## Supplementary material for LHCb-PAPER-2020-029

Figure 1 shows the invariant-mass distributions of the  $K^+\pi^-$  and  $K^-\pi^+$  samples. The data are fit with the mass model described in the paper. In Fig. 2, the decay-time distributions of the background-subtracted samples of  $B^0 \to K^+\pi^-$  decays used to determine the decay-time efficiency are shown. The result of the best fit overlaid on the data points and the shape of the decay-time efficiency is also shown with an arbitrary scale. In Fig. 3 the dependence of the slope of the combinatorial background model on decay time is shown for the  $\pi^+\pi^-$  sample. The red line represents the result of an unbinned fit, while the points are obtained by fitting the invariant mass distribution in bins of decay time.

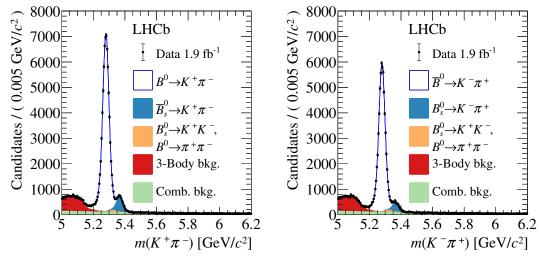


Figure 1: Distributions of the (left)  $K^+\pi^-$  and (right)  $K^-\pi^+$  invariant mass of the selected candidates in the  $K^+\pi^-$  and  $K^-\pi^+$  samples. The result of the best fit is overlaid on the data points. The various components contributing to the fit model are drawn as stacked histograms.

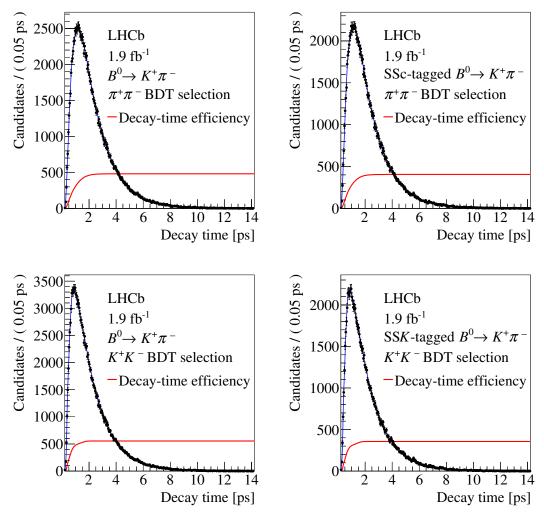


Figure 2: Distributions of decay time for the background subtracted  $B^0 \to K^+\pi^-$  samples used to determine the decay-time efficiency. The top figures show the sample satisfying the BDT selection optimised to select  $B^0 \to \pi^+\pi^-$  decays, while in the bottom figures show the sample satisfying the BDT selection optimised for  $B^0_s \to K^+K^-$  decays. On the left, the total samples are shown, while on the right only the events with a (top) SSc-tagging or (bottom) SSK-tagging assignment are used. The result of the fit is overlaid and the shape of the decay-time efficiency is also shown with arbitrary scale.

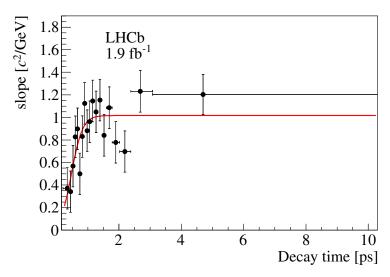


Figure 3: Dependence of the slope of the exponential function used to model the mass shape of the combinatorial background candidates on decay time for the  $\pi^+\pi^-$  sample, selected with the BDT optimised for  $B^0 \to \pi^+\pi^-$  decays. The red line is the result of a two-dimensional unbinned maximum likelihood fit to the mass and decay-time, while the points are obtained by fitting the invariant mass distribution in bins of decay time.