## Supplementary material for LHCb-PAPER-2020-038

This appendix contains supplementary material that will be posted on the public CDS record but will not appear in the paper.



Figure 1: Distributions of the charged and neutral BDT classifiers for the  $B_s^0 \to K^- \mu^+ \nu_{\mu}$  signal and various background components.

Table 1: Absolute branching fractions with uncertainty split according to the sources. The signal yield uncertainty  $\sigma(N_K)$  and the experimental systematic uncertainty (exp. syst.) are uncorrelated. The uncertainty from the normalization yield,  $\sigma(N_{D_s})$ , and all other sources are fully correlated between low and high  $q^2$  bins. The external sources (ext.) combine the uncertainties on  $\mathcal{B}(D_s^- \to K^+ K^- \pi^-)$ ,  $\tau_{B_s}$ ,  $|V_{cb}|$  and  $\mathrm{FF}_{D_s}$ .

Baseline $(\times 10^{-4})$	$\mathcal{B}(B^0_s \to K^- \mu^+ \nu_\mu)$	$\sigma(N_K)$	$\sigma(N_{D_s})$	exp. syst.	ext.
All $q^2$	1.06	0.04	0.03	0.04	0.07
$q^2 < 7 \mathrm{GeV^2/c^4}$	0.36	0.01	0.01	0.02	0.02
$q^2 > 7 \mathrm{GeV}^2/\mathrm{c}^4$	0.70	0.04	0.02	0.04	0.05

Table 2: Efficiency ratio  $\epsilon_K/\epsilon_{D_s}$  and fitted yields.

MILC2019, A. Bazavov <i>et al.</i> [1]	$\epsilon_K/\epsilon_{D_s}$	$B_s^0 \to K^- \mu^+ \nu_\mu$ Yield
All $q^2$	$0.670\pm0.008$	$13895 \pm 491$
$q^2 < 7 \mathrm{GeV}^2/\mathrm{c}^4$	$1.160\pm0.021$	$7497 \pm 325$
$q^2 > 7 \text{GeV}^2/\text{c}^4$	$0.517 \pm 0.008$	$6398 \pm 368$

Table 3: Efficiency ratio  $\epsilon_K/\epsilon_{D_s}$  and fitted yields.

HPQCD2015, C. Bouchard <i>et al.</i> [2]	$\epsilon_K/\epsilon_{D_s}$	$B_s^0 \to K^- \mu^+ \nu_\mu$ Yield
All $q^2$	$0.820\pm0.011$	$13249\pm467$
$q^2 < 7 \mathrm{GeV}^2/\mathrm{c}^4$	$1.104\pm0.017$	$6712 \pm 285$
$q^2 > 7 \mathrm{GeV}^2/\mathrm{c}^4$	$0.606 \pm 0.010$	$6537 \pm 370$

Table 4: Efficiency ratio  $\epsilon_K/\epsilon_{D_s}$  and fitted yields.

UKQCD2014, J. M. Flynn et al. [3]	$\epsilon_K/\epsilon_{D_s}$	$B_s^0 \to K^- \mu^+ \nu_\mu$ Yield
All $q^2$	$0.704 \pm 0.010$	$13726 \pm 482$
$q^2 < 7 \mathrm{GeV}^2/\mathrm{c}^4$	$1.150\pm0.020$	$7307\pm313$
$q^2 > 7 \mathrm{GeV}^2/\mathrm{c}^4$	$0.539 \pm 0.009$	$6419\pm366$

Table 5: Efficiency ratio  $\epsilon_K/\epsilon_{D_s}$  and fitted yields.

LCSR, Khodjamirian and Rusov [4]	$\epsilon_K/\epsilon_{D_s}$	$B_s^0 \to K^- \mu^+ \nu_\mu$ Yield
All $q^2$	$0.795\pm0.011$	$13412\pm466$
$q^2 < 7 \mathrm{GeV}^2/\mathrm{c}^4$	$1.120\pm0.018$	$6922 \pm 294$
$q^2 > 7 \mathrm{GeV}^2/\mathrm{c}^4$	$0.602\pm0.010$	$6490\pm362$



Figure 2: Fits to the  $K^+K^-\pi^+$  mass in intervals of  $m_{\rm corr}$  (in MeV/ $c^2$ ), in preparation to the fit to the  $B_s^0 \to D_s^-\mu^+\nu_\mu$  normalization channel.



Figure 3: Projections of the  $B_s^0 \to K^- \mu^+ \nu_{\mu}$  fit on control variables for (left) low and (right) high  $q^2$  ranges.



Figure 4:  $|V_{ub}|^{-2} \frac{d\Gamma(B_s \to K\mu\nu)}{dq^2}$  distribution for different LQCD [1–3] and LCSR [4] models.



Figure 5: PDG averages [5] (based on exclusive  $B \to \pi \ell \nu$ ,  $B \to D^* \ell \nu$  and inclusive  $B \to X_{u,c} \ell \nu$  results) and measurements of  $|V_{ub}|/|V_{cb}|$  shown in the  $(|V_{cb}|, |V_{ub}|)$  plane.

## References

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- [5] Particle Data Group, P. A. Zyla et al., Review of particle physics, Prog. Theor. Exp. Phys. 2020 (2020) 083C01.