

Figure 3: Distributions of the (top)  $K_{\rm S}^0 \pi^+$ , (middle)  $K_{\rm S}^0 K^+$  and (bottom)  $\phi \pi^+$  mass for (left)  $D_{(s)}^+$  and (right)  $D_{(s)}^-$  candidates, with fit projections overlaid. The insets in the top plots show the mass distributions zoomed around the  $D_s^+ \to K_{\rm S}^0 \pi^+$  signal region.



Figure 4: Distributions of the raw asymmetry as a function of the (top)  $K_{\rm S}^0 \pi^+$ , (middle)  $K_{\rm S}^0 K^+$ and (bottom)  $\phi \pi^+$  mass for  $D_{(s)}^+$  candidates, with fit projections overlaid. The variation of the raw asymmetry follows the relative proportions between signal, control and background decays as a function of mass and reflects the differences between the mass shapes of  $D_{(s)}^+$  and  $D_{(s)}^$ candidates.



Figure 5: Comparison between normalized and background-subtracted distributions of (top) transverse momentum, (middle) pseudorapidity and (bottom) azimuthal angle for  $D^+$  candidates from the  $D^+ \rightarrow \phi \pi^+$  and  $D^+ \rightarrow K_{\rm S}^0 \pi^+$  samples, (left) before and (right) after the kinematic weighting targeting the measurement of  $\mathcal{A}_{CP}(D^+ \rightarrow \phi \pi^+)$ . Only magnet-down 2016 data are shown.



Figure 6: Comparison between normalized and background-subtracted distributions of (top) transverse momentum, (middle) pseudorapidity and (bottom) azimuthal angle for the companion pion candidates from the  $D^+ \to \phi \pi^+$  and  $D^+ \to K_{\rm S}^0 \pi^+$  samples, (left) before and (right) after the kinematic weighting targeting the measurement of  $\mathcal{A}_{CP}(D^+ \to \phi \pi^+)$ . Only magnet-down 2016 data are shown.



Figure 7: Comparison between normalized and background-subtracted distributions of the kaon momentum for the kaon with same and opposite charge as the  $D^+$  candidate for the  $D^+ \rightarrow \phi \pi^+$  sample. Only magnet-down 2016 data are shown.



Figure 8: Comparison between  $\mathcal{A}_{CP}$  results in different data-taking years and magnet polarities. The uncertainties are statistical only. The red band corresponds to the result obtained from the integrated sample.