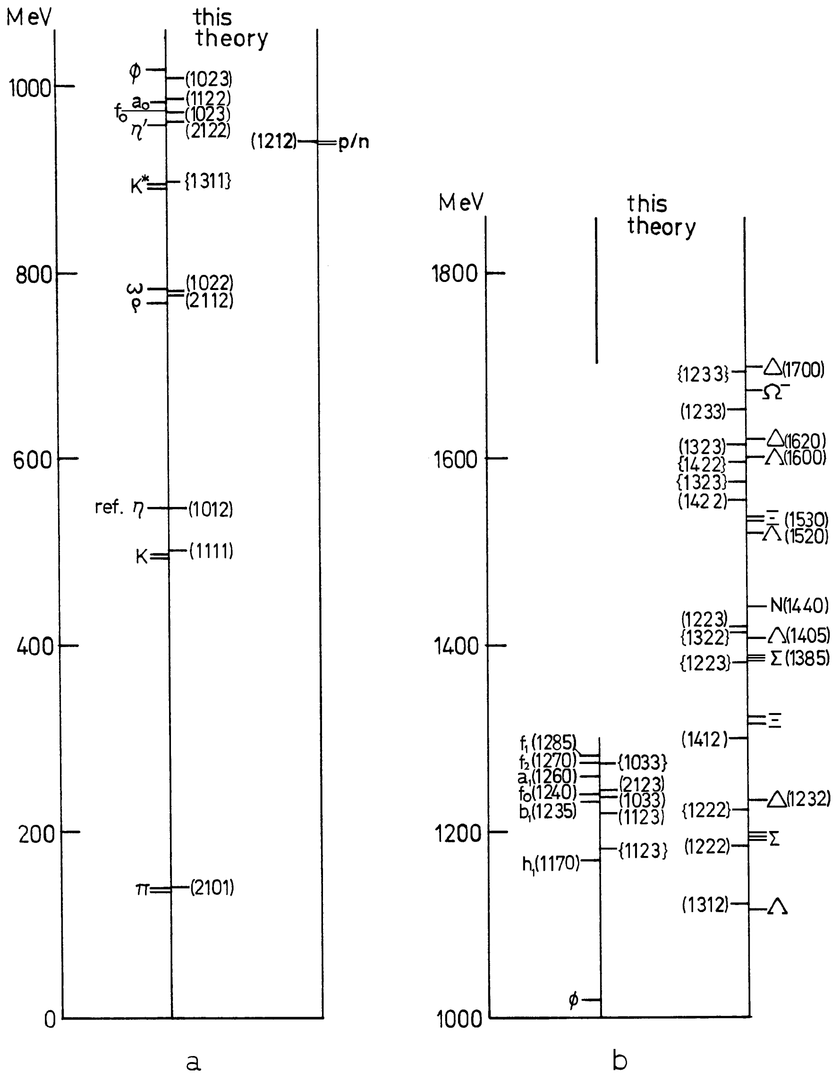

Fig. 1. – The hadron mass spectrum below 1 GeV (a) and moderately above (b), mesons to the left, baryons to the right. The middle columns show roots $(Nl\nu_j\nu_n)$ of eq. (7.11) and $\{1l\nu_j\nu_n\}$ of eq. (8.9), all multiplied with the same scale factor of 144.47 MeV which makes the (1012) root coincide with the η-meson of 547.45 MeV.

Tables II and III give lists of associations of actual mesons and baryons, respectively, with this model. The same information is shown in the form of line spectra in fig. 1. Compared with table I in sect. **7**, the lists have been enlarged by the inclusion of roots of eq. (8.9) and a few higher ones of (7.11), now all given in MeV rest energy, still with the η at 547.45 MeV as unique reference. The most convincing evidence still lies with the lightest (and most stable) particles. Above 1 GeV the

TABLE II. – *Mesons.* ($Nl\nu_j\nu_n$): root of $jn' + Nnj' = 0$; $\{1l\nu_j\nu_n\}$: root of $3jn' + nj' = 0$.

$Nl\nu_j\nu_n$	m_{th}/MeV	Particle	m_{ex}/MeV	$(m_{th} - m_{ex})/m_{ex}$
		strangeness 0		
(2101)	140.31	$\pi^{\pm}$	139.5679	+0.53%
		π^0	134.9743	+3.95%
(1012)	547.45	η	547.45	reference
(2112)	775.80	$\varrho(770)$	768.1	+1.00%
(1022)	779.22	$\omega(783)$	781.95	−0.35%
(2122)	961.99	$\eta'(958)$	957.75	+0.44%
{1023}	970.90	$f_0(975)$	974.1	−0.33%
(1122)	987.01	$a_0(980)$	982.7	+0.44%
(1023)	1008.57	$\phi(1020)$	1019.413	−1.06%
{1123}	1181.29	$h_1(1170)$	1170 ± 20	+0.96%
(1123)	1219.54	$b_1(1235)$	1232 ± 10	−1.01%
(1033)	1236.84	$f_0(1240)$	1240 ± 10	−0.25%
(2123)	1244.37	$a_1(1260)$	1260 ± 30	−1.24%
{1033}	1274.56	$f_2(1270)$	1275 ± 5	−0.03%
		$f_1(1285)$	1282 ± 5	−0.58%
		strangeness 1		
(1111)	502.51	$K^{\pm}$	493.646	+1.79%
		K^0	497.671	+0.97%
{1311}	899.48	$K^{*\pm}(892)$	891.59	+0.89%
		$K^{*0}(892)$	896.10	+0.38%
{1511}	1235.82	$K_1(1270)$	1270 ± 10	−2.7%

associations gradually become tentative as the spacing narrows and becomes comparable to the discrepancies (and finally to some of the experimental errors, not to mention the enormous widths of some resonances).

It gives reason for concern that several roots of (8.9) and a few of (7.11)—including modes that do possess radial nodes—have to be ignored for not corresponding to known particles. On the other hand, within the energy range covered by the meson table, every established particle or resonance has found a plausible allocation—mostly within 1% of mass—except that {1033} is shared by $f_2(1270)$ and $f_1(1285)$. The *Review of Particle Properties* classifies the $f_0(1240)$ meson as requiring experimental confirmation; it corresponds well with (1033). Note that among the unflavoured mesons listed, all monopole allocations correspond to isospin zero and most dipole ones to isospin one, with the notable exception of the $\eta'(958)$ and also of the very short-lived resonance $h_1(1170)$. Among the strange mesons listed, the association of $K_1(1270)$ with {1511}, not shown in fig. 1, is clearly a tentative one, this being the only decapole mode considered. It is true that any plausible allocation becomes impossible above the highest energy listed, thus above $l + \nu_j + \nu_n = 6$, there being many more observed resonances than (low-order) roots offered by eqs. (7.11) and (8.9).

The same is true for the baryons above $l + \nu_j + \nu_n = 7$, although almost complete (tentative) lists have been obtained separately for each strangeness, including $l + \nu_j +$

TABLE III. – *Baryons.* $(Nl\nu_j\nu_n)$: root of $jn' + nj' = 0$; $\{1l\nu_j\nu_n\}$: root of $3jn' + nj' = 0$.

$Nl\nu_j\nu_n$	$m_{\rm th}$/MeV	Particle	$m_{\rm ex}$/MeV	$(m_{\rm th} - m_{\rm ex})/m_{\rm ex}$
		strangeness 0: N and Δ		
(1212)	939.92	p	938.27231	+0.175%
		n	939.56563	+0.037%
{1222}	1222.43	Δ(1232)	1232 ± 2	−0.78%
(1223)	1419.97	N(1440)	1440^{+30}_{-10}	−1.4%
(1323)	1613.37	Δ(1620)	1620^{+55}_{-5}	−0.41%
{1233}	1692.36	Δ(1700)	1700^{+70}_{-30}	−0.45%
		strangeness −1: Λ		
(1312)	1121.93	Λ	1115.63	+0.56%
{1322}	1412.01	Λ(1405)	1407 ± 4	+0.36%
(1422)	1555.10	Λ(1520)	1519.5 ± 1	+2.34%
{1422}	1596.43	Λ(1600)	1600^{+100}_{-40}	−0.22%
		strangeness −1: Σ		
(1222)	1182.99	Σ^+	1189.37	−0.54%
		Σ^0	1192.55	−0.80%
		Σ^-	1197.43	−1.21%
{1223}	1381.01	$\Sigma^+(1385)$	1382.8	−0.13%
		$\Sigma^0(1385)$	1383.7	−0.19%
		$\Sigma^-(1385)$	1387.2	−0.45%
		strangeness −2: Ξ		
(1412)	1299.0	Ξ^0	1314.9 ± 0.6	−1.21%
		Ξ^-	1321.32 ± 0.13	−1.69%
{1323}	1573.66	$\Xi^0(1530)$	1531.8 ± 0.32	+2.73%
		$\Xi^-(1530)$	1535.0 ± 0.6	+2.52%
		strangeness −3: Ω		
(1233)	1653.76	Ω^-	1672.43	−1.12%

$\nu_n = 8$, which is also the allocation of the stable Ω^-. The one absence from completeness within its short list is a Δ(1600) resonance which carries three-star status in ref. [5].

The following argument is a first tentative step towards an analysis of higher orders of excentricity ε. The excentricity of the two foci by $\pm\mathbf{e}\varepsilon R$ creates a first-order pressure perturbation $\delta p(r, \theta, \varphi)$ expressed by

$$\delta p(r, \theta, \varphi) = \varepsilon R\left[\mathbf{e}\cdot\nabla\left(a_2 j_l(u)\, Y_l^m(\theta, \varphi)\right) - \mathbf{e}\cdot\nabla\left(\left(a_1 j_l(u) + Nb n_l(u)\right) Y_l^m(\theta, \varphi)\right)\right], \tag{8.10}$$

explicitly by

$$(8.11)\qquad \frac{\delta p}{\varepsilon KR} = \left[(a^- j_l' - Nbn_l')\, Y_l^m e_r + \frac{1}{u}(a^- j_l - Nbn_l)\left(\frac{\partial Y_l^m}{\partial \theta} e_\theta + \frac{\partial Y_l^m}{\sin\theta \partial\varphi} e_\varphi \right) \right]_{KR},$$

$$(8.11a)\qquad e_r = (e_x \cos\varphi + e_y \sin\varphi)\sin\theta + e_z\cos\theta\,,$$

$$(8.11b)\qquad e_\theta = (e_x \cos\varphi + e_y \sin\varphi)\cos\theta - e_z\sin\theta\,,$$

$$(8.11c)\qquad e_\varphi = (-\, e_x \sin\varphi + e_y \cos\varphi)\,,$$

where $a^- \equiv a_2 - a_1$. Since **e** has arbitrary direction, the mode can be oriented so as to make $Y_l^m = \cos m\varphi P_l^m$, where the last function is an associated Legendre polynomial. The eight angle-dependent terms on the right-hand side of (8.11) are all orthogonal to Y_l^m. This confirms the absence of a perturbation at the resonance orders (l, m) to first order in ε but indicates the excitation of neighbouring orders. Since the off-order modes excited by δp have excentric wave centres as well, their excitation produces, in turn, a reaction at the original surface-harmonic, but one that is proportional to ε^2.

Coupling of modes due to this effect of finite excentricity may be expected to become noticeable whenever adjacent frequencies are close to each other. This might be held responsible for part of the deviations of (7.11) and (8.9) from reality, possibly also for the proliferation of resonances at higher frequencies. The first conjecture is supported by the following observation. Plotting the roots of (7.11) and (8.9) as a line spectrum (fig. 1) shows that neighbouring orders are grouped in clusters of common $l + \nu_j + \nu_n$. The experimental spectrum, as associated, visibly follows the same pattern except that the mass values appear to repel each other within each group, so that the discrepancies—theory minus experiment—are positive at the lower end of a group (the Ξ with $l + \nu_j + \nu_n = 7$ being the only exception) and negative at the upper end.

Two serious problems remain and only hints of possible solutions can be suggested here. The first problem is that no model for the inhabitants' concept of 'spin'—oscillatory phase advance per geometrical rotation—has emerged so far, besides the suggestion that three-dimensional modes correspond to hadronic fermions and one- and two-dimensional ones to bosons. Secondly, inhabitants situated far enough from the centre of their universe to observe deviations from eq. (7.11) should also observe anistropy of their mass spectrum with respect to their axis **e**.

A common basis for the solution of both problems is created by assuming that all normal modes with respect to the axis **e** are always excited and that the observed rest frequency is an average. The beat between these geometrically normal oscillations will trace out a Lissajous pattern. Uncoupled dipole modes of slightly different frequency along and transverse to **e** cover all relative phases and, thus, give zero average 'spin'. Conservation of classical angular momentum (as seen from outside) might lock these modes with a permanent relative phase shift. This would give 'integer spin'. (In inhabitants' terms two geometrically orthogonal oscillations at $\pi/2$ relative oscillatory phase constitute 'unity spin'.) Note that any finite classical angular momentum which the excentric oscillations considered here may possess must be due to the motion in outer regions of the universe, the central region of $n'(r)\, Y(\theta, \varphi)$ carrying no linear momentum. Finite spin of monopole modes must be assumed due to their residual dipole component at finite excentricity.

The three normal modes of excentric sectorial oscillations will trace out a three-dimensional Lissajous pattern. This creates a prerequisite, at least, for the basic feature of half-integer spin of requiring two full rotations about an axis before the initial pattern is re-established. The model also suggests that such modes cannot be superimposed.

Two objections will be raised against this association of particle masses with roots of (7.11) and (8.9). The first one is that little or no order concerning other particle properties than mass and no manifestation of the quark model (beyond the association of mesons with $l=0$ and 1 and of baryons with $l \geqslant 2$) is perceptible. Clearly, there is no obvious correlation of these features with the spherical Bessel and Neumann functions invoked to calculate 'mass'.

It should be noted, however, that (7.11) and (8.9) are formulae for *mass*, not particle properties, and that the two are strikingly uncorrelated in reality as well. The model not only reflects this fact but also offers an explanation as follows.

Each resonant configuration considered here possesses a granular wave centre where potential and kinetic energy is concentrated, the pressure scaling with power $l+1$ and the velocity with power $l+2$ of inverse radius. Since 'particles' in this model are associated with propagating dislocations, it must be expected that 'particle' properties other than mass are *local* features, related to the granular wave centre. For 'electric charge' in terms of this model this is, indeed, the case (cf. sect. **9** and beyond). Moreover—and in close relation as will be shown—any dipole motion of the central molecular cluster controls the barycentre of the entire model—universe as dipole displacements (for $r \ll R$) scale with the cube of inverse radius.

By contrast, the 'mass' spectrum $(KR)_i$ has been derived from continuum dynamics *at the outer boundary of the model universe*, without any consideration of the granular wave centre. Continuation of $|s| \propto n'(Kr)$ to zero radius is implied, a configuration which does not entail any net transport of mass along an axis even for $l=1$.

The reason why this non-local treatment is correct (in first approximation) is that near the wave centre the *ratio* of wave pressure p to displacement $\mathbf{s}$—the local *spring constant* contributing to resonance—tends to zero although pressure and displacement separately tend to infinity (everything remaining finite, however, with finite granularity).

Therefore, in this model as in reality, 'mass' is apart from particle-like behaviour. It is a *non-local* property in being controlled by bulk dynamics at the border of the universe and only this dynamics is related to spherical Bessel and Neumann functions.

A second possible objection is the existence of gaps in the associations: the fact that not *all* roots of (7.11) *and* (8.9)—with $N=1$ *and* 2 where it applies—correspond to known particles. There are two kinds of such gaps. Firstly, modes which do not possess a radial node of pressure $(\nu_j = \nu_n = 1)$ fail *systematically* (with the exceptions discussed below) to correspond to real particles. The probable explanation is that the absence of a radial node prevents the balance of residual barycentre motion due to wave centre excentricity. If this is the explanation, then it cannot apply to odd-order zonal modes whose angular distribution $P_l(\cos\theta)$ is inherently self-balancing. This, then, is perfectly consistent with the observation that the modes (1111), {1311} and tentatively {1511} do have plausible associations.

In addition there are, within the energy range covered, nine gaps out of 36 modes which do have radial pressure nodes. Note, however, that for given order $(l\nu_j\nu_n)$ the configurations described by (8.9) and by the two variants of (7.11) are merely the result of a split of one basic mode: the angular distributions are identical, the radial ones have the same topology and the frequencies are close to each other too. One can hardly expect, therefore, that all variants of the same order $(l\nu_j \nu_n)$ are sufficiently stable to be observable in isolation. In fact, some experimental resonances in the energy range considered are very short-lived (with full widths Γ up to 300 MeV) too.

However, as far as fig. 1 reaches in energy, all orders $(l\nu_j \nu_n)$ correspond to at least one established particle-mass so that, in this wider sense, there are no gaps.

9. – Maxwell's equations and the nature of electric charge

The model universe proposed here is a standing-wave resonator for compressional waves where the irrotational term of eq. (7.2) gives rise to the spectrum of rest frequencies. The solenoidal term alone, with $\operatorname{div}\mathbf{s} = 0$ everywhere, designates dispersionless shear waves propagating freely with velocity $c = \sqrt{\mu/\varrho}$—clearly the equivalent of electromagnetic waves in vacuum. Indeed, the solenoidal part of (7.2) can be written in the form of the following pair of coupled equations:

$$\mu \nabla \times \mathbf{s} = -\dot{\mathbf{L}}, \tag{9.1}$$

$$\nabla \times \mathbf{L} = \varrho \dot{\mathbf{s}} \tag{9.2}$$

which are identical with Maxwell's in vacuum. The left-hand side of (9.1) gives (half) the shear-induced torque on a rotated volume element, identifying the auxiliary quantity **L** as an angular momentum per unit volume. Note that the pair (9.1)-(9.2) is 'Lorentz invariant' while (seen from outside) it expresses Newton's second law in an incompressible medium.

Historically, Maxwell's equations were preceded by just this model for the propagation of light. It changed, however, into complicated ether mechanisms [1] of unacceptable artificiality, brought about by various attempts at explaining coupling to the macroscopic "ponderable bodies" which were supposed to constitute our reality.

The ultimate reason for rejecting any Newtonian model of electricity—an invalid one as is claimed in sect. **3** and **4**—was special relativity. Another problem appeared to be the absence of partial conversion to longitudinal waves at skew incidence upon the interface between different media. In this model there is only one medium (the 'material' with which electromagnetic waves interact being formed by molecular wave centres therein) and the existence of travelling longitudinal waves is prohibited by the closed reflecting boundary. A third seemingly fundamental problem, the existence of electric charge [6], has also disappeared as the following argument shows.

The only plausible way of interpreting eqs. (9.1) and (9.2) as Maxwell's is to associate the mechanical displacement **s** with the 'electric displacement' **D** by putting

$$\mathbf{D} \equiv \varepsilon_0 \mathbf{E} \equiv q\mathbf{s}, \tag{9.3}$$

where q is a scale factor. The consequence is inescapable then that the 'electric charge density' ϱ_e is associated with excess or deficit of ether volume, since the identity of $\varrho_e = \mathrm{div}\,\mathbf{D}$ implies

$$\varrho_e = q\nabla\cdot\mathbf{s}\,. \tag{9.4}$$

This ether (like its original 19th-century predecessor) is incompressible over any distance much below the size of the universe. However, the concept of molecular dislocations, already introduced in the context of the mass spectrum, offers a most natural explanation for compressionless volume divergence, manifesting itself via the finite rigidity μ alone. The model is in agreement with reality: the electric charge is a particle property which does not occur without associated 'mass' and it is quantized—into $e = qV_0$ in this model if V_0 is the molecular volume, counted negative for a hole. Our deeply rooted concept of indestructible material is conserved but refers to electric charge rather than 'mass'—in agreement also with experimental reality.

Macroscopically, then, the 'electric charge density' is formed by the number density ν_0 of excess or deficit dislocations, that is

$$\varrho_e = q\langle\mathrm{div}\,\mathbf{s}\rangle = q\nu_0 V_0 \tag{9.5}$$

and the (area) density $\mathbf{J}$ of 'conduction current' must be taken as

$$\mathbf{J} = q\nu_0 V_0 \mathbf{v}_g \tag{9.6}$$

since it is at the group velocity $\mathbf{v}_g = \nabla_k \omega$ that the dislocations propagate. Continuity in the absence of compression requires that the diffusion velocity $\mathbf{v}_g$ of a total dislocation volume $\nu_0 V_0$ per reference volume be accompanied by an average material velocity $\langle\dot{\mathbf{s}}_\mathbf{J}\rangle$ in that volume. Therefore $\langle\dot{\mathbf{s}}_\mathbf{J}\rangle = \nu_0 V_0 \mathbf{v}_g$ and the conduction current $\mathbf{J}$ is accompanied by an internal displacement velocity

$$\langle\dot{\mathbf{s}}_J\rangle = \frac{\mathbf{J}}{q}\,. \tag{9.7}$$

Adding this to the right-hand side of eq. (9.2), multiplying everything with the scale factor q and defining the 'magnetic field' by $\mathbf{H}/q = \mathbf{L}/\varrho$—the mass density of angular momentum—yields the complete equations

$$c^2\nabla\times\mathbf{D} = -\dot{\mathbf{H}}\,, \tag{9.8}$$

$$\nabla\times\mathbf{H} = \dot{\mathbf{D}} + \mathbf{J}\,. \tag{9.9}$$

The auxiliary status of the axial vector $\mathbf{H}$ and the concomitant asymmetry between the two equations reflect reality; there are no magnetic monopoles.

Moreover, since the shear stress in an incompressible solid must break down at some maximum shear angle ϑ (the deviation from 90° in an infinitesimal fiducial square with a radial diagonal), every central volume perturbation V_0 will be surrounded by a stress-free spherical volume whose radius r_d is given by

$$r_d^3 = \frac{3V_0}{4\pi\vartheta} = \frac{r_0^3}{\vartheta} = -\frac{2r_0^3}{3\varepsilon_d} > 0\,, \tag{9.10}$$

where r_0 (of either sign) is the molecular radius and $\varepsilon_d = [\partial s_r/\partial_r]_d$ is the radial disruption strain (which differs from ϑ since there is shear in two orthogonal directions).

If, as is plausible, the disruption angle ϑ is small, the disruption radius r_d is larger than the molecular radius r_0. An association of r_d with (half the) 'classical electron radius' $r_\mathrm{e}/2$—a cut-off radius for the accumulation of electrostatic 'energy'—suggests itself here and will be consolidated in the next two sections in which the disruption phenomenon plays a central role. It seems worth stressing, therefore, that it is not an additional *ad hoc* hypothesis but the inevitable consequence of the introduction of granularity and of dislocations in the medium, requiring the formulation of two additional material constants—r_0 and ϑ.

It will be noted that 'electricity' in this model represents the residues of material motion—microscopic displacement from rest position and macroscopic migration of molecular defects.

The spatial localization of isolated photons and γ-particles—one of the problems sidestepped by orthodox physics—asks for an explanation. Clearly, here, a photon is a solenoidal wave packet resulting from localized generation. In the direction of propagation it does not diffuse as it suffers no interaction with the boundary of the universe and, hence, no dispersion. Transverse localization must be due to the presence of a discrete molecular perturbation. A suitable plane wave consists of a *cylindrical* dipole Neumann function whose axis is formed by displaced molecules strung along the direction, z, of propagation and an amplitude spectrum $\exp[i\omega(t-z/c)]$.

10. – Disruption modes—the leptons

The primary feature of the compressional resonances associated with hadrons (sect. **6**, **7** and 8) is the n-mode (spherical Neumann function) component. Physical reality requires that the divergence of $n_l'(r)$ be interrupted by a discontinuity of the medium near a wave centre and it was assumed that this takes place at a radius comparable to a molecular radius r_0. A self-adjusting mechanism of central intermolecular forces, responding to large displacements, has to be invoked to explain the balancing of dynamic pressure near r_0. This model is made tenable by the fact that the ratio $\omega^2\varrho r_0/(l+1)$ of dynamic n-mode pressure to concomitant radial displacement decreases linearly with r_0.

By contrast, what is considered in this section are dipole modes in which a transition from n-type to singularity-free j-type motion takes place at the *disruption boundary* of radius r_d, necessarily formed in the presence of a permanent volume dislocation. Since the small-argument j_1-mode is identical with rigid-body motion (along a vertical z-axis, say), these modes are completely devoid of intermolecular distortion inside the radius r_d.

Near r_d the spherical Bessel functions (6.2) and (6.3) are given by the asymptotic terms of their power series expansions [7]

$$j_l(u) = \frac{u^l}{(2l+1)!!}\left(1-\frac{1}{2}u^2\,\frac{1}{1!(2l+3)}+\left(\frac{1}{2}u^2\right)^2\frac{1}{2!(2l+3)(2l+5)}\,\cdots\right), \tag{10.1}$$

$$n_l(u) = -\frac{(2l-1)!!}{u^{l+1}}\left(1-\frac{1}{2}u^2\,\frac{1}{1!(1-2l)}+\left(\frac{1}{2}u^2\right)^2\frac{1}{2!(1-2l)(3-2l)}\,\cdots\right). \tag{10.2}$$

Substitution into (7.4) and (7.5) gives the dynamic dipole pressure amplitude just inside and outside the disruption boundary as

$$p_{1\mathrm{d}}^{j} = \omega^2 \varrho r_\mathrm{d} \hat{s}_z \cos\theta \,, \tag{10.3}$$

$$p_{1\mathrm{d}}^{n} = -\frac{1}{2} \omega^2 \varrho r_\mathrm{d} \hat{s}_z \cos\theta \,, \tag{10.4}$$

respectively, if $\hat{s}_z$ is the common peak north-pole $(r_\mathrm{d},\, 0)$ displacement at $r = r_\mathrm{d}$. The two dynamic pressures do not balance but add to a surface density of free radially outward force given by

$$f_\mathrm{d} = \frac{3}{2} \omega^2 \varrho r_\mathrm{d} \hat{s}_z \cos\theta \,. \tag{10.5}$$

It equals the inertial surface pressure of a sphere of effective mass

$$m_\mathrm{eff} = 2\pi \varrho r_\mathrm{d}^3 \tag{10.6}$$

oscillating bodily with frequency ω and amplitude $\hat{s}_z$ along the z-axis, one third of this being due to the n-mode motion outside r_d.

The first case to be considered is the (2101) or '*pion*' mode, modified in such a way as to account for an n-j transition at radius r_d. The effect of this transition must be expected to be a loading of the (2101) resonator by the effective mass (10.6) of the disrupted sphere and, therefore, a decrease of frequency. A tentative association with the muon suggests itself at once. It reflects the close relation between pions and muons—charged pions decaying almost exclusively to muons at about 25% loss of 'mass'—as well as the fundamental difference of presence *vs.* absence of 'strong interaction'—represented here by the presence *vs.* absence of near divergent intermolecular distortion and concomitant stress at a molecular wave centre.

In calculating the resonance of this loaded (2101) mode—which will be called $(2101)^\mathrm{d}$—two assumptions are made. The first one is that, for $r > r_\mathrm{d}$, the mode is formed by simple addition of the dipole n-mode pressure created by the oscillating sphere of radius r_d to the unperturbed (2101) pressure field defined by this mode's j_1-to-n_1 ratio a/b. The second assumption is that the resulting frequency remains far above the resonance formed by the interaction of rigidity with the effective mass (10.6).

Since the dynamic pressures (10.3) and (10.4) do not balance out, the second assumption implies that the vibration of the disrupted sphere must create an n-mode pressure field which very nearly cancels the unperturbed elastic n-mode pressure. The finite rigidity (which will be held responsible for the very low-frequency resonance discussed below) guarantees the presence of a small residual n-mode at $r > r_\mathrm{d}$, enough to define a 'particle'. However, the near cancellation of the unperturbed n-mode with that created by the n-j transition at r_d makes the elastic n-mode pressure nearly zero everywhere. What is left, as seen from inside the moving system, is the apparent inertial surface pressure $bn_1(KR)\cos\theta$ implied by $N = 2$ and this must cancel with the j-mode, as stated by (7.6). In other words, the resonant value $(KR)_{2101}^\mathrm{d}$ coincides with the node of elastic pressure—given by $aj + bn = 0$—of the unperturbed (2101) mode.

The (2101) pressure node is easily found by inserting $(KR)_{2101} = 0.973508$ (the lowest finite root of (7.11) with $N = 2,\, l = 1$) into (7.7) so as to obtain $a/b = 9.789344$ and then solve (7.6) with this value of a/b. The result of this entirely analytical manipulation, devoid of any free parameter, is

$$\frac{(KR)_{2101}}{(KR)_{2101}^\mathrm{d}} = 1.321267 \,.$$

The experimental mass ratio of the charged pion to the muon,

$$1.320931\,,$$

is lower by only 2.5×10^{-4} relative error!

In the absence of rigidity the next lower mode would have zero frequency. This (2100) mode is indeed unique in making the left-hand side of (7.11)—namely $-3(KR)^3/10$ for $KR\ll1$—vanish to third order in KR. A small but finite rigidity must be expected to give this mode, which will be called $(2100)^{\mathrm{d}}$, a finite frequency. An identification of this low and isolated resonance with the *electron* is almost inescapable at this point. In the terms of this model it forms a unique 'light lepton' whose 'mass' is entirely 'electric' in being independent of κ.

At first sight it might appear that the frequency in question is essentially equal to c/r_{d}. The analysis presented in appendix B shows that this would indeed be the case, with a numerical factor $\sqrt{6}$ if the boundary of rigid-body motion remained fixed in the oscillating medium. Such behaviour would, however, be at variance with the assumption that the boundary is created by disruption. As, by definition, the normal force at the disruption boundary cannot exceed the disruption stress, any finite momentum acquired by the mass of the sphere will make it break through the surrounding medium *almost* freely. It does not matter in this context that the disruption boundary propagates through the medium with $1/\vartheta$ times the material velocity (cf. the *caterpillar* motion discussed in appendix B) so that an individual molecule cannot follow free ballistic motion for more than a very short time. Every volume element overtaken by the boundary transmits its axial momentum forward via central elastic collisions between neighbouring molecules, the propagation—at compressional wave velocity—being essentially instantaneous. In the absence of interaction with the boundary of the universe this mechanism keeps the *j-n* boundary (the boundary of rigid-body motion) concentric with the disruption and there is no net force. This, in fact, is the case for macroscopic transport of 'electric charge' which cannot have any net interaction with the boundary of the universe.

For isolated oscillatory motion a restoring force proportional to $|\vartheta|$ does result from the impossibility of net transport of excess or deficit material *within the finite volume of the universe.* If $KR\ll1$, as was the basic assumption for the mode under discussion, the ether must be considered incompressible all the way out to the boundary radius R. Since, therefore, a translation of the monopole field associated with an isolated 'charge' would displace the entire universe by an amount inversely proportional to its mass ϱR^3 (cf. appendix B), the disruption cannot follow the oscillating mass at the centre of the (2100) mode in question. As a consequence of the resulting excentricity, the finite dipole force given by (B.19) acts on the effective mass (10.6) although there is no such force at the disruption boundary. The result is the resonance frequency given by (B.20), namely

$$\omega^{\mathrm{d}}_{2100}=2\,|\vartheta|^{1/2}\,\frac{c}{r_{\mathrm{d}}}\,. \tag{10.7}$$

Note that this 'mass', like all others, is non-local since it is due to the interaction of the wave centre with the boundary of the model universe although the value of $\omega^{\mathrm{d}}_{2100}$ does not depend on KR to the approximation in which it is treated here. Reversibility of disruption is assumed, here as well as in sect. **11**. The tentative model of rigidity proposed at the end of appendix B does have this feature.

If r_d (the cut-off radius of ether rigidity) is associated with half the classical electron radius $r_e/2$ (a cut-off for 'electric energy') the association of ω_{2100}^{d} with the electron makes $|\vartheta|^{1/2}$ equal to one quarter of the fine-structure constant $\alpha = 1/137.04$. Additional support for this association will be forthcoming in sect. **11**. It follows that $|\vartheta| = 3.328$ μrad $r_0 = 2.104 \times 10^{-17}$ m and $q = 4.109 \times 10^{30}$ As/m^3. Such relatively coarse granularity may be considered uncomfortable. Note again, however, that r_0 does not form a cut-off wavelength, since resonance and coupling are due to the participation of a large number of molecules. The physical ether displacement s with respect to the boundary of the universe amounts to 2.15×10^{-33} m for an electric field of 10^9 V/m and to 1.56×10^{-21} m at half the classical radius.

In this model, single electron creation from baryon decay (such as $n \rightarrow pe^- \bar{\nu}_e$) corresponds to the creation of a zonal mode from a sectorial one and of concomitant axial momentum. A similar problem of momentum conservation also occurs in the $(2101) \rightarrow (2101)^d$ conversion (modelling $\pi^\pm \rightarrow \mu^\pm \nu_\mu$) since the latter mode carries axial momentum at its centre and the former does not. It would appear, therefore, that both types of conversion require the simultaneous creation of a companion dipole mode of opposite displacement, a mode which must possess an n_1-j_1 transition at radius r_d although it contains no permanent dislocation.

The necessity of these classical companion modes clearly finds its counterpart in reality in the necessity of *neutrino* production as a byproduct of single charged lepton creation. The self-balancing frozen dipole stress calculated in appendix B suggests itself as a model. This will develop if an ether sphere of radius r_d is disrupted and displaced, with attendant development of dipole stress at $r > r_d$, then fused again with its sourroundings. The boundary can propagate the n_1-j_1 transition by local curl bridging. As this entails no irrotational elastic force, zero rest frequency results.

In summary, then, the modes analysed in this section represent *leptons* by virtue of the absence of any distortion and associated elastic stress within the disruption radius r_d which is half the inhabitants' 'classical electron radius'. The 'charge' is represented by an excess or deficit molecule but its place within r_d is undefined and the corresponding expansion or contraction of volume uniformly distributed within a sphere of liquid. Neutrinos also have a bodily oscillating core but contain no excess or deficit volume, zero rest frequency being a necessary consequence.

The point-like nature of leptons corresponds to the geometrical centre of the structureless disruption sphere. The 'strong force', by implication, must be related to the nearly divergent intermolecular distortion associated with n-mode penetration down to molecular size.

What remains unexplained is the absence of higher-mode versions of the muon and the difference between different varieties of neutrinos. Also unexplained is the τ-lepton with its mass above that of the stable hadrons (and above the range of quantitative success of the hadron model of sect. **7** and 8).

The association of leptons with dipole oscillations seems to be inconsistent with the result of sect. **7** and **8** where hadronic fermions were associated with three-dimensional modes. It is, however, a characteristic feature of the modes considered here that the bodily moving disruption sphere carries local classical momentum and, hence, angular momentum $2\pi\varrho r_d^2 \varepsilon R\mathbf{e} \times \dot{\mathbf{s}}$ at the excentric position $\varepsilon R\mathbf{e}$. The effect is independent of r_d since $|\mathbf{s}| \propto r_d^{-3}$. Since all modes of this section are of $N = 2$ type, they are subject to inertial coupling to a partner via the surface of the universe (non-locality!). The partner might be the proton 'charge' in a hydrogen atom; it will be the partner-in-creation as long as a newly formed lepton pair remains undisturbed. The coupling of two discrete, excentric and non-coincident oscillatory masses does appear to contain the necessary element of three-dimensionality. This is not true, by contrast, of the zonal modes associated with hadrons—not even for $N = 2$ dipole modes—as an n-mode reaching all the way to its centre contains no localized momentum there. The statement includes 'charged' modes since the excess or deficit molecule is not supposed to be localized within the disruption sphere it creates.

11. – The Coulomb force

An explanation of the electrostatic 'force' in terms of ether rigidity will cover one of the four 'forces' of nature; it will also form the keystone for the model of classical electricity presented in sect. **9**.

For convenience, the words positive or negative charge (without quotation marks) will be used for one excess or deficit molecule, respectively, without commitment as to the polarity of the corresponding electric 'charge'. Two charges at distance z_{12} from each other define a vertical z-axis, the upper one being called test charge, the lower one source charge, notwithstanding the obvious symmetry of the situation.

It will be sufficient to show that the ether perturbation produced by the source charge acts on the test charge in such a way as to impart on any wave centred therein a rate of change of wave number given by

$$\frac{\mathrm{d}k}{\mathrm{d}t} = \pm \alpha \frac{c}{z_{12}^2}, \tag{11.1}$$

the upper sign (repulsion) being valid for equal signs of the two charges. It will be sufficient, moreover, to demonstrate this in the rest frame of the test charge.

It might be thought that the model attempted to lend (imagined) reality to the concept known as Maxwell stress tensor. This concept is a phenomenological one, however, already concerned with the inhabitants' 'force' $\hbar\partial\omega/\partial z$. In fact, the Faraday-Maxwell concept of 'electric stress' supposed to exist even in the absence of a test 'charge' and in unbounded 'vacuum' does not find a direct counterpart in this model's ether stress. The monopole stress created by an isolated volume perturbation decreases with the inverse third power of distance—its gradient with inverse fourth—and cannot be invoked. And the mutual body force created by direct interaction of two monopole stresses in an unbounded medium is zero.

The Coulomb force must be associated, therefore, with the *displacement* of the medium surrounding the test charge with respect to the closed boundary of the universe, representing 'absolute space'. Since the total volume of this universe is statically incompressible, such an effect is bound to exist and it is the one already invoked to explain the electron rest frequency. In what follows, it will be assumed at first that the test 'charge' is in fact an electron, the dislodged molecule forming a $(2100)^{\mathrm{d}}$ mode.

The unperturbed ether displacement due to the source charge amounts to

$$s_{z0} = \vartheta_1 \frac{r_{\mathrm{d}}^3}{3z_{12}^2} \tag{11.2}$$

with the sign of ϑ_1.

The test charge cannot follow the displacement (11.2) directly. This results in an n-mode strain of (north-pole) amplitude

$$\hat{s}_z = -\vartheta_1 \vartheta_2 \frac{r_\mathrm{d}^3}{3 z_{12}^2} \tag{11.3}$$

at the disruption radius—as if the sphere were displaced by $-\vartheta_2 s_{z0}$. Substitution of $\hat{s}_z$ into eq. (B.17) gives the total force

$$F_z = 4\pi\mu\vartheta_1\vartheta_2 \frac{r_\mathrm{d}^4}{z_{12}^2}\ , \tag{11.4}$$

upward (repulsive) for equal signs of ϑ_1 and ϑ_2. The mechanism is similar to that generating the $(2100)^\mathrm{d}$-mode ('electron') restoring force: it creates a net force F_z at constant disruption stress.

The force F_z does not relax the strain directly but sets in motion the *caterpillar migration* discussed in appendix B and sect. **10**. Now, however, the effective mass is only that of the volume perturbation V_0, hence only ϑ_2 times m_eff of (10.6), so that the material inside the disruption sphere suffers an acceleration in z-direction of

$$\dot{u} = 2\vartheta_1 c^2 \frac{r_\mathrm{d}}{z_{12}^2}\ . \tag{11.5}$$

As is shown in appendix B (cf. the arguments leading to (B.18)) the propagation of charge in the bounded incompressible medium requires that the disruption limit propagates at $v = u/\vartheta$. This gives

$$\dot{v} = 2\frac{\vartheta_1}{\vartheta_2} c^2 \frac{r_\mathrm{d}}{z_{12}^2} = \pm 2c^2 \frac{r_\mathrm{d}}{z_{12}^2}\ , \tag{11.6}$$

the upper sign—repulsion—being valid for equal signs of the two charges.

The free propagation of the $(2100)^\mathrm{d}$ mode, as of all others, is due to the solenoidal (double curl) term of eq. (7.2). Since the rigidity μ is zero inside the disruption boundary, the only way its contents can follow the group motion is by means of the divergence-free migration velocity $v = u/\vartheta$ which involves classical momentum, transmitted by elastic intermolecular collisions. Since, therefore, v is identical with the group velocity $c^2 k/\omega$ of the electron, (11.6) is identical with

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{k}{\omega}\right) = \frac{1}{\omega\gamma^2}\frac{\mathrm{d}k}{\mathrm{d}t} = \pm 2\frac{r_\mathrm{d}}{z_{12}^2}\ . \tag{11.7}$$

For the inhabitant observer in the electron rest frame, for whom the Lorentz transformations of time and distance are valid and ω appears as the rest frequency ω_0, (11.7) reads

$$\frac{\mathrm{d}k'}{\mathrm{d}t'} = \pm 2\frac{\omega_0 r_\mathrm{d}}{z_{12}^2}\ . \tag{11.8}$$

Dropping the dashes and substituting (10.7) for ω_0 yields, indeed, Coulomb's law in the form of

(11.9) $$\frac{\mathrm{d}k}{\mathrm{d}t} = \pm 4\,|\vartheta|^{1/2}\,\frac{c}{z_{12}^2}\ .$$

The same result is obtained if the inhabitants' view is taken directly and their law for the addition of colinear relativistic velocities is employed for adding velocity increments $\dot{v}\,\mathrm{d}t$ to v_g.

Equation (3.7) identifies (11.9)—obtained from group acceleration—with a refractive frequency gradient given by

(11.10) $$-\frac{\partial\omega}{\partial z} = \pm 4\,|\vartheta|^{1/2}\,\frac{c}{z_{12}^2}\ .$$

It is instructive to derive this refraction directly, by means of the following argument [8].

In an ether frame, the material velocity u at the disruption centre is zero, but there is a finite acceleration $\partial u_z/\partial t = \dot{u}$. In a group rest frame (still seen by the outside observer) a central flow velocity $\mathbf{v}_\mathrm{f} = -\mathbf{v}_\mathrm{g}$ and a flow acceleration $\mathrm{d}\mathbf{v}_\mathrm{f}/\mathrm{d}t = \dot{\mathbf{u}} - \dot{\mathbf{v}}$ are observed. Continuity in this frame requires the inclusion of a convection term $(\mathbf{v}_\mathrm{f}\cdot\mathrm{grad})\,\mathbf{v}_\mathrm{f}$ so that (in z-direction)

(11.11) $$\dot{u} - \dot{v} = \frac{\mathrm{d}v_\mathrm{fz}}{\mathrm{d}t} = \frac{\partial v_\mathrm{fz}}{\partial t} + \mathbf{v}_\mathrm{f}\cdot\nabla v_\mathrm{fz} = \dot{u} - \mathbf{v}_\mathrm{g}\cdot\nabla v_\mathrm{fz}$$

and thus, with $\mathbf{v}_\mathrm{g} = c^2\mathbf{k}/\omega$ and $\dot{v}$ from (11.6):

(11.12) $$\mathbf{k}\cdot\nabla v_\mathrm{fz} = \frac{\omega}{c^2}\,\dot{v} = \pm 2 r_\mathrm{d}\,\frac{\omega}{z_{12}^2}\ .$$

The product of a wave number and a velocity gradient produces a negative Doppler gradient of local frequencies $\omega(z)$ (whose Doppler-shifted transformations in the group frame all equal the carrier frequency ω), thus

(11.13) $$-\frac{\partial\omega(z)}{\partial z} = \pm 2 r_\mathrm{d}\,\frac{\omega}{z_{12}^2}\ .$$

Since there cannot be a discontinuity of phase in the medium, (11.13) must be identical with $-\partial\omega/\partial z$ in the neighbourhood of the wave centre, so that (3.7) gives the associated 'force' $\mathrm{d}k/\mathrm{d}t$. So far, this is the outsider's point of view; but since it is already taken in an 'electron' rest frame the only change for the inhabitants is their interpretation of ω at the right-hand side of (11.13) as their electron rest frequency ω_0, to be identified with $\omega_{2100}^{\mathrm{d}}$ of (10.7). Again, Coulomb's law is the result if r_d is associated with half the classical radius.

The argument exposes a Doppler gradient as the translator from the outside observer's classical dynamics to the wave kinematics which form the inhabitants' 'dynamics'. The factor ω in the numerator of (11.13) is intriguing—a force proportional to test-*mass* as in gravity. It is, however, cancelled by the factor r_d.

The result (11.9) establishes mutual agreement among the three claims made in connection with disruption:

– that the disruption radius is the cut-off radius $r_e/2$;

– that the $(2100)^d$ mode is the electron;

– that the square root of the disruption shear angle $|\vartheta|$ is one quarter of the fine-structure constant α.

An extension of this 'Coulomb force' to an arbitrary *host* mode carrying a permanent molecular dislocation at its centre may be based on the observation that the caterpillar migration appears to form the only possible way of charge transport. The corresponding $(2100)^d$ motion cannot be freely propagating but must be locked to the host mode, whatever its configuration. A mechanism for this is given by the fact that the disruption limit forms a common boundary for finite-rigidity (solenoidal) propagation of both modes. If the host-mode wave centre shifts with respect to the caterpillar momentum centre, a force analogous to (B.19) appears and imparts a rate of change of momentum to the medium inside r_d. The mechanism does have the characteristic of a phase-lock system, a wave number correction (dk/dt) produced by a phase error ($k\delta z$).

In the presence of a source charge at distance z_{12}, the force (11.4) adds the acceleration $\dot{v}$ of (11.6) to the $(2100)^d$-propagation. The resulting acceleration of disruption is not equal to $\dot{v}$, however, but reduced by the synchronizing force in such a way as to equalize the rates of change of wave number of the two locked modes. This implies that

$$\omega^{d}_{2100}\dot{v} = \omega_x \dot{v}_{gx}\,, \tag{11.14}$$

where the index x is given to the host mode. It follows from this and either one of the derivations of eqs. (11.9) and (11.10) that they remain valid for any charge-carrying mode.

If an ether molecule could be spontaneously created, the associated monopole displacement would spread with compressional velocity, thus 'instantly', just as a radial electric field would emerge instantly from the fictitious creation of an isolated electric charge. It is a fundamental feature of this model as well as of reality that this is impossible. After pair creation, a molecular dislocation can only migrate with $v_g < c$. This dipole motion does not create compression and the attendant displacement field can only spread with the shear velocity c, creating the familiar 'retardation'.

Whether actual Coulomb scattering at impact parameters comparable with or smaller than r_d is properly replicated remains to be studied and may become a test for the range of validity of this simple model. At least the absence of a hard core is properly represented.

It must be admitted that the analysis of disruption given here requires consolidation even within the realm of classical mechanics, since, increasingly, as the arguments advanced, the derivations relied on intuition and plausibility for want of rigour. Perhaps more seriously, it may be objected that continuum concepts are being employed throughout although the disruption sphere contains a finite number of molecules. While this appears justified for analysing average behaviour, the coarse granularity of the propagating medium might be taken as a suggestion that the model—at least in its naive simplicity—has been stretched to its limits and cannot be expected to render finer details or non-hadronic (non-compressional) phenomena substantially above 10 GeV, where $\omega \gg c/r_0$. On the other hand, the coarseness of the medium, permitting continuum analysis for averages but not for individual events, reflects the 'indeterminacy' of reality.

An ether model which replicates the three bulk constants μ/ϱ, κ/ϱ, ϑ in a natural way may be constructed from hard frictionless spheres which are held in nearly close packing by a hydrostatic pressure but prevented from self-locking by the presence of dislocations. This construction will be briefly sketched at the end of appendix B. It should be considered a mere proof of existence.

12. – The Schrödinger equation

This model universe consists of waves which propagate in a medium possessing a very large ratio of bulk modulus κ to rigidity μ, a simple molecular structure and a large spherical boundary. It has been shown that in undisturbed regions near the centre of this universe vibrational displacements $\mathbf{s}(r, t)$ from rest position are governed by the differential equation (6.1), namely ([3])

(12.1) $$\frac{\partial^2}{\partial t^2}\mathbf{s}(\mathbf{r}, t) = c^2\nabla^2\mathbf{s}(\mathbf{r}, t) - \omega_0^2\mathbf{s}(\mathbf{r}, t),$$

in agreement with the 'free space' dispersion relation (3.1), namely

(12.2) $$\omega^2 - \omega_0^2 = k^2c^2\,.$$

Here $\mathbf{r} = x_i$, $k = |\mathbf{k}|$ and ω_0 is one of the rest frequencies or 'masses' of which representative examples have been calculated in sect. **7**, **8** and **10**.

The model supports free wave packets which form around molecular dislocations migrating with group velocity $\mathbf{v}_\mathrm{g} = \mathrm{grad}_k\,\omega$. The central Fourier component is given by

(12.3) $$\mathbf{s}(\mathbf{r}, t) = \mathbf{s}_0(\mathbf{r})\,e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}\,.$$

In non-relativistic approximation, that is to first order in $(\omega - \omega_0)/\omega_0 \ll 1$, the free-wave dispersion takes the form

(12.4) $$\omega_\Delta \equiv (\omega - \omega_0) = \frac{k^2c^2}{2\omega_0}$$

and this expression designates the inhabitants' 'kinetic energy over $\hbar$ of a free particle'. With this dispersion (corresponding to a linearized group velocity $v_\mathrm{g} = kc^2/\omega_0$) the wave (12.3) satisfies the differential equation

(12.5) $$-i\frac{\partial}{\partial t}\mathbf{s}(\mathbf{r}, t) = \frac{c^2}{2\omega_0}\nabla^2\mathbf{s}(\mathbf{r}, t) - \omega_0\mathbf{s}(\mathbf{r}, t)$$

containing only the first derivative with respect to time.

([3]) In this section the convention of using dots for material-motion time derivatives will be dropped in favour of $\partial/\partial t$. Also, the phase rotation for positive frequency will be taken as clockwise (thus i replaced by $-i$ with respect to all other sections) so as to conform to general usage in the context of quantum mechanics.

The proximity of another wave centre can be expected to perturb the medium in such a way as to create *refraction* for nearby waves and this may be expressed in terms of a local perturbation $\delta\omega_0(\mathbf{r}, t)$ of the rest frequency. The simplicity of the model offers only few possibilities for the formation of a refractive gradient $\partial\omega_0/\partial x_i$, but the 'forces' of our real world are in fact limited to just four also, one of which is explained in terms of this model in sect. 11.

Adding a refractive frequency shift $\delta\omega_0$ (the result of integrating (11.10) from infinity to $\mathbf{r}$ for the Coulomb 'potential') to ω_0 changes (12.5) to

$$-i\frac{\partial}{\partial t}\mathbf{s}(\mathbf{r}, t) = \frac{c^2}{2\omega_0}\nabla^2\mathbf{s}(\mathbf{r}, t) - (\omega_0 + \delta\omega_0(\mathbf{r}, t))\,\mathbf{s}(\mathbf{r}, t). \tag{12.6}$$

The simple addition of $\delta\omega_0$ is justified provided the gradients and rates of change of $\delta\omega_0(\mathbf{r}, t)$ are adiabatic, that is the fractional change per radian of oscillation remains small.

Taking the divergence on both sides of (12.6), multiplying with $-\kappa$ and remembering that $-\kappa\,\mathrm{div}\,\mathbf{s}$ is the pressure p, one obtains

$$-i\frac{\partial}{\partial t}p(\mathbf{r}, t) = \frac{c^2}{2\omega_0}\nabla^2 p(\mathbf{r}, t) - (\omega_0 + \delta\omega_0(\mathbf{r}, t))\,p(\mathbf{r}, t). \tag{12.7}$$

It may be noted that taking the divergence of (12.1) results in Schrödinger's relativistic (or Klein-Gordon) equation. In either case, the change from the vector field $\mathbf{s}$ to the scalar p discards all information about the solenoidal part of the motion, contained in the double curl component of the vector Laplacian. Equation (12.7) cannot be expected, therefore, to represent intrinsic angular-momentum terms like the 'magnetic field'.

The condition of adiabatic variation imposed on $\delta\omega_0(\mathbf{r}, t)$ permits factoring out the steady-state time dependence by putting $p(\mathbf{r}, t) = \widehat{p}(\mathbf{r}, t)\exp[-i\omega_0 t]$ making $\widehat{p}(\mathbf{r}, t)$ the distribution in space and time of a complex pressure *amplitude*. With this, eq. (12.7) takes the form of

$$-i\frac{\partial}{\partial t}p(\mathbf{r}, t) = \frac{c^2}{2\omega_0}\nabla^2\widehat{p}(\mathbf{r}, t) - \delta\omega_0(\mathbf{r}, t)\,\widehat{p}(\mathbf{r}, t) \tag{12.8}$$

or of

$$-\nabla^2\widehat{p}_s(\mathbf{r}) = 2\,\frac{\omega_0}{c^2}(\omega_\Delta - \delta\omega_0(\mathbf{r}))\,\widehat{p}_s(\mathbf{r}) \tag{12.9}$$

if $\delta\omega_0$ does not depend on time at all so that $p(\mathbf{r}, t) = \widehat{p}_s(\mathbf{r})\exp[-i\omega t]$.

Clearly, (12.8) and (12.9) are the time-dependent and stationary versions, respectively, of the Schrödinger equation if the amplitude $\widehat{p}$ of ether pressure is taken as the wave function, an association which is in fact very plausible. As $|\widehat{p}|^2/\kappa$ is the density of compressional energy, the wave function $\widehat{p}$ can be normalized by a division by the total compressional energy. In the non-relativistic regime considered here this is nearly equal to the total wave energy and, hence, constant in any independent system. Moreover, even for $\omega > \omega_0$ the absence of linear coupling between the irrotational and solenoidal terms of eq. (7.1) suggests invariance of compressional energy in a closed system.

The association equals a particle's 'position probability density' with the density of compressional wave energy and the 'expectation value' of a measurable quantity with a volume average weighted with this energy. Note that here the term *energy* stands for the classical dynamic quantity seen by the outside observer, *not* the inhabitants' 'energy'. And it is to be expected that enlightened inhabitants will employ the kinematic operators $i\partial/\partial t$, $-i$ grad and $-i\mathbf{r}\times$grad for their 'abstract' description of hidden but imagined amplitude transients in a world of patterns where the corresponding eigenvalues, the steady-state kinematic entities *frequency, wave number* and *angular wave number* appear to them as 'dynamic' and are called (total) 'energy', 'momentum' and 'angular momentum'.

In the presence of several interacting wave centres, the total pressure amplitude $\widehat{p}_{\mathrm{t}}$ is the phasor sum of the partial pressures. Since the refraction $\delta\omega_0(\mathbf{r}, t)$, called 'potential', depends on the coordinates of all participating centres, eqs. (12.8) and (12.9) become multidimensional. If it is true, then, that the compressional wave energy supplies a weighing function, the cross products of partial pressures contained in $\widehat{p}_{\mathrm{t}}\widehat{p}_{\mathrm{t}}^*$ give it for the complete configuration.

Nothing new can be deduced from this model for a free particle, where the abstract notion of a wave function has been merely replaced by an (imagined) wave pressure. There are, however, two fundamental problems, concerning particle interaction and bound states, which have to be faced here although orthodox physics refuses to do so.

The orthodox interpretation of the 'wave function' has to insist on its fictitious-probabilistic nature, reverting to 'reality' only upon its instantaneous reduction to a specific result by the act of 'observation'—an act which necessarily involves the interaction of propagating dislocations in this model. It seems indeed very plausible, then, that the creation of new wave centres at the expense of original ones, implied by such interaction, occurs at positions whose probability is proportional to the original energy density, but also that such molecular relocation, or *jump*, will occur at compressional wave velocity (of the order of the pion rest energy times $R/\hbar$), thus *instantaneously*. The claim remains a qualitative one here, but the process appears to be accessible to quantitative simulation.

A second and related problem occurs in the description of bound states. It is in fact an essential part of orthodoxy to *abstain* from forming models of the four ingredients of Coulomb bound states, namely the law of dynamics (3.1), the existence of an elementary electric charge, the Coulomb force and its continued action when the point charge has been spread out into a standing-wave pattern. Since this theory proposes a model covering the three former facts and as migrating molecular perturbations play a fundamental role therein, it becomes necessary to accept their continued presence in bound states as well.

Naively, therefore, the electron in a ground-state hydrogen atom might be imagined to be identical with an unmodified disruption sphere carrying out orbital migrations, at the Bohr radius r_{e}/α^2 and with group velocity αc, around a 'charged' compressional (1212) wave centre, the refraction (11.10) making the phase advance, at velocity c/α, curl around and close in itself. Spherical symmetry must then be explained by continuous precession of the orbital plane.

This return to pre-1926 orbiting electrons is unsatisfactory but it is also partly at variance with the present model and suggestive of the following modification. The radial gradient of tangential group velocity $\partial^2\omega/\partial r\,\partial k_\theta$ associated with (11.10) must be expected to spread out the disruption volume into spherical shells. In the ground state there is one such shell (presumably occupying a fractured molecular layer). In the absence of any preferred direction, tangential (classical) momentum in the layer and concomitant group velocity outside, average out to zero. The oscillations are purely radial and so synchronized as to form the breathing motion associated with $l = m = 0$. Although the perturbation of molecular order associated with 'charge' is spread around the (1212) wave centre, it is still due to exactly one excess or deficit molecule. It will, thus, coalesce again into a regular disruption sphere at the centre of a free $(2100)^{\mathrm{d}}$ mode as soon as the refraction is cancelled due to interaction with a third particle. The 'ionizing potential' of 13.6 eV/$\hbar$ is the frequency shift (12.4) with respect to $\omega_0 = \omega^{\mathrm{d}}_{2100}$, taken at the Bohr radius $a_0 = \lambda\!\!\!^{-}_{\mathrm{e}} / \alpha = r_{\mathrm{d}} / 8\vartheta$, defined by the phase closure $a_0 k_\theta = 1$. The same result is obtained by integrating (11.10) from a_0 to infinity and taking into account the self-shielding which reduces the effective proton charge (the radial displacement due to the excess or deficit molecule at the (1212) centre) by one half at the middle layer of the $(2100)^{\mathrm{d}}$ disruption shell.

It remains to be demonstrated—but appears quite likely at this point—that the model is able to replicate the (partly disconnected) charge distributions of excited states as well. The coarse granularity of the model ether precludes the formation of smooth patterns for individual atoms but not on average over a large number and this is consistent with the randomness of individual events. Spontaneous mode conversion by molecular rearrangement will be at compressional velocity, hence 'instantaneous'. The matter will not be pursued further here but proposed for quantitative simulation, one of the many occasions to test this model which is supposed to expose what the inhabitants cannot observe, and (*a fortiori*) what happens when they do not look.

The formation of nuclei, the only other case of stable bound states occurring in nature, must be explained by the mutual coupling of (1212) modes via the strong refractive gradients which may plausibly be assumed to exist near n-mode wave centres. The 'force' must be attractive, which means that $\partial\omega/\partial r$ must be positive, if a bound state is to be formed. The concomitant positive gradient of phase velocity at given and constant oscillation frequency ω, namely

$$\left[\frac{\partial v_{\mathrm{p}}}{\partial r}\right]_{\omega\,=\,\mathrm{const}} = \frac{v_{\mathrm{p}}}{2\omega_{\Delta}}\,\frac{\partial\omega_0}{\partial r}\,,$$

makes propagating (1212) modes curl into each other.

A quantitative analysis is beyond the scope of this paper but might yield new information and constitute another test for the model. Qualitatively, there is no problem. A number n of coupled (1212) resonators will exhibit a comb of n closely spaced modes whose central frequency is close to but below the free (1212) frequency. The inhabitants add the n frequencies and call the sum (contained in the cross-products of partial pressures) the 'nuclear mass'.

13. – Summary and conclusions

It is claimed that—quite unexpectedly and in violation of a dogma—the most elementary features, at least, of our reality can be quantitatively represented by a classical mechanical model of striking simplicity. The features of reality treated here do not extend to the subjects of state-of-the-art research but they do include special relativity, elementary quantum mechanics, a mass spectrum splitting into hadrons and leptons and classical electricity.

In considering the model, a clear distinction must be made between the point of view of its external observer—generally taken here—and that of hypothetical model inhabitants who are claimed to experience their universe as we experience ours.

The only material constituent of this model is a single and universal medium for the propagation of classical mechanical waves. The assumed medium is subject to the combination of Newton's and Hooke's laws—expressed by eq. (7.1)—with invariable mass density. Thus, *when viewed from outside*, this model is rigorously governed by the Newtonian-Galilean dynamics which we believe to experience in the limits of low velocity and large dimensions. The medium is assumed to be devoid of any structure other than being composed of *very* small molecular constituents and being contained within a *very* large closed boundary. It is also assumed to be devoid of any motion beyond sub-molecular perturbative displacements from rest position. There are, in particular, *no* ballistic particles.

The model universe formed on this macroscopically homogeneous and stationary medium, including the inhabitants of this universe and *everything* available to them, is to *consist* entirely of propagating perturbations so that *group velocity of wave patterns* is the only manifestation of motion.

It is a consequence of this construction that any model inhabitants must necessarily *live inside* a purely kinematic world of patterns, where time and space are their only meaningful entities. The inhabitants are, of course, free to define a system of 'dynamics' of their own by multiplying the kinematic quantities frequency ω (2π over period) and wave number k (2π over wavelength)—which can be counted against the system of internal references introduced below—with a universal scale factor $\hbar$ and call the results 'energy' and 'momentum', respectively. It follows at once, however, that this system of 'dynamics' must be subject to the familiar *uncertainty relations*—the mathematical fact, applied to conjugate variables, that the rms width of a distribution times the width of its Fourier transform cannot be smaller than one half—since these relations, called bandwidth theorem in classical wave physics, impose a fundamental and unsurmountable limit on the transmission of any information by wave patterns.

Even a solely kinematic world requires references for time and space in order to be recognizable. Since the absence of material structures is to be a fundamental feature of the medium, these references are assumed to be formed, instead, by a universal propagation velocity c combined with a spectrum of intrinsic resonance frequencies ω_{0i}—called 'rest energies' $\hbar\omega_{0i}$ or 'masses' $\hbar\omega_{0i}/c^2$ by the inhabitants. Since the model must have these features anyhow in order to match reality, this construction, which makes the *mass spectrum the only reference of the inhabitants' time*, is the most economic one possible.

It is pointed out, then, that the dispersion relation $\omega(k)$ of eq. (3.1), which fully creates for the model inhabitants what we call *special relativity*, is indeed the dispersion of a classical medium consisting of a three-dimensional ensemble of identical coupled harmonic oscillators, provided that each oscillator has a resonance spectrum ω_{0i} matching our rest energies, that the coupling is frequency-independent and transverse to wave propagation and that the medium exhibits no graininess to inhabitant observers. These required properties can be summarized by the statement that the medium must possess an appropriate spectrum ω_{0i} of intrinsic resonances in addition to the more familiar properties of finite mass density and rigidity, zero viscosity and almost vanishing compressibility.

Thus, already at this stage, relativistic dynamics, the Lorentz transformations (cf. sect. 4 and appendix A) and the uncertainty relations emerge as deducible consequences. In short, all apparently 'material' structures of our world are declared illusions while reality is assigned to the seemingly 'abstract' kinematic entities of quantum wave mechancis. At first sight, little seems to have been gained at this point, since there remain the problems, not only of constructing a plausible medium with the required properties, but also of accounting for the manifest existence in reality of particle-like interaction centres and for the probabilistic nature of any description by waves. The first problem might be expected to lead to the introduction of unacceptable contrivances, reminiscent of 19th-century ether models. The second and third one appear to have been the reasons why all particle-free pure-wave models have been discarded long ago and almost from the outset.

Here it is claimed, however, that a quantitatively suitable classical-model medium does exist, and that it is of seemingly barren simplicity. In fact, all that is required in addition to finite rigidity μ and mass density ϱ is a very large but finite modulus of compression κ (so that κ/μ amounts to many tens of orders of magnitude), a free spherical boundary at a very large radius R—*the radius of the model universe*—and a very small but ordered molecular structure, however simple, in which *dislocations* can develop and propagate.

Explicitly, this model universe may be imagined as a large spherical assembly (a bag of radius R) of small frictionless spheres whose hardness is such as to produce the bulk modulus κ. A coherence pressure p_0 compresses these spherical molecules to a packing order just short of coordination number 12. In fact, the bulk density reaches the maximum permitted by the presence of dislocations which prevent the configuration from locking into complete 12-connection. This *bag model* exhibits a disruptible and reversible rigidity μ (see the end of appendix B) which is roughly equal to p_0 and, thus, unrelated to the arbitrarily large bulk modulus κ.

Strikingly, no further constructions or assumptions are required. Yet, what unfolds from these spare if unorthodox ingredients is a broad-brush picture of elementary physics. It includes most, though not all, of what was known around 1960 and, although there is no representation of subsequent developments, there is no flagrant contradiction either.

In this model the '*rest energies*' ω_{0i} are formed by *standing compressional waves* which shuttle between opposite boundaries of the entire universe in times commensurate with $\hbar$ over rest energy. They form a spectrum of cavity resonances $\omega_{0i} = (KR)_i c_\kappa/R$, where $c_\kappa = \sqrt{\kappa/\varrho}$ is the compressional wave velocity and $(KR)_i$ a spectrum of numerical factors. Molecular dislocations, called 'particles' by the inhabitants of this universe, form compressional wave centres where pressure and displacement nearly diverge but remain finite due to the finite molecular size. For wave centres not too far from the centre of this spherical cavity-universe the spectrum of compressional resonances is approximated by the roots $(KR)_i$ of eqs. (7.11) and (8.9); they relate the spherical Bessel and Neumann functions j_l, n_l with their derivatives and contain no free parameter.

An association of these spherical-cavity resonances with the low-energy hadron spectrum is depicted in fig. 1 (sect. 8). The agreement—at 1% level—is striking at first sight up to the ϕ-meson and still discernible as far as the listing goes via the correlated clustering of masses and of resonances which share a common sum $l + \nu_j + \nu_n$, where l is the order of the spherical Bessel and Neumann functions and ν_j, ν_n are the orders of neighbouring zeros of these functions (cf. the discussion in sect. 7 and 8).

To the author's knowledge this representation of the low-energy mass spectrum by an elementary analytical formula was hereto unknown. Two objections are likely to be raised. The first one is the absence of clear correlation with any particle properties *other than mass* (and beyond the definite association of mesons with $l=0$ and 1 and of baryons with $l \geqslant 2$). The second objection is the presence of gaps in the associations—the fact that not *all* roots of (7.11) *and* (8.9) correspond to known particles. Both objections are answered at the end of sect. **8**. In any case and in spite of all open questions the concordance appears too good to be dismissed as coincidence and it is further strengthened by the (0.025%) success of a closely related prescription for calculating the pion-to-muon mass ratio emerging from the lepton model discussed in sect. **10** and summarized below. The correlation shown in fig. 1 is, therefore, taken as justification for pursuing the concept from which it has emerged.

As is discussed at the beginning of sect. **7**, the combination of travelling shear waves—which would be free of dispersion and have wave velocity $c=\sqrt{\mu/\varrho}$ if the medium had no boundary—with the compressional cavity resonances ω_{0i} produces the waveguide-like dispersion (3.1) and concomitant group velocity $v_{\mathrm{g}}=\partial\omega/\partial k<c$. This dispersion forms, in every respect, an exact representation of *special relativity* if, obviously, the free shear velocity c is taken as the velocity of light. The Lorentz transformations result unambiguously from this dispersion and the absence of any references for the inhabitants' time and space besides ω_{0i} and c (sect. **4** and appendix A).

Although the field of dynamic wave pressure associated with each propagating dislocation fills the entire sphere (a 'wave function' associated with every 'particle' fills the 'universe'), the dislocation finds essentially infinite space for migration since c is very much smaller than c_κ. And although the pressure fields of all 'particles' are superimposed within the same closed boundary, they do not interact except in the vicinity of the wave centres where the (Neumann-function) near-divergence of pressure and stress makes such interaction very likely. The interaction creates the inhabitants' '*force*' in the form of mutual diffraction $\mathrm{d}\mathbf{k}/\mathrm{d}t=-\operatorname{grad}\omega$ and can lead to bound states. It is true that only one such interaction—a long-range one which is found to be mediated by the boundary at radius R—has been studied here (sect. **11**): it turns out to be in complete agreement with the *Coulomb force*.

Particle-like objects are introduced by this notion of molecules. But, in contrast to ballistic particles, they are macroscopically stationary and do not violate the basic assumption of there being nothing but waves. Moreover, as the cavity resonances ω_{0i} and the shear-wave velocity c are both bulk properties, involving a very large number of molecules, the granularity of the medium is not reflected in the dispersion relation (3.1). There is, in particular, no π-mode cut-off of propagation at a wavelength approaching the molecular size.

On the other hand, any dynamic interaction of wave centres at close distance to each other must be expected to produce small relocations of molecules, thus *jumps* of wave centres. These jumps will occur in the granular medium at compressional wave velocity c_κ, hence 'instantaneously'. They are necessarily limited to microscopic regions of high wave pressure near dislocations.

It turns out (sect. **12**) that an association of this wave pressure (a complex entity since its Fourier components include phase) with the *wave function* of orthodox physics leads (for $hc \ll \omega_0$) directly to the Schrödinger equation (in multidimensional configuration space if several coupled wave centres participate). It may be said, therefore, that this concept of non-local bulk waves in a granular medium assures the model's success at the most fundamental level. The concept implies at once that waves, filling the model universe, on whose large-scale properties they depend alone, completely determine all probabilistic behaviour, while perturbations of microscopic granularity define exact (but statistically fluctuating) phase space coordinates of dislocations and associated wave centres. Recognition of these coordinates by model inhabitants consisting of waves—and deriving their structure of time and space from the bulk-wave properties ω_{0i} and c alone—is bound to be subject to the uncertainty relations, since these relations control the information content of modulated waves.

The concept of compressional resonances filling the model universe implies vanishing compressibility over any distance much smaller than R. Therefore, local configurations of pure shear waves, in which all displacements $\mathbf{s}$ from rest position form closed loops, have $\operatorname{div}\mathbf{s} = 0$. They do not interact with the boundary and propagate at velocity c without dispersion. As might have been expected, such configurations conform to Maxwell's equations in vacuum if $\mathbf{s}$ is associated with the inhabitants' 'electric field'. It turns out that the concomitant 'magnetic field' is (half) the mass density of angular momentum carried in the medium. This and the two subsequent statements are derived in sect. **9**.

A permanently dislocated molecule, as well as the hole it leaves behind elsewhere, creates in the surrounding medium a localized source of radial displacement which falls off with the inverse square of distance. Clearly, therefore, the volume perturbation created by such an excess or deficit molecule forms a most natural replica of *electric charge*. The concept explains at once the quantization of electric charge, its two exactly annihilating polarities, its conservation under all other circumstances and its attachment to wave centres ('particles'). It also completes Maxwell's equations by representing average 'charge density' as $\langle \operatorname{div}\mathbf{s} \rangle \neq 0$ and thus as divergence of 'electric field' and by adding the associated 'conduction current'.

Moreover, it is a necessary consequence of this identification of quantized 'charge' with dislocated molecules, creating compression-free strain and stress in their neighbourhood, that each one must be surrounded by a spherical volume of definite radius within which the maximum elastic shear angle of the medium is exceeded and its rigidity is disrupted. The bag model invoked above does have this feature, the disruption strain being defined as the one required to break up the molecular order of nearly dense packing so as to destroy its rigidity. The bag model also features automatic re-establishment of rigidity behind a propagating dislocation, in such a way that the sphere of disruption follows the dislocation.

In the terms of the model, the disruption radius formed by this inescapable process—a cut-off radius for elastic energy—suggests itself as a model for the *classical radius*—a cut-off of electrostatic energy—and this association is fully validated in yielding an '*electron mass*' and a '*Coulomb force*' in quantitative mutual agreement. The elastic forces in the medium which are responsible for these results are subtle ones and their existence depends on the medium's containment in a boundary. This is the subject of sect. **10** and **11**, the more technical part being deferred to appendix B.

Generally, '*leptons*' in this model are associated with dipole modes in which the Neumann-function distributions of dynamic pressure and displacement do not penetrate into the molecular radius, as was implied for the 'hadrons' of sect. **7** and 8 and fig. 1, but end at the much larger disruption radius. Inside this radius, the medium, having lost its molecular order, is free of any strain and stress, so that these modes are *coreless*. The second lowest mode of this kind is being associated with the *muon*. Combining this association and that of the pion with the lowest root of eq. (7.11)—as shown in fig. 1—implies a unique analytical prescription (sect. **10**) for the *charged-pion–to–muon mass ratio*. Its 0.025% agreement with reality lends strong support to both this lepton model and to the hadron model of sect. 8. The model also offers an immediate explanation for the predominant decay mode of charged pions.

Finally, it emerges that the mode conversions which, in this model, correspond to single charged-lepton creation from baryon decay require—in order to conserve classical momentum—the creation of a companion mode which possesses a disruption radius, surrounded by permanent stress frozen into the medium (appendix B), although it contains no 'charge'. An association with the *neutrino* strongly suggests itself although there is, so far, no quantitative confirmation beyond vanishing 'mass' ($\omega_0 = 0$), and no explanation for the existence of different varieties of neutrino in reality.

Clearly, in this model, an isolated dislocation, forming a discrete wave centre, represents a free particle. The complicated configurations of bound-state wave functions must be represented by distributed patterns of molecular perturbation, held together by mutual refraction, each participant creating sufficient refractive gradient $\text{grad}\,\omega(r)$ in its neighbourhood to make a partner's wave propagation close in itself and all partners' propagations curl into each other. Here, only the simplest example by far, the ground-state hydrogen atom, has been explicitly related to the model. Following the demonstrations that the model does render the Coulomb 'potential' $\partial\omega_0 = \alpha c/r$ and Schrödinger's equation applied to pressure waves in the medium, only the smeared-out status of the electron charge remains to be explained. It is argued that this does in fact correspond to a pattern of molecular disruption which, although caused by exactly one displaced molecule, is sheared into a spherical shell by the radial gradient of tangential group velocity $\partial\omega/\partial r\,\partial k_\theta$. The concept is readily extendible to excited states, including the concomitant existence of disconnected regions populated by smeared-out fragments of 'elementary charge'. The re-emergence of a localized wave centre (a 'free electron') following escape from the refractive gradient $\alpha c/r^2$ is properly represented.

Thus, this model of non-local bulk waves in a granular bounded medium (a large bag of small spherical grains) offers detailed concepts for the nature of hadrons and leptons—including a classification in mesons and baryons for the former and in charge carriers and neutrinos for the latter—as well as a naturally emerging explanation for classical electricity. This explanation, which associates the 'electric field' with what little material displacements from rest position (or 'vacuum') are present, includes Maxwell's equations, quantized charge surrounded by a cut-off radius for electrostatic 'energy', the electron and the Coulomb 'force'. In agreement with reality, it is 'charge', not 'mass', which represents indestructible material in this model universe, whose limited size and non-local behaviour therein are decisive features in all respects.

Admittedly, only an indirect and qualitative hint is given for the nature of the strong force, none for the weak force, and only conjectures for the nature of spin (end of sect. 8).

Quantitative results are analytical prescriptions for the mass spectrum of hadrons and for the pion-to-muon mass ratio. The former gives the mass ratios of the quasistable hadrons π, K, η, p/n, Λ, Σ, Ξ, Ω^- and of many more light particles and resonances with 1% rms accuracy. The latter is correct within 2.5×10^{-4} relative error. The electron mass resulting from this model is in exact agreement with the concomitant explanations for the classical electron radius and the Coulomb force.

Finally, the model yields a straightforward derivation of the Schrödinger equation and this gives rise to the following epistemological remarks. The model contains two kinds of hidden variable. The first kind concerns bulk dynamics of the medium, the most prominent example being its dynamic pressure which is, however, no more hidden from inhabitants' recognition (and less from their imagination) than its orthodox equivalent, the 'wave function'. The second kind concerns molecular kinematics, such as the exact position of a dislocation. It appears that the latter parameters are indeed hidden more deeply from *wavy* inhabitants' observation than atoms ever were from chemists or molecular trajectories from thermodynamicists. This, however, does not prevent the model from exposing itself generously to potential falsification in confrontation with quantitative detail and with more advanced subjects where it may—or may not—lose its naive simplicity or fail altogether.

It has to be investigated yet, in fact, whether the model can account for all the numerous phenomena and quantitative relationships which are the subjects of contemporary physics and it may well fail to do so. It seems unlikely even then, however, that its validity should stop abruptly at the elementary but surprisingly complete level reached here without offering potentially useful guidance into more advanced subjects.

For the time being, the richness of its contents is remarkable and at variance with the dogma that such models are doomed to fail from the outset. Therefore, the most important result in the author's opinion is re-established hope that the world of which we are part might yet be accessible to common-sense logic and visual imagination, even though the model's barren simplicity may appear shocking.

Appendix A

The Lorentz transformations

In a universe formed by nothing but propagating oscillations a meaningful frame of reference can only consist of a coherent wave pattern moving with group velocity v_g. Another pattern—for instance the single pulse constituting an elementary 'particle'—moving with the same group velocity in the same direction must be considered 'at rest' in that frame. A comoving inhabitant observer has no choice, therefore, but to consider that particle's apparent carrier frequency (the central frequency of the observed Fourier spectrum) as the 'rest frequency' in that frame. The inhabitant observer has no difficulty in doing so. Since all patterns at rest in the moving frame have the same value of $\gamma = \omega_i/\omega_{0i}$ in the ether frame, their mutual ratios of frequencies remain unchanged.

A pattern moving in the direction of the positive x-axis (seen in the ether frame and from outside) with group velocity $v_g = \beta c$ may be taken as a modulated wave whose carrier is of the form $\exp[i(\omega t - kx)]$. With the help of eqs. (3.3) and (3.4) the exponent can be written as

$$\varphi = \gamma\omega_0\left(t - \frac{\beta}{c}x\right). \tag{A.1}$$

This is the phase difference between two oscillating ether elements at distance x and time difference t. For $x = v_g t$ this reduces to $\gamma\omega_0 t(1-\beta^2)$ and thus to

$$\varphi_0 = \frac{\omega_0 t}{\gamma} \tag{A.2}$$

and it does not matter that the inhabitants can count φ_0 in terms of half-cycles only. (Note that $x = v_g t$ is indeed the place of maximum contribution to the Fourier spectrum since it designates the *stationary phase* [9] in the sense of $\partial\phi/\partial\omega = \partial\varphi/\partial k = 0$.)

Observers moving through structureless ether with group velocity v_g cannot know that the waves of which they and their surroundings consist penetrate fresh volume elements continuously. They have no choice but to consider φ_0 as the variation of phase 'at a fixed location' in their frame and—in the absence of any other reference of time—equate φ_0 with $\omega_0 t'$, defining their time t' by the nominal value assigned to ω_0. Equating $\omega_0 t'$ with the right-hand side of (A.2) yields the time dilatation at fixed position x' in the moving frame as

$$t = \gamma[t']_{x'=\text{const}} \,. \tag{A.3}$$

Moreover, the phases of a comoving wave, observed 'simultaneously' but at different locations x' in the moving frame, are all identical, confirming that the observed wave is indeed a rest-frequency (0-mode) oscillation. This may be argued as follows.

'Simultaneity' of events at points x_1' and x_2' (say) in the moving system can only be defined by an exchange of signals—light pulses for instance. Thus, light pulses simultaneously leaving an intermediate point at $x' = 0$ (say) in opposite directions are reflected at x_1' and x_2' 'simultaneously' if they return to $x' = 0$ in coincidence (a situation which also defines 'equidistance' $x_1' = -x_2'$). Assuming coincidence of coordinate systems ($x = x' = 0$) at the time $t = 0$ of light emissions, the reflections may be said to occur at x_1, t_1 and x_2, t_2 as seen in the ether frame. While the light signals travel, the moving observer's origin has shifted a distance $x = v_g t$, where t is the round-trip travel time—the same for both signals since they travel from coincidence (at $x = 0$) to coincidence (at $x = v_g t$) in a stationary medium. It follows that the locations and times of the two reflections, as seen in the ether frame, are given by

$$2x_1 = -(1-\beta)\,ct\,, \tag{A.4}$$

$$2x_2 = (1+\beta)\,ct\,, \tag{A.5}$$

$$ct_1 = -x_1 > 0\,, \tag{A.6}$$

$$ct_2 = x_2 > 0\,. \tag{A.7}$$

Since $\varphi = \omega t - kx$ is the phase of the light wave travelling in the direction of $x_2 > 0$, one finds

$$2\varphi_1 = 2(\omega t_1 - kx_1) = -2x_1\left(\frac{\omega}{c} + k\right) = ct(1-\beta)\left(\frac{\omega}{c} + k\right), \tag{A.8}$$

$$2\varphi_2 = 2(\omega t_2 - kx_2) = +2x_2\left(\frac{\omega}{c} - k\right) = ct(1+\beta)\left(\frac{\omega}{c} - k\right) \tag{A.9}$$

and, therefore, with the help of eqs. (3.3) and (3.4)

$$\varphi_1 = \varphi_2 = \frac{\omega_0 t}{2\gamma}\,, \tag{A.10}$$

the same for both arbitrarily chosen points x_1' and x_2' in the moving frame, confirming the 0-mode feature of any comoving wave pattern.

Since, therefore, the observed wave number k' is zero in any comoving wave the entire expression (A.1) must be interpreted as $\omega_0 t'$. This yields the Lorentz transformation of time

$$t' = \gamma\left(t - \beta\frac{x}{c}\right). \tag{A.11}$$

According to the arguments of sect. **3**, 'distance' in a frame moving with group velocity v_g in the ether is half the round-trip time t' of a reflected-light pulse times the assigned nominal value c. Let L be such a distance (the end points moving with v_g) as seen from the privileged position of the outside observer. The times of forward and return travels of the light pulses are given by

$$ct_1 = L + v_g t_1\,, \tag{A.12}$$

$$ct_2 = L - v_g t_2\,, \tag{A.13}$$

respectively, the moving frame having advanced by $v_g t_1$ during the forward travel and by $v_g(t_1 + t_2)$ on the return to the moving origin—all this seen from outside. This yields

$$ct = 2\gamma^2 L \tag{A.14}$$

for the round-trip time since $\gamma^2 = 1/(1+\beta)(1-\beta)$. For the outside observer the Galilei transformation $L = x - \beta ct$ is valid. For inhabitants who are stationary in the moving frame the time is $t' = t/\gamma$. Moreover,

$$x' \equiv \frac{ct'}{2} \tag{A.15}$$

defines the distance x' for them, since they have no other reference but this, with the nominal value of c. What follows is

$$x' = g(x - \beta ct)\,, \tag{A.16}$$

the Lorentz transformation of distance.

Finally, the group properties of (A.11), (A.16) extend their validity to all transformations between inhabitants' frames in constant relative motion. These frames, therefore, are indistinguishable for them.

Formally, the arguments given above are familiar and elementary. The point is that they are being made from an entirely Galilean position, concerning an entirely Newtonian system: no *ad hoc* 'principle' is being invoked; the indistinguishable equivalence of inertial frames and the universality of a velocity limit are results rather than starting points of the analysis.

Appendix B

Disruption dynamics

Spherical coordinates r, θ, φ will be used, the axis being defined by the dipole motion under discussion. The terms *pole*, *meridian* and *equator* will designate locations on the spherical disruption limit of radius $r_{\rm d}$, the point $(r_{\rm d}, 0)$ being called *north pole*.

For radii outside $r_{\rm d}$ (but much smaller than R), monopole and dipole displacements can be derived as gradients of (fictitious) potentials ϕ^{n0} and ϕ^{n1} which are proportional to the small-argument asymptotes of $n_0(r)$ and $n_1(r)\cos\theta$, respectively. Explicitly, these potentials, the displacements $\mathbf{s}^{nl}(r, \theta)$ and strain tensors $\varepsilon^{nl}_{ik}(r, \theta)$ are given by

(B.1) $$\phi^{n0} = -\vartheta\,\frac{r_{\rm d}^3}{3r}\,,$$

(B.2) $$\phi^{n1} = -\hat{s}_z\,\frac{r_{\rm d}^3}{2r^2}\cos\theta\,,$$

(B.3) $$s_r^{n0} = \vartheta\,\frac{r_{\rm d}^3}{3r^2}\,,$$

(B.4) $$s_r^{n1} = \hat{s}_z\,\frac{r_{\rm d}^3}{r^3}\cos\theta\,,$$

(B.5) $$s_\theta^{n0} = 0\,,$$

(B.6) $$s_\theta^{n1} = \hat{s}_z\,\frac{r_{\rm d}^3}{2r^3}\sin\theta\,,$$

(B.7) $$\varepsilon^{n0}_{ik} = \frac{2}{3}\,\vartheta\,\frac{r_{\rm d}^3}{r^3}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix},$$

(B.8) $$\varepsilon^{n1}_{ik} = 3\hat{s}_z\,\frac{r_{\rm d}^3}{r^4}\begin{bmatrix} -\cos\theta & -\sin\theta/2 & 0 \\ -\sin\theta/2 & \cos\theta/2 & 0 \\ 0 & 0 & \cos\theta/2 \end{bmatrix},$$

where $\vartheta \equiv r_0^2/r_{\rm d}^3$ (including the sign of r_0), $\hat{s}_z$ is the north-pole dipole displacement $s_r^{n1}(r_{\rm d}, 0)$ and $s_\varphi = 0$ everywhere. Note that $s_\theta > 0$ means downward displacement at the equator. The monopole fields are created by an excess or deficit molecule of radius r_0 contained in the disruption sphere. The medium inside $r_{\rm d}$ may be taken as liquid and the volume perturbation uniformly spreads among randomly distributed molecules.

Offsetting the monopole centre by a small amount δz with respect to a given coordinate system (by removing the volume perturbation and recreating it uniformly shifted) creates a situation which is identical with a superposition of the original monopole and a dipole field which is the gradient of a potential perturbation

(B.9) $$\delta\phi^{n1} = -\,\delta z(\cos\theta,\ -\sin\theta,\ 0)\cdot\nabla\phi^{n0}\,.$$

This is equivalent to a dipole (north pole) displacement

(B.10) $$\delta\hat{s}_z = \frac{2}{3}\vartheta\delta z\,.$$

Thus the offset monopole creates a dipole strain tensor as given by substituting $\delta\hat{s}_z$ for $\hat{s}_z$ in (B.8).

The dipole motion for $r < r_\mathrm{d}$ is the uniform vertical displacement

(B.11) $$s_r^{j1} = \hat{s}_z \cos\theta\,,$$

(B.12) $$s_\theta^{j1} = -\hat{s}_z \sin\theta\,.$$

The θ-component is the opposite double of $s_\theta^{n1}(r_\mathrm{d})$ implying slippage at the boundary surface.

The area density of elastic force across the surface $r = r_\mathrm{d}$ is the scalar product of the surface-normal vector $(2\mu, 0, 0)$ with the appropriate strain tensor. Thus the monopole strain creates a radial force density of $-4\mu\vartheta/3$, inward, producing a positive pressure of equal magnitude for $\vartheta > 0$, that is for the presence of an excess molecule. (Note that the vanishing divergence of $\mathbf{s}$ does not exclude pressure, since $\kappa \to \infty$).

If, in a dipole motion, the disruption surface remains locked in the medium, the dipole strain creates a skew force density across $r = r_\mathrm{d}$, the radial and meridian components being given by

(B.13) $$f_r^{n1} = -6\mu\hat{s}_z \frac{\cos\theta}{r_\mathrm{d}}\,,$$

(B.14) $$f_\theta^{n1} = -3\mu\hat{s}_z \frac{\sin\theta}{r_\mathrm{d}}\,.$$

For $\hat{s}_z > 0$ the radial force points downward at both poles but the meridian force points upward at the equator. Separate integration of the z-components $f_r \cos\theta$ and $-f_\theta \sin\theta$ over the surface $r = r_\mathrm{d}$ gives the total axial forces due to f_r^{n1} and f_θ^{n1} as

(B.15) $$F_z^r = -8\pi\mu r_\mathrm{d}\hat{s}_z\,,$$

(B.16) $$F_z^\theta = -F_z^r\,,$$

respectively. Therefore, in a situation in which the tangential force is actually transmitted across $r = r_\mathrm{d}$ the total force is zero indicating static stability. A configuration of this kind is formed if a solid but disrupted sphere *not* containing an excess or deficit molecule fuses with its sourroundings after having suffered a dipole displacement. Such *frozen dipole stress* is being associated with the neutrino.

The force F_z^θ cannot act across a disruption at $r = r_\mathrm{d}$, however. Instead, it is effectively reversed and halved by the slippage between boundaries. That this must be so can be seen by comparing the total elastic-dipole energy $6\pi\mu r_\mathrm{d}\hat{s}_z^2$, found by integrating $\mu\varepsilon_{ik}^{n1}\varepsilon_{ik}^{n1}$ over the entire volume outside r_d, with the work $4\pi\mu r_\mathrm{d}\hat{s}_z^2$ done by displacing the sphere against F_z^r alone. Therefore, if the assumption of a disruption surface locked to the medium (a solid sphere displaced within an elastic medium to which it does not stick) could be maintained, a total axial restoring force

(B.17) $$F_z = -12\pi\mu r_\mathrm{d}\hat{s}_z$$

would result. Balancing this against $\omega^2 \hat{s}_z$ times the effective mass (10.6) would yield a resonance $\omega = \sqrt{6}\, c/r_\mathrm{d}$.

The disruption boundary cannot remain locked to the medium, however, if indeed ϑ is the disruption shear angle, a definition which also implies that the normal force at the disruption surface remains constant at $-4\mu\vartheta/3$ and the restoring force (B.17) cannot occur. Instead, any momentum acquired by the mass of the disrupted sphere makes it break through the surrounding medium almost freely—completely so if the medium had no boundary—in spite of the fact that individual molecules are not free to follow ballistic motion.

The motion, for $\vartheta > 0$, say, can be visualized as follows. An infinitesimal material displacement s_z of the disrupted sphere, due to its momentum, drains a trailing hemispherical meniscus of thickness $\Delta z = s_z/\vartheta$ of its share ϑ of fractional excess volume and inflates a corresponding meniscus in front by the same fractional amount ϑ. (Here the term *meniscus* designates a thin shell bounded by two hemispheres, both of radius r_d, whose centres differ by Δz.) The motion is identical with a shift of the excess volume and of the associated spherical disruption boundary by Δz. For $r > r_\mathrm{d}$ the motion is an n-mode of north-pole amplitude $2s_z/3$, as given by (B.10). This provides the necessary backflow across the equatorial plane, so as to maintain continuity at zero divergence, the entire process being the exact equivalent of removing the excess molecule and recreating it at the new position Δz. For $\vartheta < 0$ the motion is the same except that the material displacement s_z and the (much larger) shift Δz of disruption are in opposite directions. This motion is free of divergence and curl. It resembles the advance of a *caterpillar*: since ϑ is small, the main body stays almost at rest (it even retreats slightly for $\vartheta < 0$) while, continuously, a head is stretched out and a tail retracted.

Thus, if the uniform material velocity for $r < r_\mathrm{d}$ is $\mathbf{u}$, the propagation of volume perturbation and disruption equals

(B.18) $$\mathbf{v} = \frac{\mathbf{u}}{\vartheta}\,.$$

For either sign of ϑ, molecules continuously leave the sphere at the rear of the group motion $\mathbf{v}$ while new ones enter in front. However, the molecules which—having been overtaken by the rear boundary—come to rest in an incompressible dissipationless medium, cannot but transmit their momentum (of either sign) forward with respect to the direction of $\mathbf{v}$ and at the near infinite compressional propagation velocity $\sqrt{\kappa/\varrho}$. The result would be force-free concentric propagation of the j-n boundary, the momentum centre, the volume perturbation and the disruption boundary—*if the ether had infinite extension.*

For $KR \ll 1$ the medium must, however, be considered incompressible over its entire volume out to the boundary at R. In this situation, an isolated excess (or deficit) ether volume V_0 cannot migrate with respect to this boundary, as the equivalent n_1-motion would displace the entire universe by an amount inversely proportional to its mass ϱR^3. On the other hand, the oscillatory displacement s_z of the effective mass (10.6) is not prohibited since it is balanced by opposite motion elsewhere, this being an essential part of the mode in question as expressed by $N = 2$. The resulting excentricity s_z of the disruption limit to the centre of the moving mass and eq. (B.10) give an equivalent n-mode amplitude $\delta\hat{s}_z = 2\vartheta s_z/3$. Substituting this instead of $\hat{s}_z$ into (B.17) gives a total force

(B.19) $$F_z = -8\pi\mu r_\mathrm{d}\,|\vartheta|\,\hat{s}_z\,.$$

When balanced against $\omega^2 \hat{s}_z$ times the effective mass (10.6), this yields a resonance frequency given by

$$\omega^2 = 4\,|\vartheta|\,\frac{c^2}{r_d^2} \tag{B.20}$$

which is being associated with the electron. The caterpillar migration invoked above does remain the mechanism for macroscopic charge propagation or 'current' which, being closed in itself, does not interact with the boundary of the universe.

The following explicit molecular model appears to replicate the three bulk constants μ/ϱ, κ/ϱ, ϑ in a natural way. It consists of an ensemble of hard frictionless spheres pressed into dense packing by a coherence pressure p_0 which might be of external origin or due to central intermolecular attraction including non-contacting near-neighbours. The packing density η is, therefore, a maximum, plausibly the apparent (if mathematically unconfirmed) absolute maximum of 0.7405 of a (near)12-connected configuration (face-centred cubic, hexagonal or mixed). Ideally such a configuration is self-locking. But 12-connection of identical incompressible spheres forms an overconstrained system which changes to a smaller coordination number, and thus gains freedom for small deformations, by the presence of the slightest lattice perturbation somewhere within the long range of negligible compressibility. The presence of dislocations guarantees this.

For small shear angles $\xi = (\partial s_i/\partial x_k + \partial s_k/\partial x_i)$ the bulk volume perturbations are, therefore, of second order in ξ, so that

$$\frac{\partial}{\partial\xi}(\nabla\cdot\mathbf{s}) = \eta_0\,\frac{\partial}{\partial\xi}(\eta^{-1}) = g\xi\,, \tag{B.21}$$

where g is a numerical factor of order unity. Multiplication of $\nabla\cdot\mathbf{s}$ with p_0 gives a shear-dependent energy density $W(\xi)$ which defines a rigidity μ via the relation

$$\mu \equiv \frac{\partial^2}{\partial\xi^2}\,W(\xi) = gp_0\,. \tag{B.22}$$

The quiescent deviation from 12-connection may be characterized by a small average distortion angle. The dynamic shear ξ beyond this enforces a change of configuration such that the dependence of volume—and thus of energy $W(\xi)$—changes to first order in ξ, making the second derivative zero. The size of this disruption angle, small but not infinitesimal, is related to the density of dislocations averaged over the universe, or at least a large fraction of it. The economy of constructions is potentially improved over the assumption of separate bulk constants μ and ϑ, as p_0 merely replaces μ while ϑ is at least conceptually related to the presence of dislocations which is already a fundamental ingredient of the model. The concept requires, however, that α^2 be related to a long-range average of 'particle' density and this remains to be studied.

An alternative model might be built on the assumption of a local minimum of packing density (as in a body-centred or simple cubic configuration), due to repulsion between non-contacting neighbours. However, coherence would require an attractive contact force in addition and the smallness of the disruption angle ϑ would remain unexplained.

There remains the striking fact that the numerical value of the hadron scale length r_s of eq. (7.12) (the radius of an equivalent universe with compressional wave velocity c), extracted from table I, is nearly equal to the value of the disruption radius r_d

implied by its association with $r_e/2$. The agreement is within 3%. It implies a relation

$$\frac{\kappa}{\mu} \approx \left(\frac{R}{r_d}\right)^2, \tag{B.23}$$

which points to further economy in free parameters. However, a plausible model for this has not been found so far.

REFERENCES

[1] WHITTAKER E. T., *A History of the Theories of Aether and Electricity* (Dover Publications Inc., New York) 1989, two vol. in one (first edition of Vol. **1**: Dublin University Press Series, 1910).

[2] BELL J. S., *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press) 1987 and references therein, in particular: BOHM D., *Phys. Rev.,* **85** (1952) 166, 180; DE BROGLIE L., *Tentative d'interpretation causale et nonlinéaire de la mécanique ondulaire* (Gauthier-Villars, Paris) 1956.

[3] LIGHTHILL J., *Waves in Fluids* (Cambridge University Press) 1978, p. 317.

[4] ELMORE W. C. and HEALD M. A., *Physics of Waves* (Dover Publications Inc., New York) 1969, p. 465.

[5] *Review of Particle Properties*, *Phys. Rev. D*, **45** (1992).

[6] SOMMERFELD A., *Mechanik der deformierbaren Medien* (Nachdruck der 6. Auflage Verlag Harri Deutsch) 1978, p. 100.

[7] ABRAMOWITZ M. and STEGUN I. A. (Editors), *Handbook of Mathematical Functions* (Dover Publications Inc., New York) 1970, p. 437.

[8] In ref. [3], p. 325.

[9] In ref. [3], p. 249.