## Supplementary material for LHCb-PAPER-2020-040

This appendix contains supplementary material that will posted on the public CDS record but will not appear in the paper. Figure 1 shows the fit to  $B^+ \to J/\psi K^+$  decays used to determine nuisance asymmetries. Figure 2 is the invariant mass of all candidates after the trigger and stripping, before additional cuts and BDT selection. Figures 3-4 are critical variables to the event selection, used in the trigger, BDT, or both. The signal component corresponds to simulated  $B^+ \to K^+\pi^0$  decays, background is data from the mass sidebands ( $m(K^+\pi^0) < 4860 \text{ MeV}/c^2$ ,  $m(K^+\pi^0) > 5700 \text{ MeV}/c^2$ ). Figure 11 shows the raw  $B^+ \to K^+\pi^0$  asymmetry between years and magnet polarities. Figure 12 shows the topology of the  $B^+ \to K^+\pi^0$  decay.



Figure 1: Invariant mass distribution of  $B^+ \to J/\psi K^+$  candidates used to correct for nuisance asymmetries. The data is divided by the charge of the *B* meson, with  $B^+ \to J/\psi K^+$  shown on the left and  $B^- \to J/\psi K^-$  on the right.



Figure 2: Invariant mass distribution of  $B^+ \to K^+ \pi^0$  candidates after the initial candidate selection. The roll-off at high mass is due to differences in the online and offline reconstruction, particularly of neutral pions.



Figure 3: Signal and background distributions of DOCA- $\chi^2$  after the initial candidate selection, normalized to unit area.



Figure 4: Signal and background distributions of multiplicity of tracks in a cone of  $\Delta R = 1.7$ around the  $B^+$  candidate trajectory after initial event selection, normalized to unit area.



Figure 5: Signal and background distributions of  $p_T$  asymmetry in a cone of  $\Delta R = 1.7$  around the  $B^+$  candidate trajectory after the initial candidate selection, normalized to unit area.



Figure 6: Signal and background distributions of the smallest  $\chi^2$  of vertex formed by adding one additional track to the  $K^+$  after the initial candidate selection, normalized to unit area.



Figure 7: Signal and background distributions of the smallest  $\Delta \chi^2$  of vertex formed by adding a second track to the lowest  $\chi^2$  vertex formed by adding one additional track to the  $K^+$  after the initial candidate selection, normalized to unit area.



Figure 8: Signal and background distributions of the smallest change in  $\chi^2$  of the PV when including the  $K^+$  in the vertex fit after the initial candidate selection, normalized to unit area.



Figure 9: Signal and background distributions of  $K^+ p_T$  after the initial candidate selection, normalized to unit area.



Figure 10: Signal and background distributions of multiplicity of vertices having small  $\chi^2$  after the initial candidate selection, normalized to unit area.



Figure 11: Raw asymmetries for  $B^+ \to K^+ \pi^0$  separated by year and whether the magnetic field is aligned vertically upwards (MU) or downwards (MD)



Figure 12: Diagram of the  $B^+ \to K^+ \pi^0$  decay topology. The solid blue line represents the  $K^+$  track, and the wide dashed purple line represents the reconstructed  $\pi^0$  momentum. They are combined to form the  $B^+$  momentum trajectory shown as a narrow red dashed line, which is assumed to originate from the primary vertex also shown in red.