

1 Supplementary material for LHCb-PAPER-2020-047

3 This appendix contains supplementary material that will be posted on the public CDS
4 record but will not appear in the paper.

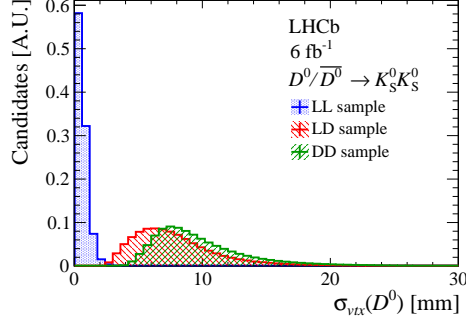


Figure 1: D^0 decay vertex uncertainty distribution for $D^0 \rightarrow K_S^0 K_S^0$ decays. The distributions are plotted in blue for the LL sample, in red for the LD sample and in green for the DD sample.

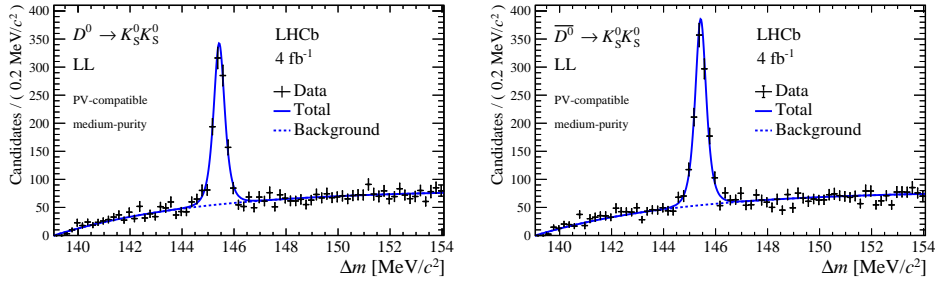


Figure 2: Distributions and fit projections of Δm for D^0 (left) and \bar{D}^0 (right) decays for the 2017–2018 PV-compatible LL sample.

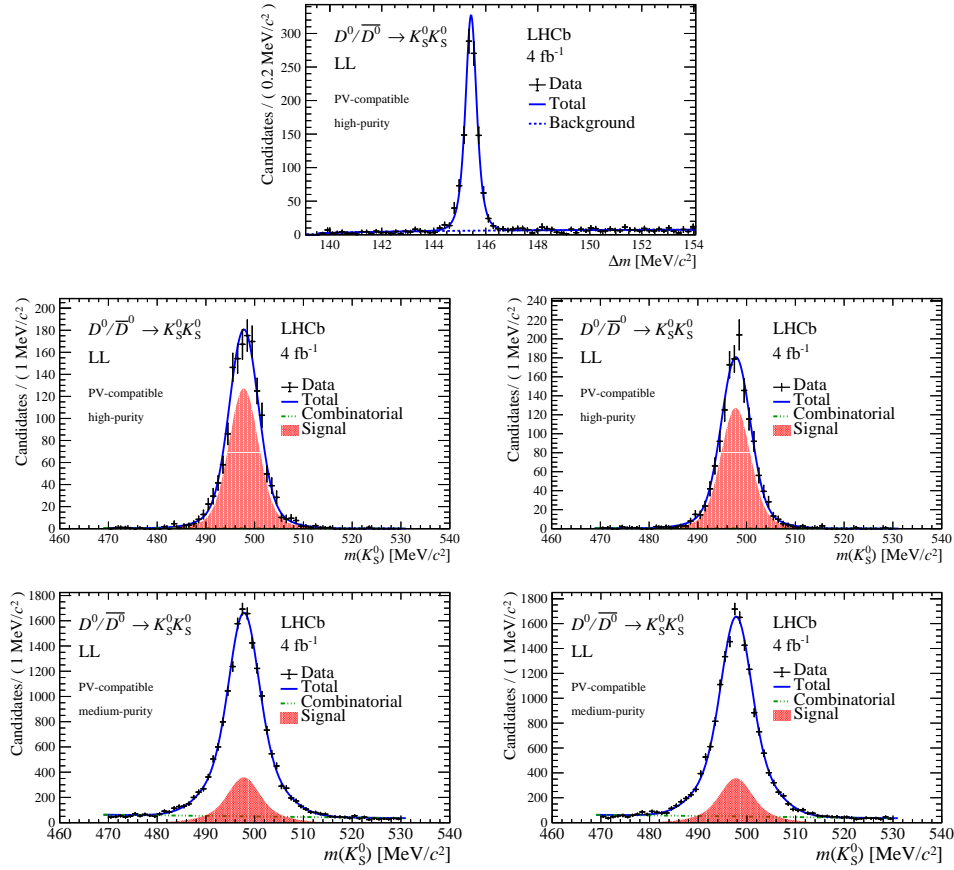


Figure 3: Charge-integrated mass distributions and fit projections for LL PV-compatible sample (2017–2018 data). Top: Δm distribution. Middle and bottom: $m(K_S^0)$ distributions for the two K_S^0 candidates (left, right) in the two ranges of classifier output. The mass range for L K_S^0 candidates is limited to $[470, 530]$ MeV/c^2 by a trigger requirement.

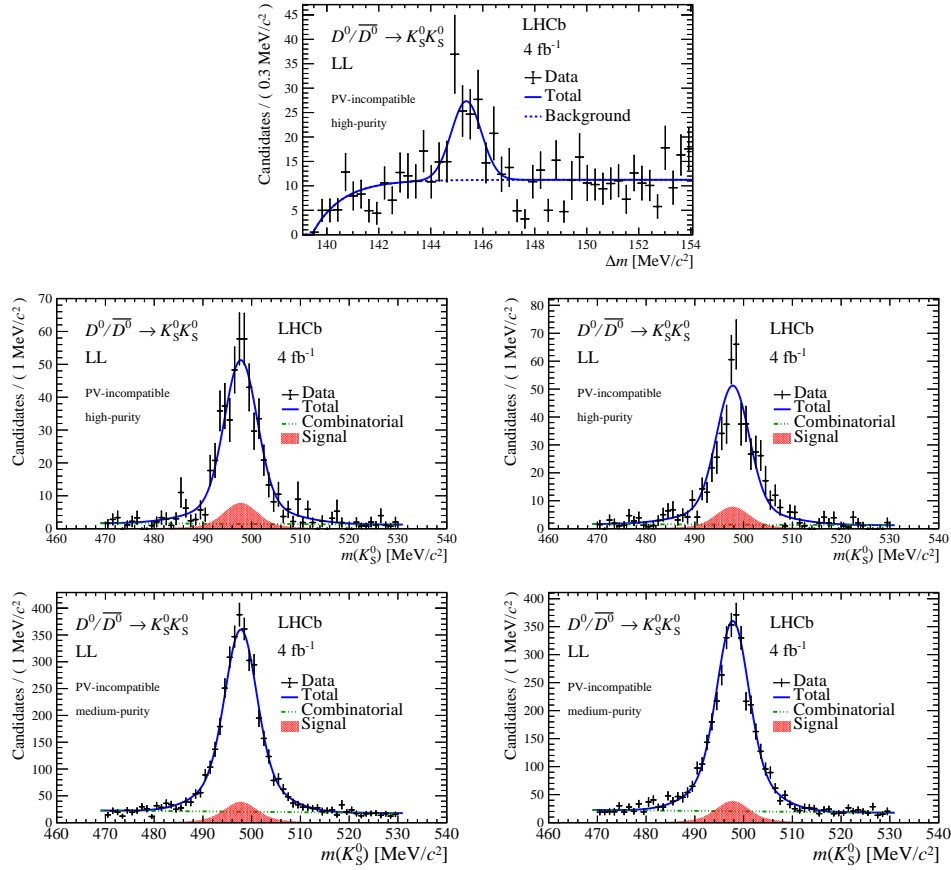


Figure 4: Charge-integrated mass distributions and fit projections for LL PV-incompatible sample (2017–2018 data). Top: Δm distribution. Middle and bottom: $m(K_S^0)$ distributions for the two K_S^0 candidates (left, right) in the two ranges of classifier output. The mass range for L K_S^0 candidates is limited to $[470, 530]$ MeV/c^2 by a trigger requirement.

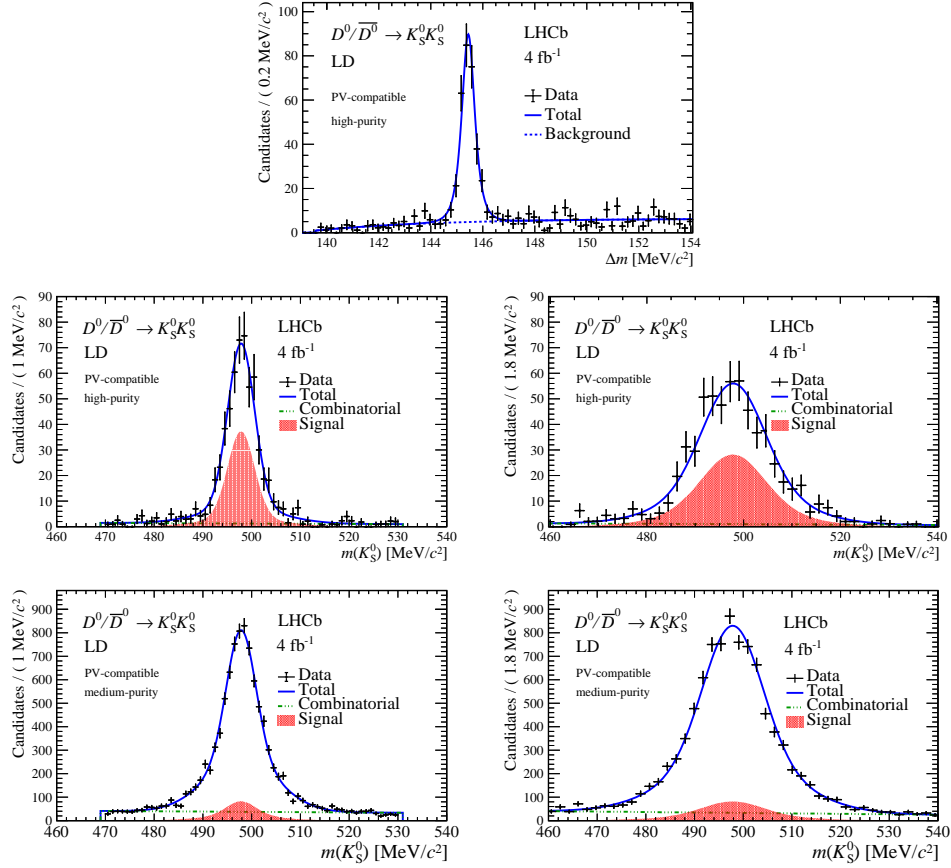


Figure 5: Charge-integrated mass distributions and fit projections for LD PV-compatible sample (2017–2018 data). Top: Δm distribution. Middle and bottom: $m(K_S^0)$ distributions for L (left) and D (right) K_S^0 candidates in the two ranges of classifier output. The mass range for L K_S^0 candidates is limited to $[470, 530]$ MeV/c^2 by a trigger requirement.

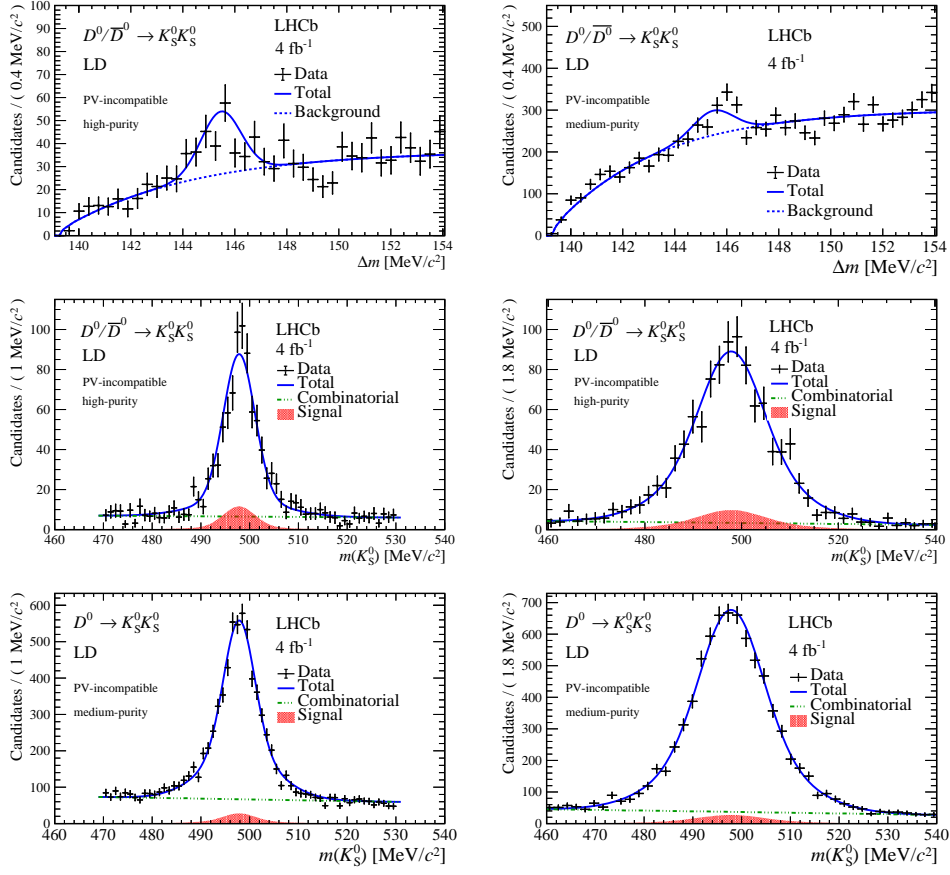


Figure 6: Charge-integrated mass distributions and fit projections for LD PV-incompatible sample (2017–2018 data). Top: Δm distribution for the two ranges of classifier output. Middle and bottom: $m(K_S^0)$ distributions for L (left) and D (right) K_S^0 candidates in the two ranges of classifier output. The mass range for L K_S^0 candidates is limited to $[470, 530]$ MeV/c^2 by a trigger requirement.

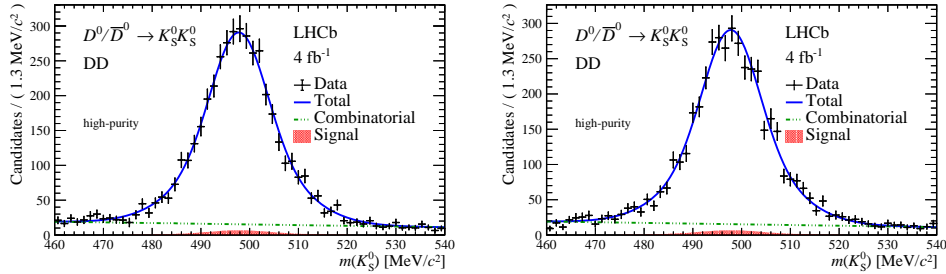


Figure 7: Charge-integrated mass distributions for the two K_S^0 in the DD sample (2017–2018 data).

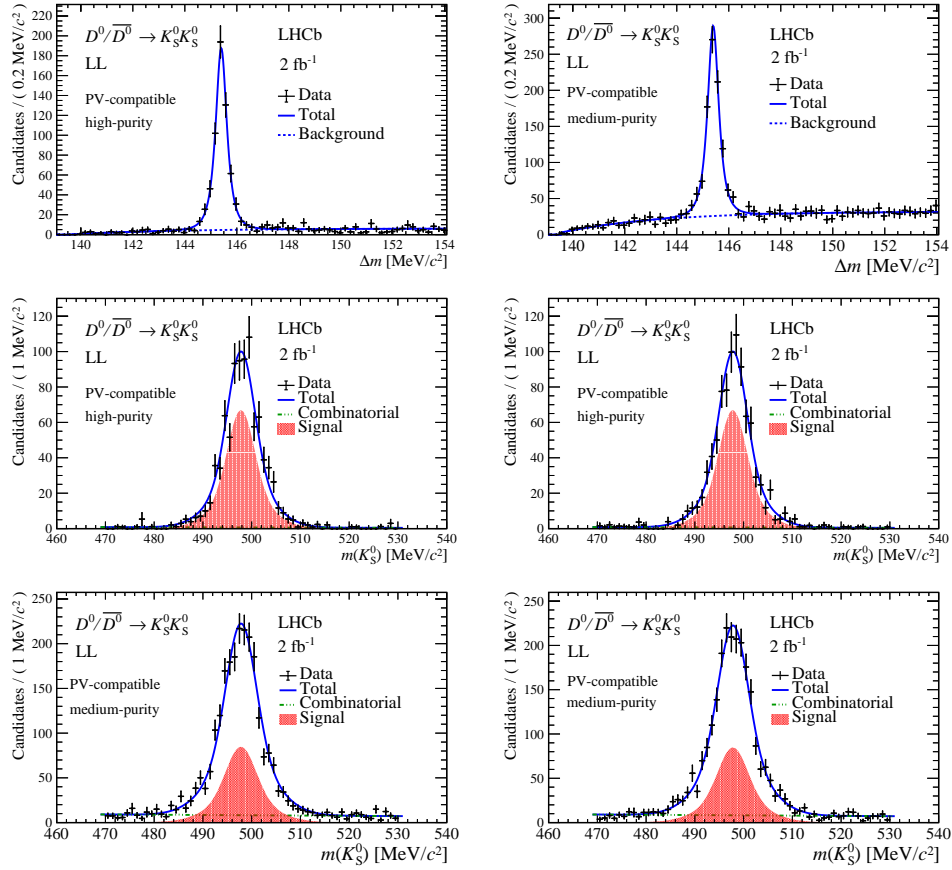


Figure 8: Charge-integrated mass distributions and fit projections for LL PV-compatible sample (2015–2016 data). Top: Δm distributions for the two ranges of classifier output. Middle and bottom: $m(K_S^0)$ distributions for the two K_S^0 candidates (left, right) in the two ranges of classifier output. The mass range for L K_S^0 candidates is limited to $[470, 530]$ MeV/c^2 by a trigger requirement.

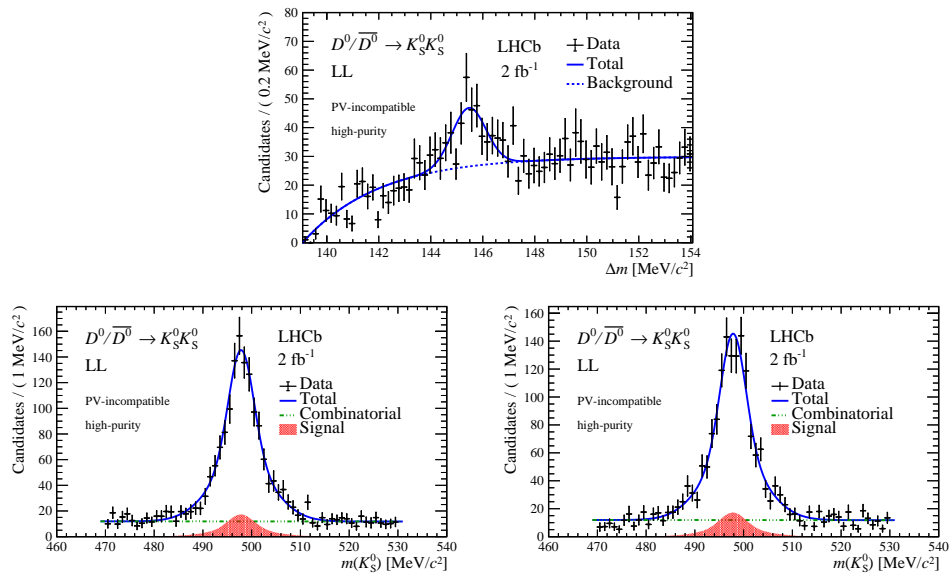


Figure 9: Charge-integrated mass distributions and fit projections for LL PV-incompatible sample (2015–2016 data). Top: Δm distribution. Bottom: $m(K_S^0)$ distributions for the two K_S^0 candidates. The mass range for L K_S^0 candidates is limited to $[470, 530]$ MeV/c² by a trigger requirement.

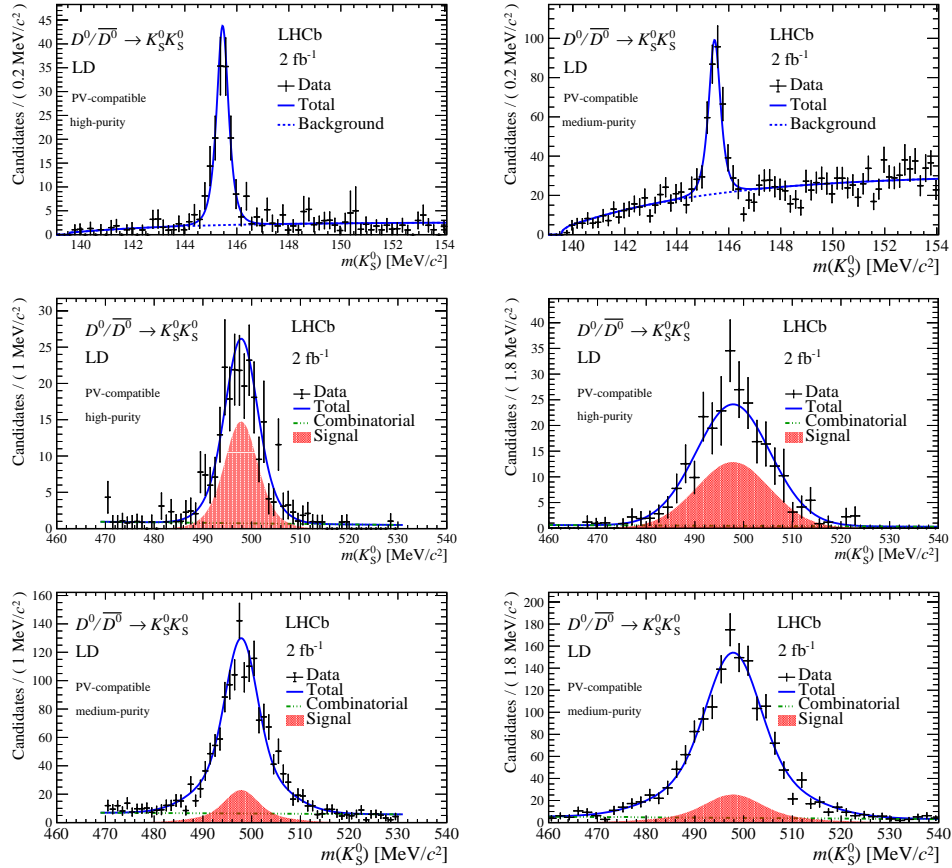


Figure 10: Charge-integrated mass distributions and fit projections for LD PV-compatible sample (2015–2016 data). Top: Δm distributions for the two ranges of classifier output. Middle and bottom: $m(K_S^0)$ distributions for L (left) and D (right) K_S^0 candidates in the two ranges of classifier output. The mass range for L K_S^0 candidates is limited to $[470, 530]$ MeV/c^2 by a trigger requirement.

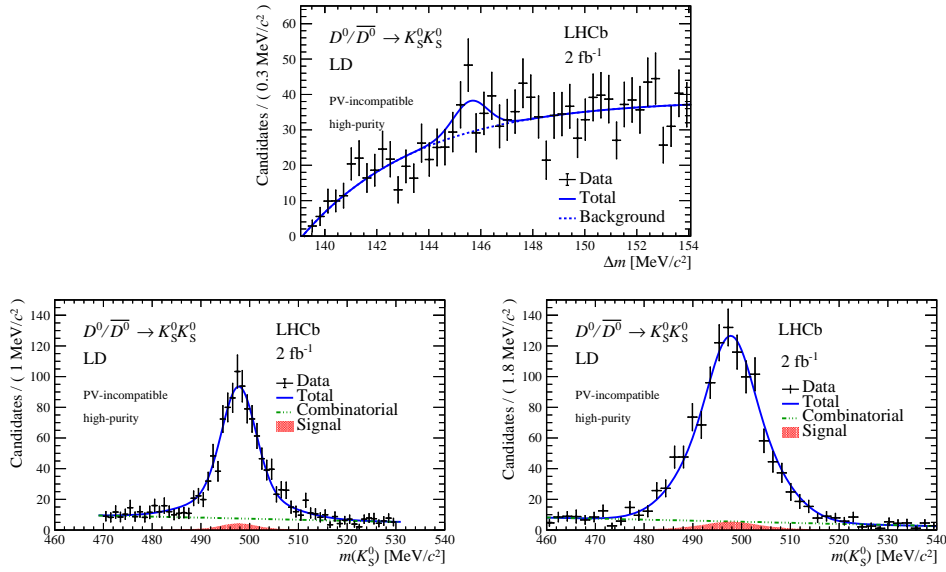


Figure 11: Charge-integrated mass distributions and fit projections for LD PV-incompatible sample (2015–2016 data). Top: Δm distribution. Bottom: $m(K_S^0)$ distributions for L (left) and D (right) K_S^0 candidates. The mass range for L K_S^0 candidates is limited to [470,530] MeV/c² by a trigger requirement.

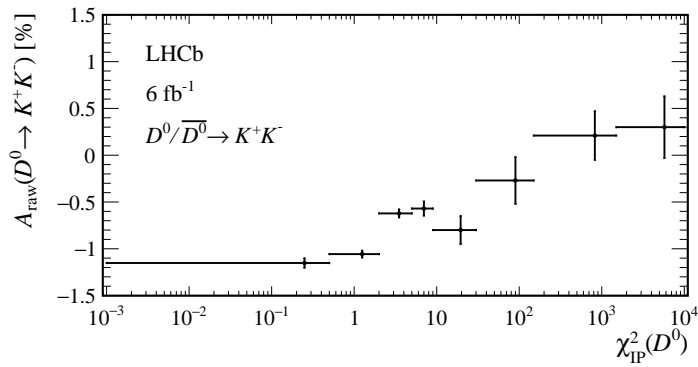


Figure 12: Raw asymmetry, evaluated on the $D^0 \rightarrow K^+ K^-$ calibration sample, as a function of $\chi_{\text{IP}}^2(D^0)$.

5 Here a step-by-step derivation of Eq. (3) is given. Start by noting that weighting
6 events with the weight defined by Eq. (3), in the limit of large numbers, leads to a total
7 number of events given by:

$$N_W^\pm = \int \frac{1 \pm \mathcal{A}^{CP}(K^+K^-)}{2n_C^\pm(\vec{p}_0)} [n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)] n_S^\pm(\vec{p}_0) d\vec{p}_0 \quad (1)$$

8 This is effectively a Monte Carlo approximation to the above integral.

9 The integrand in Eq. (1) can be rewritten as:

$$\frac{[1 \pm \mathcal{A}^{CP}(K^+K^-)][n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)]}{2N \int \epsilon_C^\pm(\vec{p}_0, \vec{p}_*) p_C^\pm(\vec{p}_0, \vec{p}_*) d\vec{p}_*} n_S^\pm(\vec{p}_0), \quad (2)$$

10 where N is the number of pp collisions, $\epsilon_C^\pm(\vec{p}_0, \vec{p}_*)$ is the overall efficiency for detecting a
11 $D^{*\pm}$ in the calibration channel, and $p_C^\pm(\vec{p}_0, \vec{p}_*)$ is the probability density for producing a
12 $D^{*\pm} \rightarrow D^0 \pi^\pm$ decay, where the $D^{*\pm}$ and the D^0 have respectively momentum \vec{p}_* and \vec{p}_0
13 and the given charge, with the subsequent $D^0 \rightarrow K^+K^-$ decay (the ‘‘C’’ suffix stands for
14 ‘‘calibration’’).

15 It can be noted that $\epsilon_C^\pm(\vec{p}_0, \vec{p}_*) = \epsilon^\pm(\vec{p}_*) \epsilon_C(\vec{p}_0)$, where $\epsilon_C(\vec{p}_0)$ is the efficiency for
16 detecting a D^0 decaying into the calibration channel and $\epsilon^\pm(\vec{p}_*)$ is the (relative) efficiency
17 for detecting a $D^{*\pm}$. The latter is assumed to be independent of D^0 kinematics and decay
18 mode (D^0 final states are charge-symmetric), but is allowed to have arbitrary dependency
19 on \vec{p}_* , and be different for each charge.

In addition, the following equality holds:

$$p_C^\pm(\vec{p}_0, \vec{p}_*) = p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) B_C (1 \pm \mathcal{A}^{CP}(K^+K^-)) / 2,$$

where $p_\pm(\vec{p}_*)$ is the probability density for producing a $D^{*\pm}$ of momentum \vec{p}_* in a single
 pp collision, $p_0(\vec{p}_0 | \vec{p}_*)$ is the probability density for producing a D^0 or \bar{D}^0 candidate with
a momentum \vec{p}_0 , conditional on \vec{p}_* , and B_C is the branching ratio. Therefore, the quantity
in Eq. (2) can be rewritten as

$$\begin{aligned} & \frac{[1 \pm \mathcal{A}^{CP}(K^+K^-)][n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)]}{2N \int \epsilon_C^\pm(\vec{p}_0, \vec{p}_*) p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) B_C / 2 (1 \pm \mathcal{A}^{CP}(K^+K^-)) d\vec{p}_*} n_S^\pm(\vec{p}_0) = \\ & = \frac{2n_S^\pm(\vec{p}_0)[n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)]}{2N \int \epsilon_C^\pm(\vec{p}_0, \vec{p}_*) p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) B_C d\vec{p}_*} = \\ & = \frac{2[n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)] \int \epsilon_S^\pm(\vec{p}_0, \vec{p}_*) p_S^\pm(\vec{p}_0, \vec{p}_*) d\vec{p}_*}{2B_C \int \epsilon_C^\pm(\vec{p}_0, \vec{p}_*) p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) d\vec{p}_*} \\ & = \frac{[n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)] \epsilon_S(\vec{p}_0) \int \epsilon^\pm(\vec{p}_*) p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) B_S (1 \pm \mathcal{A}^{CP}(K_S^0 K_S^0)) d\vec{p}_*}{2B_C \epsilon_C(\vec{p}_0) \int \epsilon^\pm(\vec{p}_*) p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) d\vec{p}_*}, \end{aligned}$$

20 where the ‘‘S’’ suffix labels the quantities already defined above, but for the ‘‘signal’’
21 $D^0 \rightarrow K_S^0 K_S^0$ sample. This can be further rewritten as

$$\frac{[n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)] \epsilon_S(\vec{p}_0) B_S (1 \pm \mathcal{A}^{CP}(S))}{2B_C \epsilon_C(\vec{p}_0)} \times \frac{\int \epsilon^\pm(\vec{p}_*) p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) d\vec{p}_*}{\int \epsilon^\pm(\vec{p}_*) p_\pm(\vec{p}_*) p_0(\vec{p}_0 | \vec{p}_*) d\vec{p}_*} =$$

$$= \frac{B_S \epsilon_S(\vec{p}_0)}{2B_C \epsilon_C(\vec{p}_0)} (1 \pm \mathcal{A}^{CP}(K_S^0 K_S^0)) (n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0))$$

22 and therefore

$$N_W^\pm = \frac{B_S}{2B_C} (1 \pm \mathcal{A}^{CP}(K_S^0 K_S^0)) \int \frac{\epsilon_S(\vec{p}_0)}{\epsilon_C(\vec{p}_0)} (n_C^+(\vec{p}_0) + n_C^-(\vec{p}_0)) d\vec{p}_0$$

23 Note that the above integral is now charge-independent, so it drops out in the event
24 number asymmetry:

$$\mathcal{A}^{CP}(K_S^0 K_S^0) = \frac{N_W^+ - N_W^-}{N_W^+ + N_W^-}, \quad (3)$$

25 leading to an unbiased estimate of the CP asymmetry of the channel of interest.