

1 Supplementary material for LHCb-PAPER-2021-009

This appendix contains supplementary material that will be posted on the public CDS record but will not appear in the paper.

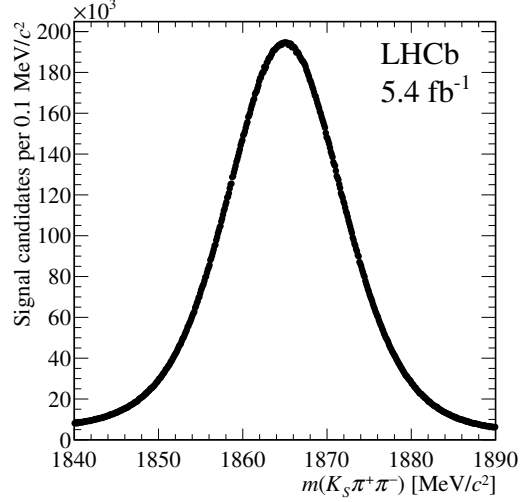


Figure 5: Distribution of $m(K_S^0 \pi^+ \pi^-)$ for the selected candidates.

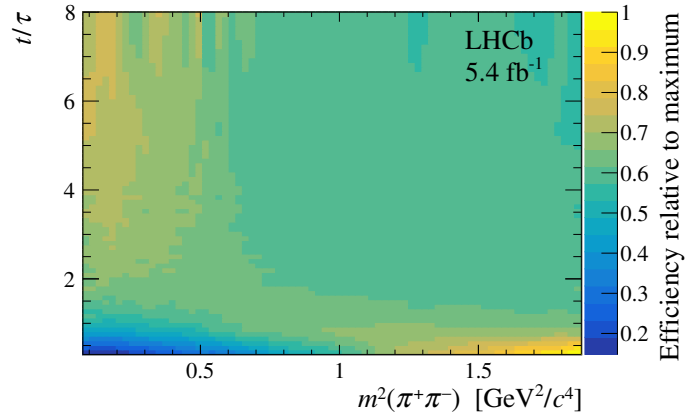


Figure 6: Smoothed efficiency as a function of the invariant mass of the two final-state pions, $m^2(\pi^+\pi^-)$, and the decay-time divided by the D^0 lifetime τ for $D^{*+} \rightarrow D^0(\rightarrow K_S^0 \pi^+ \pi^-) \pi^+$ decays, as determined from the data with downstream K_S^0 candidates. The correlation of decay-time and Dalitz-plot efficiency is predominantly in $m^2(\pi^+\pi^-)$, and is visualized by the change of color from low to high $m^2(\pi^+\pi^-)$.

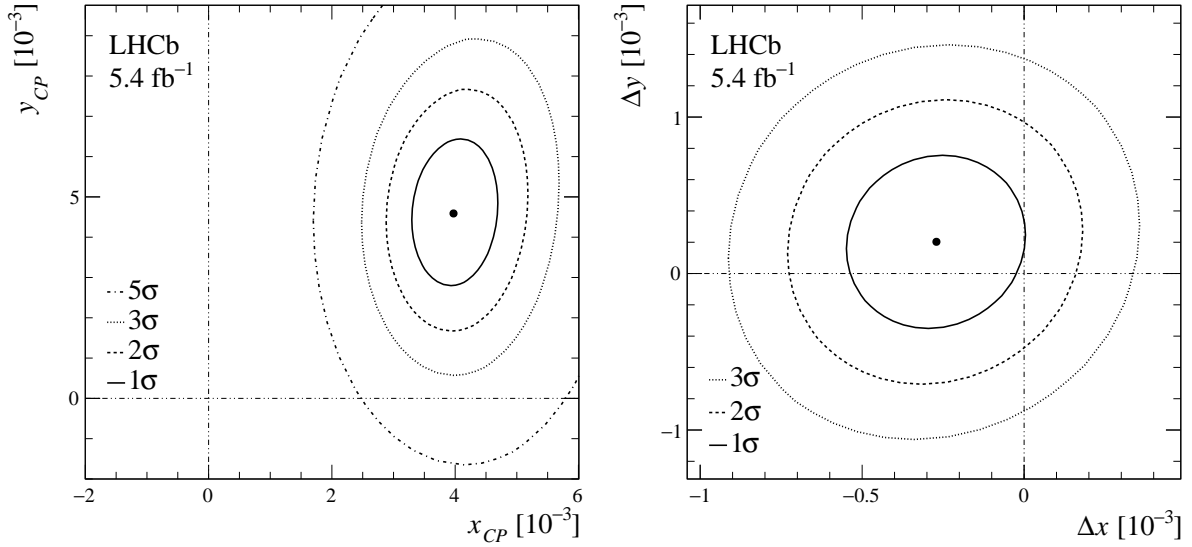


Figure 7: Two-dimensional confidence-level (CL) contours for 1, 2, and 3 Gaussian standard deviations, σ , in the (left) (x_{CP}, y_{CP}) and (right) $(\Delta x, \Delta y)$ planes considering only statistical uncertainties. In the left plot, the 5σ contour is shown in addition. The no-mixing hypothesis (indicated by the crossing lines in the left plot) is consistent with the data at 10.3σ . The CP symmetry hypothesis (indicated by the crossing lines in the right plot) is consistent at 1.1σ .

Table 6: Initial and final values and uncertainties for strong phase parameters, which are constrained in the fit to the values measured in Refs. [1,2]. The uncertainties are statistical only.

| | Initial | Final |
|-------|--------------------|--------------------|
| c_1 | 0.699 ± 0.020 | 0.702 ± 0.020 |
| c_2 | 0.643 ± 0.036 | 0.641 ± 0.036 |
| c_3 | 0.001 ± 0.047 | 0.006 ± 0.047 |
| c_4 | -0.608 ± 0.052 | -0.613 ± 0.052 |
| c_5 | -0.955 ± 0.023 | -0.955 ± 0.023 |
| c_6 | -0.578 ± 0.058 | -0.568 ± 0.058 |
| c_7 | 0.057 ± 0.057 | 0.047 ± 0.055 |
| c_8 | 0.411 ± 0.036 | 0.413 ± 0.036 |
| s_1 | 0.091 ± 0.063 | 0.014 ± 0.054 |
| s_2 | 0.300 ± 0.110 | 0.341 ± 0.094 |
| s_3 | 1.000 ± 0.075 | 0.956 ± 0.069 |
| s_4 | 0.660 ± 0.123 | 0.767 ± 0.112 |
| s_5 | -0.032 ± 0.069 | -0.073 ± 0.063 |
| s_6 | -0.545 ± 0.122 | -0.627 ± 0.106 |
| s_7 | -0.854 ± 0.095 | -0.828 ± 0.081 |
| s_8 | -0.433 ± 0.083 | -0.449 ± 0.072 |

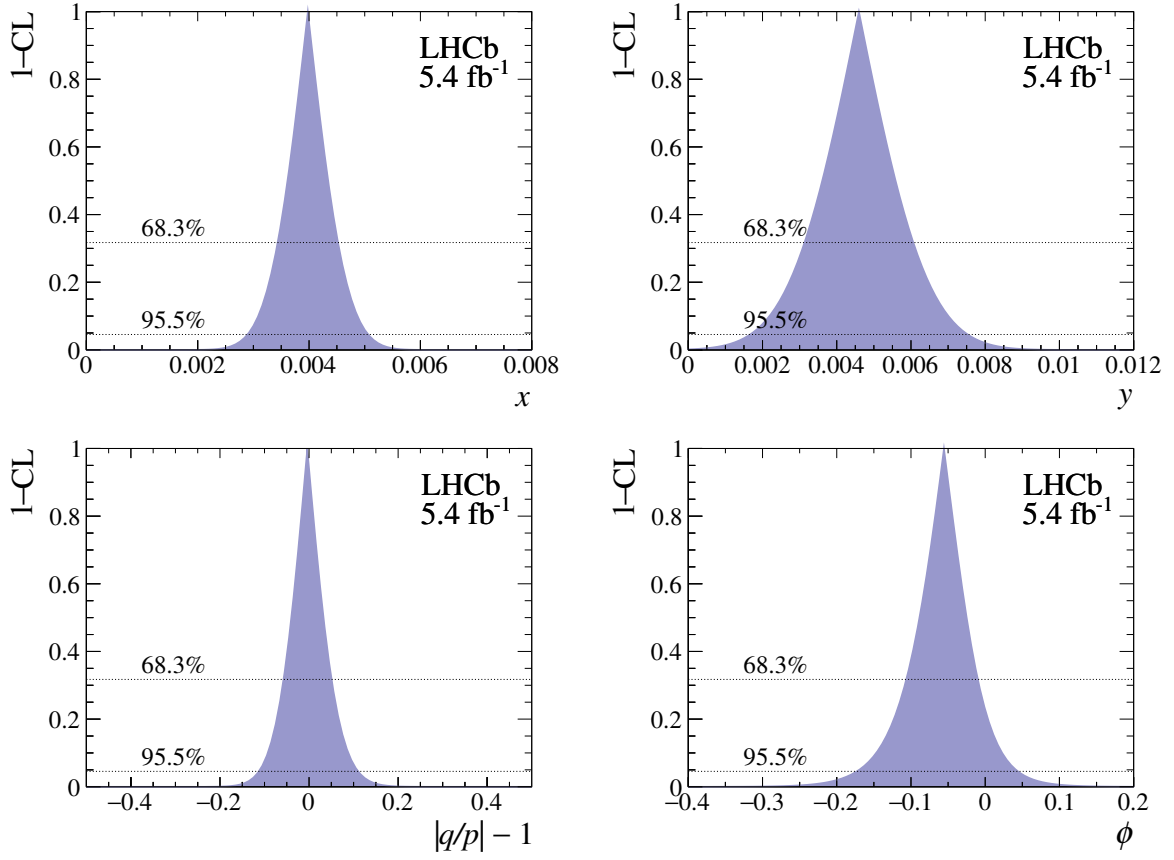


Figure 8: Distribution of $1 - \text{CL}$ for the derived parameters x , y , $|q/p|$, ϕ .

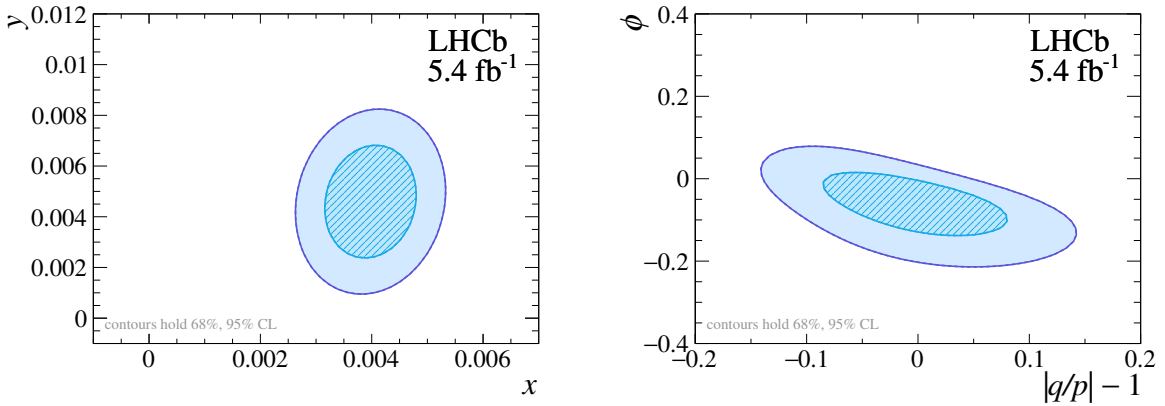


Figure 9: Two-dimensional constraints of the derived parameters (left) x , y and (right) $|q/p|$, ϕ .

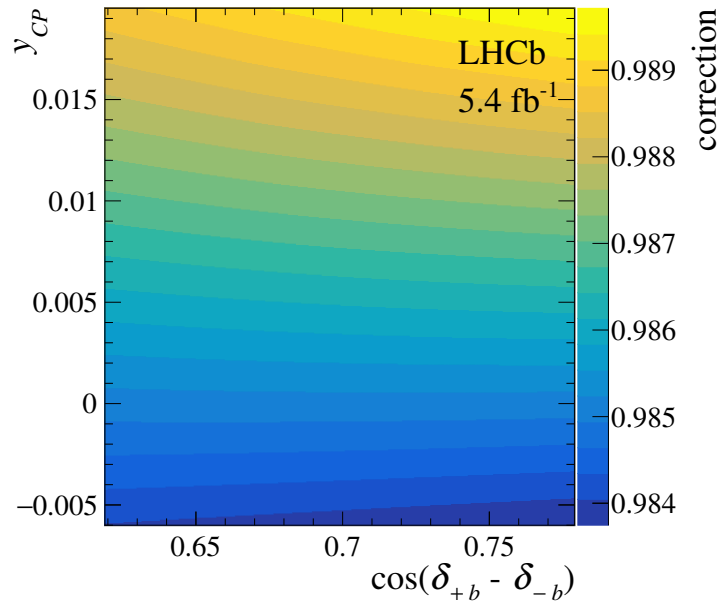


Figure 10: Example of a correction to remove the experimentally induced correlations between phase space and decay time. The shown correction corresponds to that of the first Dalitz bin, first decay time bin, and largest subsample (downstream, one-track).

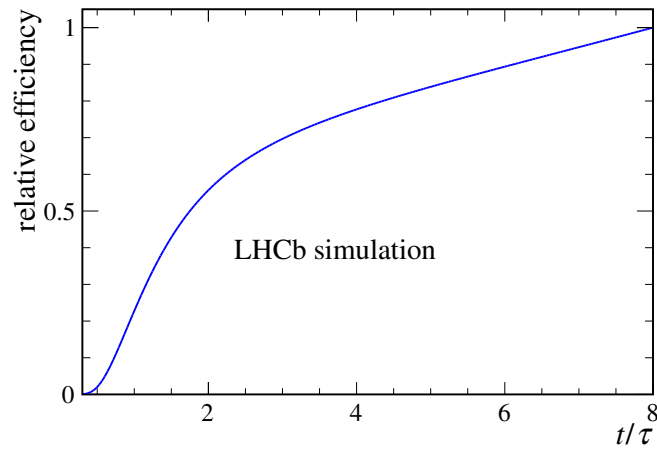


Figure 11: Relative efficiency as a function of decay time as determined from simulation for the downstream one-track sample.

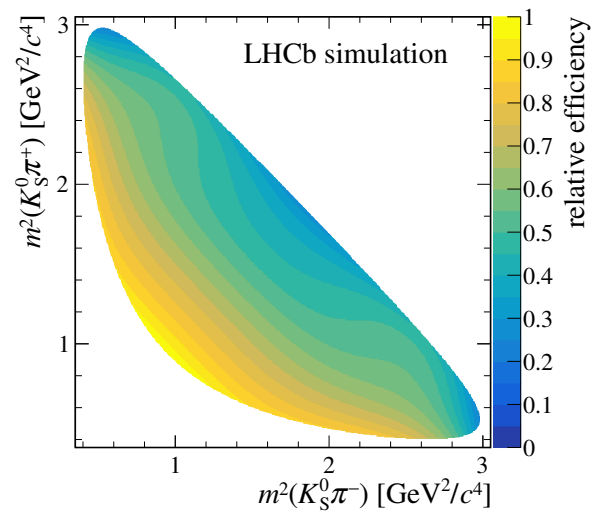


Figure 12: Relative efficiency as a function of the Dalitz-plot position as determined from simulation for the downstream one-track sample.

Updated world average of charm-mixing parameters

We combine the results presented in this Letter with current knowledge of charm-mixing parameters to assess their impact on the world average. The combination procedure follows closely the methods of the Heavy Flavor Averaging Group. In addition to the results presented in this Letter, the following measurements are included in the combination:

- LHCb collaboration, R. Aaij *et al.*, *Updated determination of D^0 - \bar{D}^0 mixing and CP violation parameters with $D^0 \rightarrow K^+\pi^-$ decays*, Phys. Rev. **D97** (2018) 031101, arXiv:1712.03220;
- Belle collaboration, B. R. Ko *et al.*, *Observation of D^0 - \bar{D}^0 mixing in e^+e^- collisions*, Phys. Rev. Lett. **112** (2014) 111801, Erratum *ibid.* **112** (2014) 139903, arXiv:1401.3402;
- CDF collaboration, T. Aaltonen *et al.*, *Observation of D^0 - \bar{D}^0 mixing using the CDF II detector*, Phys. Rev. Lett. **111** (2013) 231802, arXiv:1309.4078;
- BaBar collaboration, B. Aubert *et al.*, *Evidence for D^0 - \bar{D}^0 mixing*, Phys. Rev. Lett. **98** (2007) 211802, arXiv:hep-ex/0703020;
- CLEO collaboration, D. M. Asner *et al.*, *Updated measurement of the strong phase in $D^0 \rightarrow K^+\pi^-$ decay using quantum correlations in $e^+e^- \rightarrow D^0\bar{D}^0$ at CLEO*, Phys. Rev. **D86** (2012) 112001, arXiv:1210.0939;
- LHCb collaboration, R. Aaij *et al.*, *Measurement of the charm-mixing parameter y_{CP}* , Phys. Rev. Lett. **122** (2019) 011802, arXiv:1810.06874;
- Belle collaboration, M. Nayak *et al.*, *Measurement of the charm-mixing parameter y_{CP} in $D^0 \rightarrow K^0\omega$ at Belle*, Phys. Rev. **D102** (2020) 071102, arXiv:1912.10912;
- LHCb collaboration, R. Aaij *et al.*, *Search for time-dependent CP violation in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays*, Submitted to Phys. Rev. D (2021), arXiv:2105.09889;
- LHCb collaboration, R. Aaij *et al.*, *Updated measurement of decay-time-dependent CP asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays*, Phys. Rev. **D101** (2020) 012005, arXiv:1911.01114;
- Belle collaboration, M. Starič *et al.*, *Measurement of D^0 - \bar{D}^0 mixing and search for CP violation in $D^0 \rightarrow K^+K^-$, $\pi^+\pi^-$ decays with the full Belle data set*, Phys. Lett. **B753** (2016) 412, arXiv:1509.08266;
- LHCb collaboration, R. Aaij *et al.*, *Measurement of indirect CP asymmetries in $D^0 \rightarrow K^-K^+$ and $D^0 \rightarrow \pi^-\pi^+$ decays using semileptonic B decays*, JHEP **04** (2015) 043, arXiv:1501.06777;
- CDF collaboration, T. Aaltonen *et al.*, *Measurement of indirect CP-violating asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays at CDF*, Phys. Rev. **D90** (2014) 111103, arXiv:1410.5435;

- BaBar collaboration, J. P. Lees *et al.*, *Measurement of $D^0-\bar{D}^0$ mixing and CP violation in two-body D^0 decays*, Phys. Rev. **D87** (2013) 012004, arXiv:1209.3896;
- LHCb collaboration, R. Aaij *et al.*, *Measurement of mixing and CP violation parameters in two-body charm decays*, JHEP **04** (2012) 129, arXiv:1112.4698;
- LHCb collaboration, R. Aaij *et al.*, *Model-independent measurement of mixing parameters in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays*, JHEP **04** (2016) 033, arXiv:1510.01664;
- Belle collaboration, T. Peng *et al.*, *Measurement of $D^0-\bar{D}^0$ mixing and search for indirect CP violation using $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays*, Phys. Rev. **D89** (2014) 091103, arXiv:1404.2412;
- BaBar collaboration, P. del Amo Sanchez *et al.*, *Measurement of $D^0-\bar{D}^0$ mixing parameters using $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $D^0 \rightarrow K_S^0 K^+ K^-$ decays*, Phys. Rev. Lett. **105** (2010) 081803, arXiv:1004.5053;
- Belle collaboration, A. Zupanc *et al.*, *Measurement of y_{CP} in D^0 meson decays to the $K_S^0 K^+ K^-$ final state*, Phys. Rev. **D80** (2009) 052006, arXiv:0905.4185;
- LHCb collaboration, R. Aaij *et al.*, *First observation of $D^0-\bar{D}^0$ oscillations in $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays and measurement of the associated coherence parameters*, Phys. Rev. Lett. **116** (2016) 241801, arXiv:1602.07224;
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- BaBar collaboration, B. Aubert *et al.*, *Measurement of $D^0-\bar{D}^0$ mixing from a time-dependent amplitude analysis of $D^0 \rightarrow K^+ \pi^- \pi^0$ decays*, Phys. Rev. Lett. **103** (2009) 211801, arXiv:0807.4544;
- Belle collaboration, U. Bitenc *et al.*, *Improved search for $D^0-\bar{D}^0$ mixing using semileptonic decays at Belle*, Phys. Rev. **D77** (2008) 112003, arXiv:0802.2952;
- BaBar collaboration, B. Aubert *et al.*, *Search for $D^0-\bar{D}^0$ mixing using doubly flavor tagged semileptonic decay modes*, Phys. Rev. **D76** (2007) 014018, arXiv:0705.0704.
- LHCb collaboration, R. Aaij *et al.*, *Measurement of the mass difference between neutral charm-meson eigenstates*, Phys. Rev. Lett. **122** (2019) 231802, arXiv:1903.03074.

The results are reported in Table 7 and Fig. 13. The current world average includes the previous results [3]. The previous and current implementations of the bin-flip method share the external inputs from CLEO. This correlation is not accounted for in the presented average but is considered negligible as the precision of the previous result is much lower and the external inputs did not contribute significantly to the uncertainty.

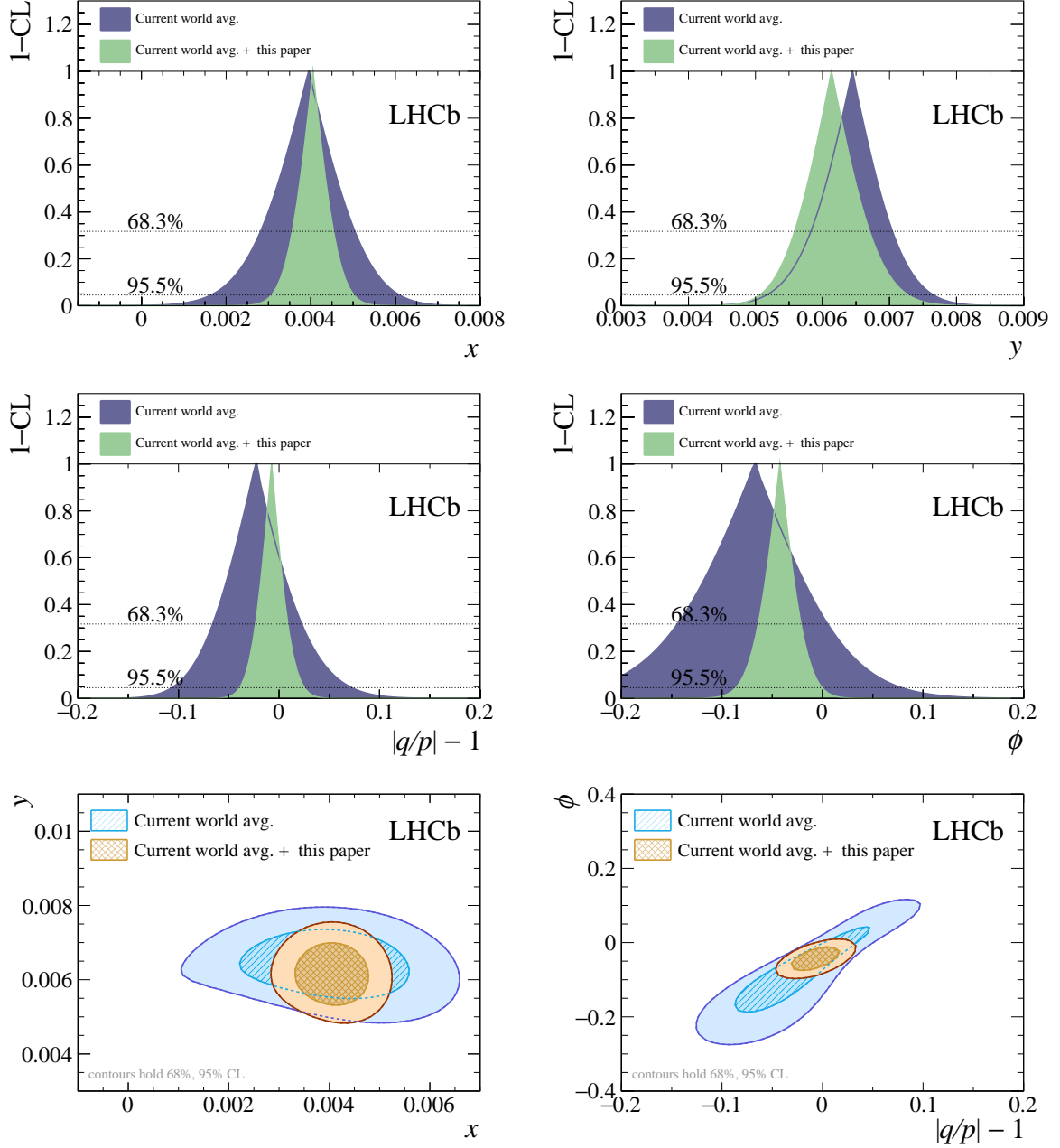


Figure 13: Impact of the results reported in this Letter on current global averages of charm-mixing parameters. The hatched and shaded areas in the bottom panels indicate the 68% and 95% confidence regions, respectively.

Table 7: Updated global combinations of charm-mixing measurements.

| Parameter | Value | Allowed interval | | |
|---------------|----------------------------|------------------|-----------------|-----------------|
| | | 68.3% CL | 95.5% CL | 99.7% CL |
| $x [10^{-2}]$ | $0.405^{+0.049}_{-0.049}$ | [0.307, 0.4502] | [0.26, 0.55] | [0.26, 0.57] |
| $y [10^{-2}]$ | $0.613^{+0.057}_{-0.055}$ | [0.558, 0.670] | [0.51, 0.73] | [0.46, 0.79] |
| $ q/p $ | $0.993^{+0.016}_{-0.016}$ | [0.977, 1.009] | [0.961, 1.025] | [0.944, 1.043] |
| ϕ | $-0.042^{+0.021}_{-0.022}$ | [-0.064, -0.021] | [-0.086, 0.000] | [-0.110, 0.021] |

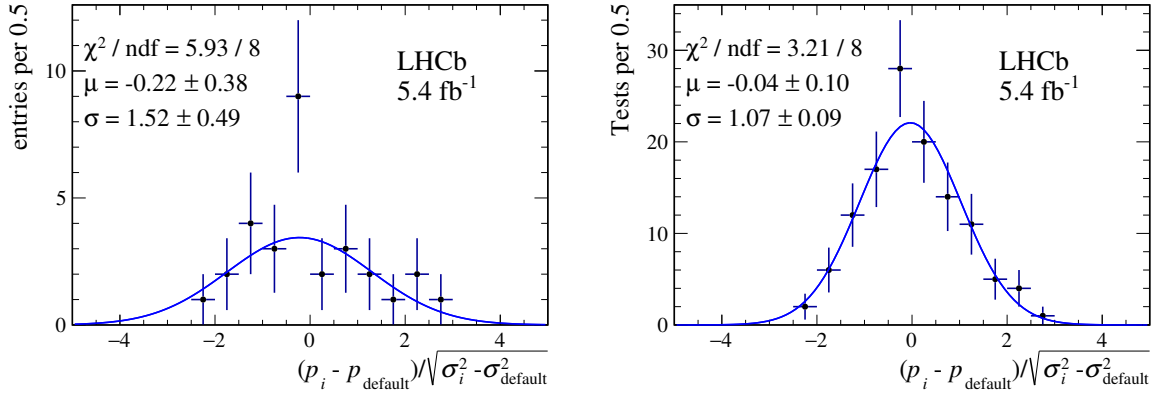


Figure 14: Pulls obtained when fitting subsamples of the data for (left) only x_{CP} and (right) x_{CP} , y_{CP} , Δx , and Δy . The subsamples are obtained by splitting in magnet polarity, trigger and K_S^0 category, data-taking period, D^{*+} meson kinematics, and impact parameter χ^2 of the D^0 candidate. The pull is calculated for each subsample by dividing the difference of the fit result, p_i , and the default result, p_{default} , with the uncorrelated part of the statistical uncertainty. The latter is obtained by the difference in quadrature of the fitted uncertainties: $\sqrt{\sigma_i^2 - \sigma_{\text{default}}^2}$. A Gaussian fit result is superimposed, and the obtained parameters and fit quality are displayed.

References

- [1] CLEO collaboration, J. Libby *et al.*, *Model-independent determination of the strong-phase difference between D^0 and $\bar{D}^0 \rightarrow K_{S,L}^0 h^+ h^-$ ($h = \pi, K$) and its impact on the measurement of the CKM angle γ/ϕ_3* , Phys. Rev. **D82** (2010) 112006, [arXiv:1010.2817](#).
- [2] BESIII collaboration, M. Ablikim *et al.*, *Model-independent determination of the relative strong-phase difference between D^0 and $\bar{D}^0 \rightarrow K_{S,L}^0 \pi^+ \pi^-$ and its impact on the measurement of the CKM angle γ/ϕ_3* , Phys. Rev. **D101** (2020) 112002, [arXiv:2003.00091](#).
- [3] LHCb collaboration, R. Aaij *et al.*, *Measurement of the mass difference between neutral charm-meson eigenstates*, Phys. Rev. Lett. **122** (2019) 231802, [arXiv:1903.03074](#).