

239 A Supplementary material

240 A.1 Alternative partitions

241 The S_{CP} distributions with alternative partitions are shown in Fig. 6 for $D^0 \rightarrow K^- K^+ \pi^- \pi^+$
 242 decays and Fig. 7 for $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays.

243 The S_{CP} distributions with alternative partitions for the $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ control
 244 channel are shown in Fig. 8, for combined magnet up and magnet down data.

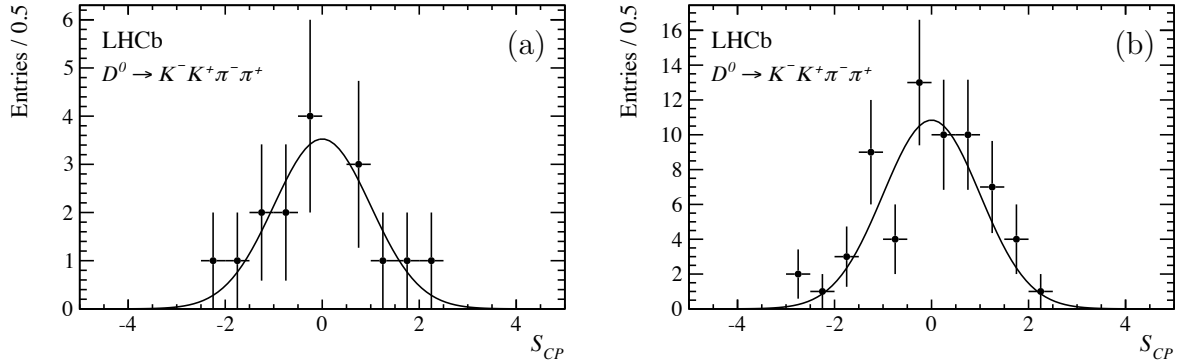


Figure 6: Distribution of S_{CP} for $D^0 \rightarrow K^- K^+ \pi^- \pi^+$ decays with (a) 16 bins and (b) 64 bins. The points show the data distribution and the solid line is a reference Gaussian distribution corresponding to the no CPV hypothesis.

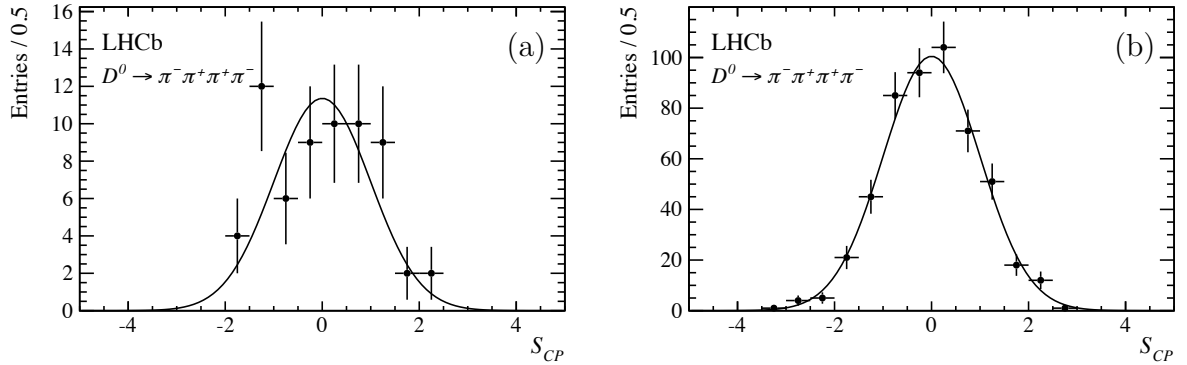


Figure 7: Distribution of S_{CP} for $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays with (a) 64 bins and (b) 256 bins. The points show the data distribution and the solid line is a reference Gaussian distribution corresponding to the no CPV hypothesis.

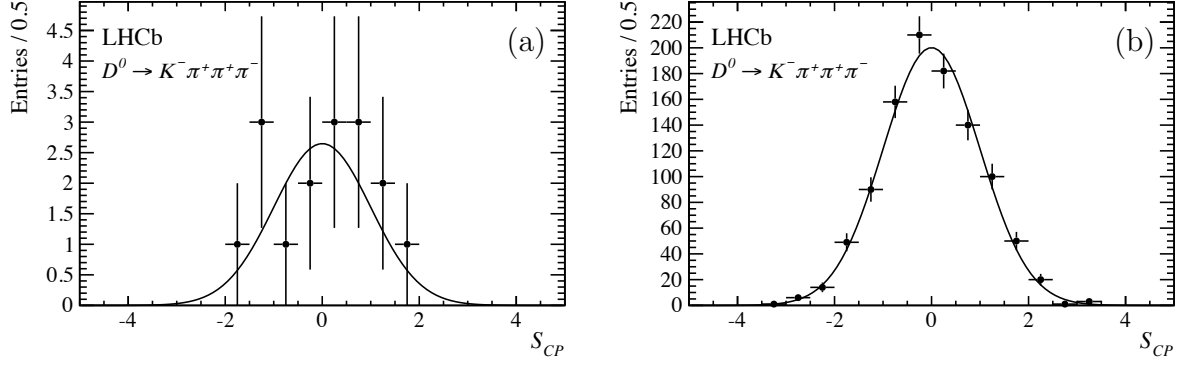


Figure 8: Distribution of S_{CP} for $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ decays with (a) 16 bins and (b) 1024 bins. The points show the data distribution and the solid line is a reference Gaussian distribution corresponding to the no CPV hypothesis.

245 A.2 Bin definitions

246 The phase-space partitions for $D^0 \rightarrow K^- K^+ \pi^- \pi^+$ and $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays are defined
 247 in terms of five invariant mass-squared combinations of the D^0 meson decay products. The
 248 definitions of default partitions in terms of these five variables are shown along with the
 249 S_{CP} value for each corresponding bin for $D^0 \rightarrow K^- K^+ \pi^- \pi^+$ decays and $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$
 250 decays in Fig. 9 and Fig. 10, respectively.

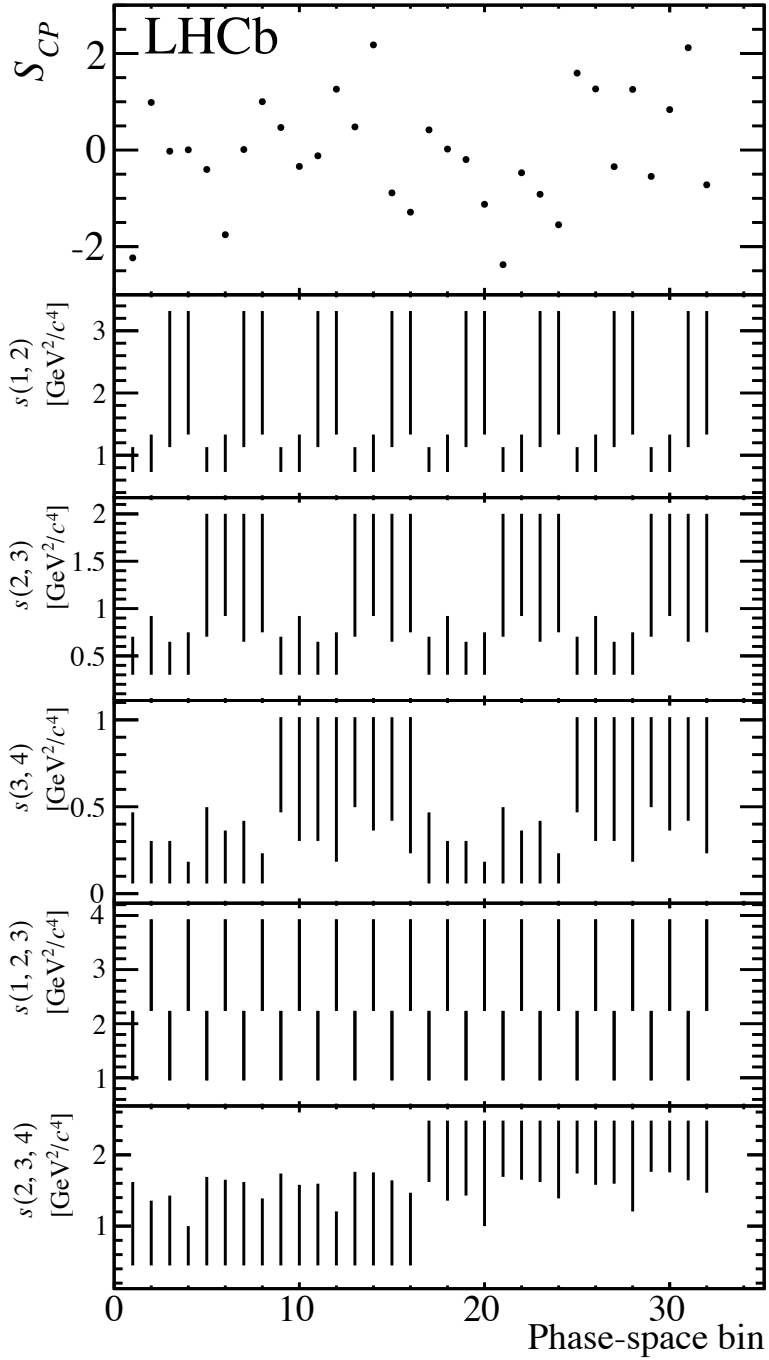


Figure 9: Definitions of the default partition of 32 bins across the five-dimensional phase space of the $D^0 \rightarrow K^- K^+ \pi^- \pi^+$ decay. The vertical lines show the range covered in the given invariant mass-squared combination in units of GeV^2/c^4 . The invariant mass-squared combinations $s(1,2)$, $s(2,3)$, $s(1,2,3)$, $s(2,3,4)$, and $s(3,4)$ correspond to $s(K^-, K^+)$, $s(K^+, \pi^-)$, $s(K^-, K^+, \pi^-)$, $s(K^+, \pi^-, \pi^+)$, and $s(\pi^-, \pi^+)$, respectively. The markers on the first plot show the value of S_{CP} for each corresponding bin.

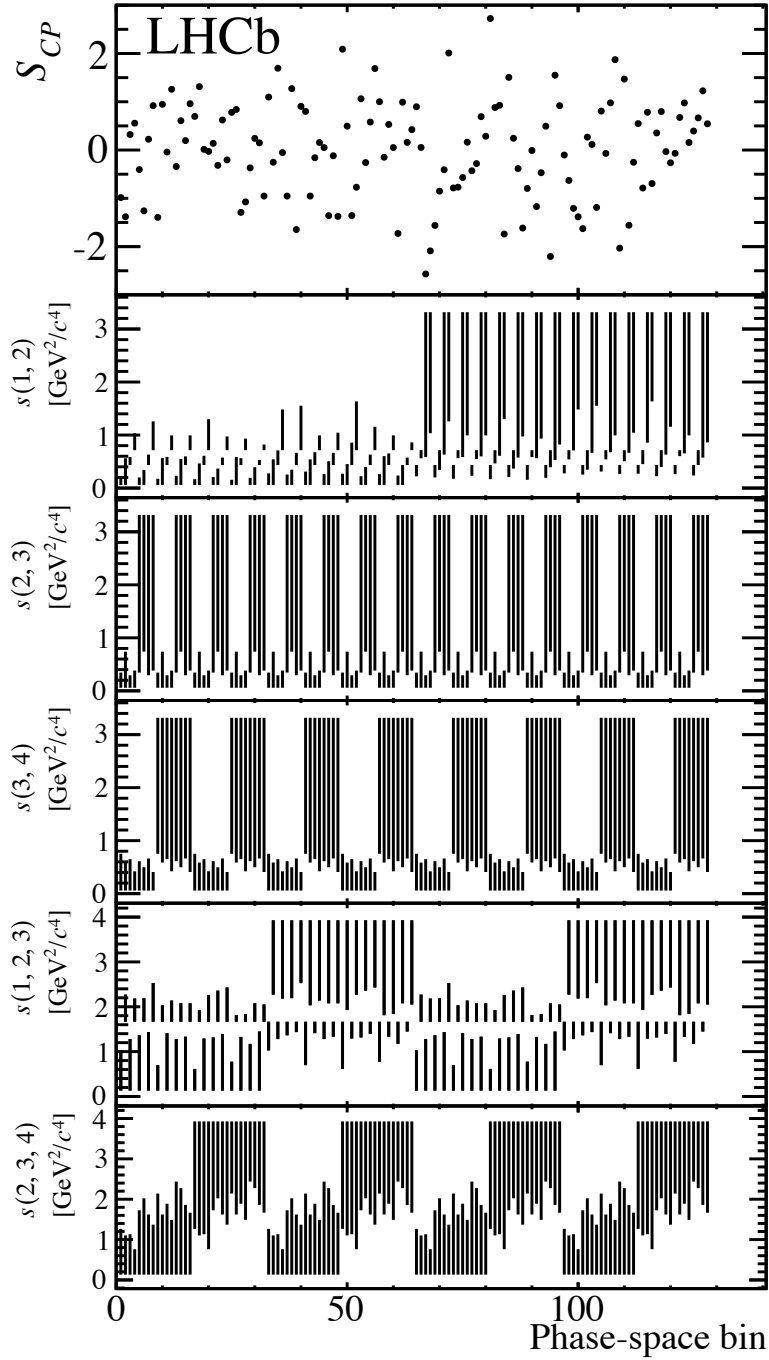


Figure 10: Definitions of the default partition of 128 bins across the five-dimensional phase space of the $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decay. The vertical lines show the range covered in the given invariant mass-squared combination in units of GeV^2/c^4 . The invariant mass-squared combinations $s(1,2)$, $s(2,3)$, $s(1,2,3)$, $s(2,3,4)$, and $s(3,4)$ correspond to $s(\pi^-, \pi^+)$, $s(\pi^+, \pi^+)$, $s(\pi^-, \pi^+, \pi^+)$, $s(\pi^+, \pi^+, \pi^-)$, and $s(\pi^+, \pi^-)$, respectively. The markers on the first plot show the value of S_{CP} for each corresponding bin.