

# Supplementary material for LHCb-PAPER-2022-016

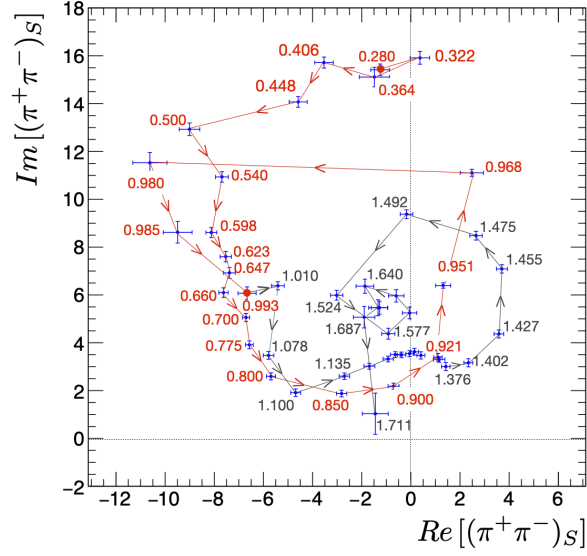


Figure 7: Argand plot of the  $\pi^-\pi^+$  S-wave amplitude as obtained from the Dalitz plot fit of  $D^+ \rightarrow \pi^-\pi^+\pi^+$  decays, with statistical uncertainties only. At each point, the number shown indicates the  $m(\pi^-\pi^+)$  value in GeV. The arrows indicate the flow of change of the S-wave amplitude. From  $m(\pi^-\pi^+) = 0.993$  GeV onward the line color changes from red to gray for better display. The point for the last knot at  $m_{\pi\pi} = 1.73$  MeV at  $(8.1 \pm 1.2, -1.5 \pm 2.3)$  is not included since it has very large uncertainty.

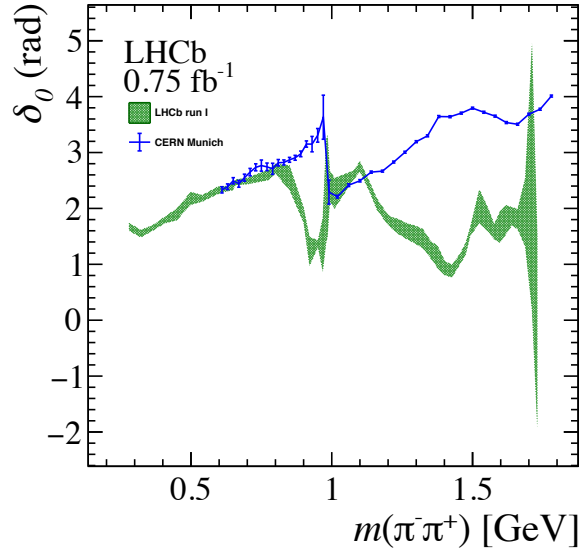


Figure 8: Comparison of the  $\pi^-\pi^+$  S-wave phase from  $D^+ \rightarrow \pi^-\pi^+\pi^+$  decay to that from  $\pi p \rightarrow \pi^-\pi^+n$ , where an offset of +1.35 rad has been added (data from the CERN-Munich experiment [7,8], summarised by W. Ochs in Ref. [6]).