## Supplementary material for LHCb-PAPER-2020-040

This appendix contains supplementary material that will posted on the public CDS record but will not appear in the paper. Figure 1 shows the fit to $B^{+} \rightarrow J / \psi K^{+}$decays used to determine nuisance asymmetries. Figure 2 is the invariant mass of all candidates after the trigger and stripping, before additional cuts and BDT selection. Figures $3 \cdot 4$ are critical variables to the event selection, used in the trigger, BDT, or both. The signal component corresponds to simulated $B^{+} \rightarrow K^{+} \pi^{0}$ decays, background is data from the mass sidebands $\left(m\left(K^{+} \pi^{0}\right)<4860 \mathrm{MeV} / c^{2}, m\left(K^{+} \pi^{0}\right)>5700 \mathrm{MeV} / c^{2}\right)$. Figure 11 shows the raw $B^{+} \rightarrow K^{+} \pi^{0}$ asymmetry between years and magnet polarities. Figure 12 shows the topology of the $B^{+} \rightarrow K^{+} \pi^{0}$ decay.


Figure 1: Invariant mass distribution of $B^{+} \rightarrow J / \psi K^{+}$candidates used to correct for nuisance asymmetries. The data is divided by the charge of the $B$ meson, with $B^{+} \rightarrow J / \psi K^{+}$shown on the left and $B^{-} \rightarrow J / \psi K^{-}$on the right.


Figure 2: Invariant mass distribution of $B^{+} \rightarrow K^{+} \pi^{0}$ candidates after the initial candidate selection. The roll-off at high mass is due to differences in the online and offline reconstruction, particularly of neutral pions.


Figure 3: Signal and background distributions of DOCA- $\chi^{2}$ after the initial candidate selection, normalized to unit area.


Figure 4: Signal and background distributions of multiplicity of tracks in a cone of $\Delta R=1.7$ around the $B^{+}$candidate trajectory after initial event selection, normalized to unit area.


Figure 5: Signal and background distributions of $p_{T}$ asymmetry in a cone of $\Delta R=1.7$ around the $B^{+}$candidate trajectory after the initial candidate selection, normalized to unit area.


Figure 6: Signal and background distributions of the smallest $\chi^{2}$ of vertex formed by adding one additional track to the $K^{+}$after the initial candidate selection, normalized to unit area.


Figure 7: Signal and background distributions of the smallest $\Delta \chi^{2}$ of vertex formed by adding a second track to the lowest $\chi^{2}$ vertex formed by adding one additional track to the $K^{+}$after the initial candidate selection, normalized to unit area.


Figure 8: Signal and background distributions of the smallest change in $\chi^{2}$ of the PV when including the $K^{+}$in the vertex fit after the initial candidate selection, normalized to unit area.


Figure 9: Signal and background distributions of $K^{+} p_{\mathrm{T}}$ after the initial candidate selection, normalized to unit area.


Figure 10: Signal and background distributions of multiplicity of vertices having small $\chi^{2}$ after the initial candidate selection, normalized to unit area.


Figure 11: Raw asymmetries for $B^{+} \rightarrow K^{+} \pi^{0}$ separated by year and whether the magnetic field is aligned vertically upwards (MU) or downwards (MD)


Figure 12: Diagram of the $B^{+} \rightarrow K^{+} \pi^{0}$ decay topology. The solid blue line represents the $K^{+}$ track, and the wide dashed purple line represents the reconstructed $\pi^{0}$ momentum. They are combined to form the $B^{+}$momentum trajectory shown as a narrow red dashed line, which is assumed to originate from the primary vertex also shown in red.

