Supplementary material for LHCb-PAPER-2023-034

This appendix contains supplementary material that will be posted on the public CDS record but will not appear in the paper.

Two examples of misidentified background contributions, which are removed in the selection are shown as invariant-mass distributions with swapped particle hypotheses in Fig. 1.

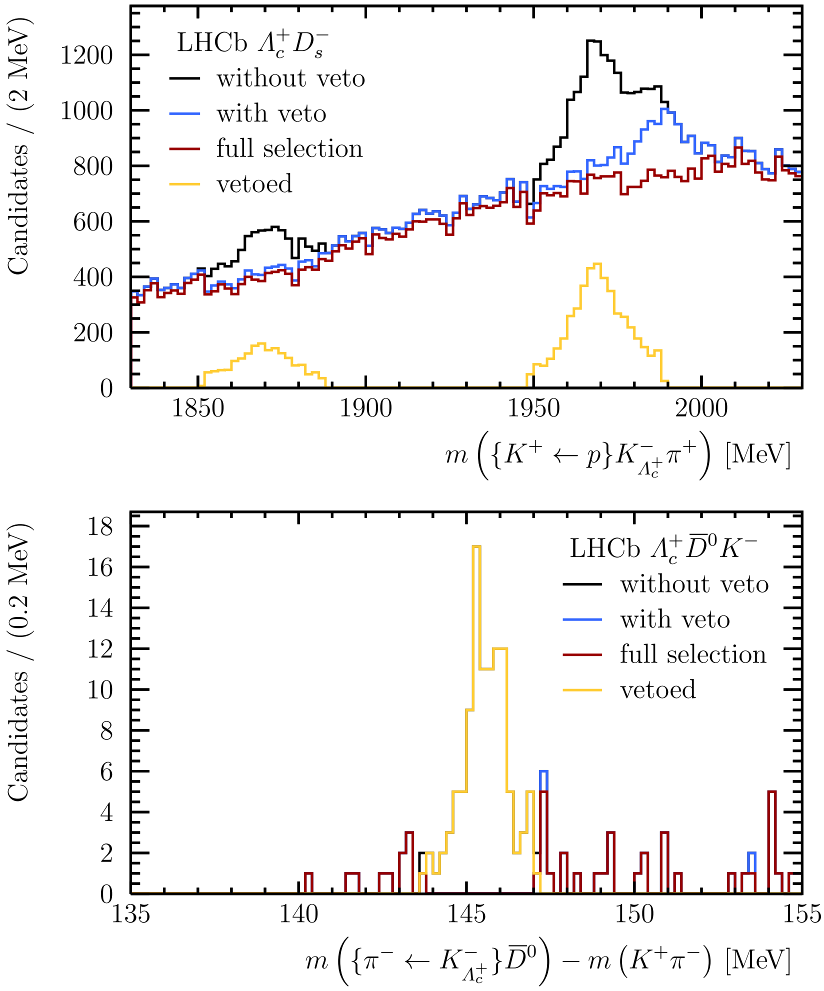

Figure 1: Invariant-mass distributions with swapped particle hypotheses showing the rejection of $D^+_{(s)} \to \{K^+ \leftarrow p\}K^-_{\Lambda_c^+}\pi^+$ candidates in the $\Lambda_c^+ D_s^-$ decay channel and $D^{*-} \to \{\pi^- \leftarrow K^-_{\Lambda_c^+}\}\overline{D}^0$ in the $\Lambda_c^+ \overline{D}^0 K^-$ decay channel.

Three-dimensional extended maximum-likelihood fits, which are carried out in a region around the exclusive $\Lambda_b^0 \to \Lambda_c^+ D_s^-$ and $\Lambda_b^0 \to \Lambda_c^+ \overline{D}^0 K^-$ peaks,are shown in Figs. 2 and 3.

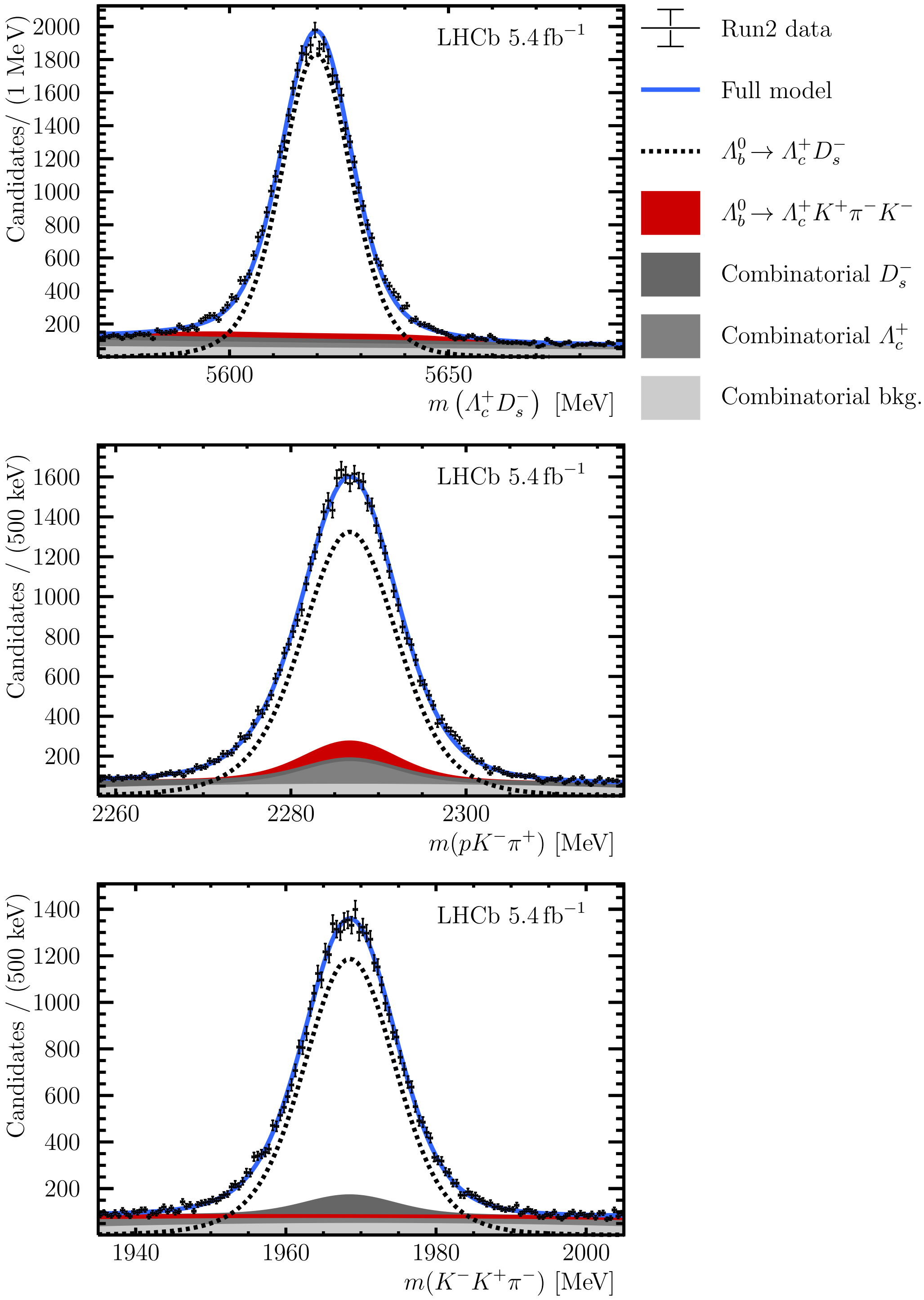

Figure 2: Distributions of (upper) $m(\Lambda_c^+ D_s^-)$, (middle) $m(pK^-\pi^+)$ and (lower) $m(K^-K^+\pi^-)$ for the $\Lambda_c^+ D_s^-$ candidates, with the fit projections overlaid.

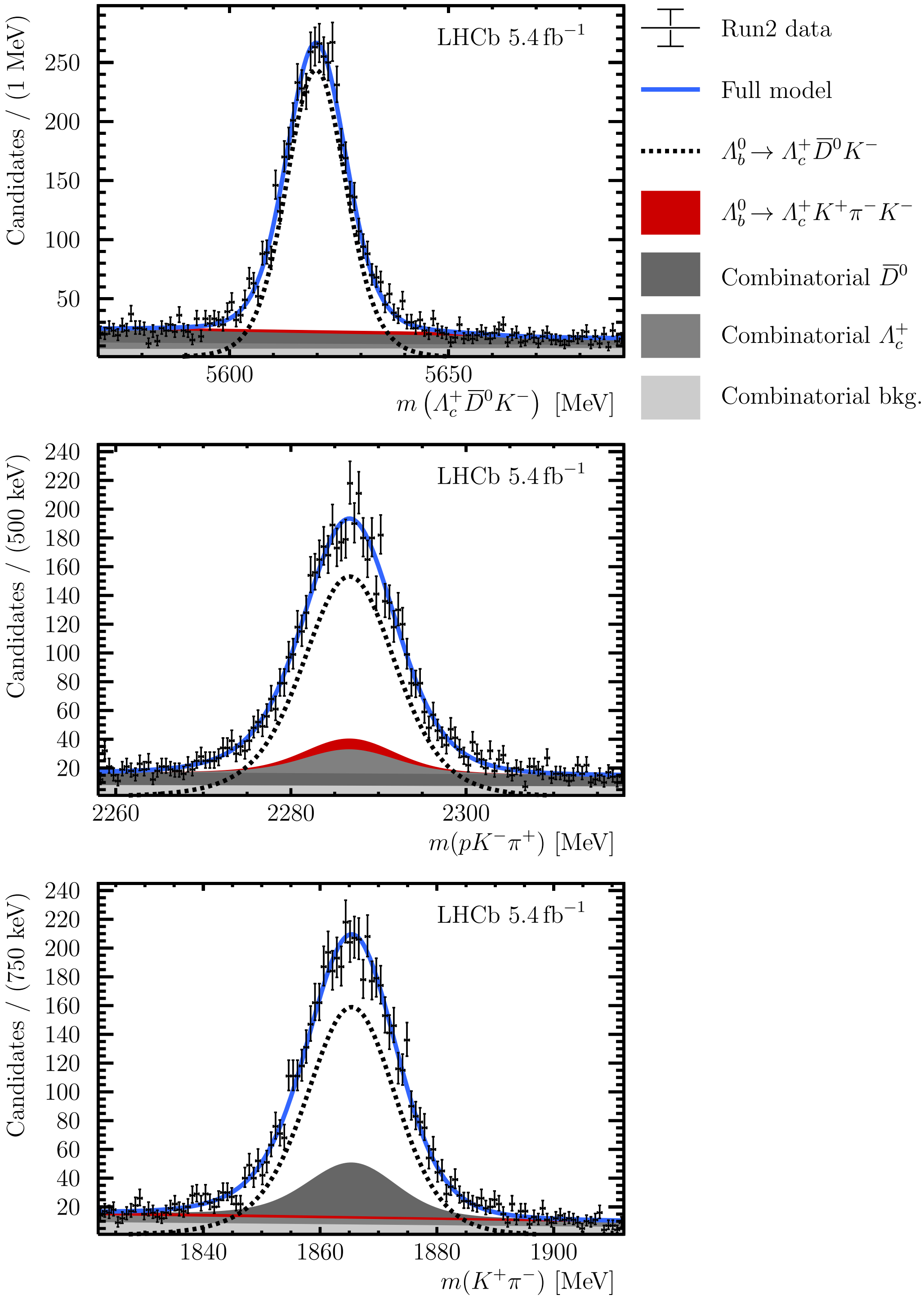

Figure 3: Distributions of (upper) $m(\Lambda_c^+\bar{D}^0K^-)$, (middle) $m(pK^-\pi^+)$ and (lower) $m(K^+\pi^-)$ for the $\Lambda_c^+\overline{D}^0K^-$ candidates, with the fit projections overlaid.

Alternative fit models have been used to determine the systematic uncertainty of the invariant-mass fits. Model-variations concern a single aspect of the baseline fit model, like the background function, and vary the choice, *e.g.* use an exponential instead of a linear function. For each alternative, a manual iteration over all other variations has been carried out and the model that minimized the corrected likelihood is taken into consideration for the discrete profiling method. In fits to $m(\Lambda_c^+ D_s^-)$, the variations have been labelled as follows:

- **A** Baseline model.
- **B** Exponential background.
- **C** 2nd order Chebychev polynomial background.
- **D** 3rd order Chebychev polynomial background.
- **E** Fix slope and D_s^{*-} branching fractions.
- **F** Alternative $\Lambda_b^0 \to \Lambda_c^+ D_s^- \pi\pi$ model.
- **G** Allow for different fraction of Box and Hill/Horns components for $\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$.
- **H** Constrain $\Lambda_b^0 \to \Lambda_c^+ K^+ \pi^- K^-$ normalization, fix its shape at about 20 % larger width.
- **I** Double-sided Crystal Ball function as signal model.

These different fits are input to a discrete profiling method that approximates likelihoods with bifurcated[1] parabolas. This offers the major advantage that the envelope can be derived analytically, and that profiling the likelihood, which would take months of CPU time in this case, is not needed. The likelihood-approximating bifurcated parabola of a fit i is given by

$$\Delta \log(\lambda)_{\text{corr}} = \begin{cases} a_{i,\text{low}} \cdot (x - \hat{\theta}_i)^2 + \Delta_i \log(\lambda)_{\text{corr}} \\ a_{i,\text{high}} \cdot (x - \hat{\theta}_i)^2 + \Delta_i \log(\lambda)_{\text{corr}} \ , \end{cases} \tag{1}$$

where $\Delta_i \log(\lambda)_{\text{corr}}$ is the corrected likelihood difference of the global best fit to the fit i, in which $\hat{\theta}$ is the best fit value of the parameter of interest, and a is the parameter characterizing the parabola. It can be obtained by inserting known points on the parabola, *i.e.* for non-pathological likelihoods, the Minos uncertainty returns the point where the profile likelihood difference $\Delta \log(\lambda) = 0.5$, such that $a = 1/2(\delta x_i)^{-2}$, where δx_i is the Minos uncertainty on the parameter of interest.

From the individual parabolas, omitting the low/high notation as they follow equivalently, one can solve the system of equations

$$a_{\text{env},i}(\tilde{x} - \hat{\hat{\theta}})^2 = a_i(\tilde{x} - \hat{\theta})^2 + \Delta_i \log(\lambda)_{\text{corr}} \tag{2}$$

$$2a_{\text{env},i}(\tilde{x} - \hat{\hat{\theta}}) = 2a_i(\tilde{x} - \hat{\theta}) \tag{3}$$

to obtain a_{env} and the point $\tilde{x}, \tilde{\Delta} \log(\lambda)_{\text{corr}}$ where the envelope tangents the "outermost" individual parabolas. The assumption here is that the global best fit is the minimum of

[1]The bifurcation accounts for asymmetric uncertainties returned by Minos.

the envelope, $\log(\lambda)_{\text{corr}}(\hat{\hat{\theta}}) = 0$, with the global best fit value of the parameter of interest $\hat{\hat{\theta}}$. The outermost parabola is obtained by computing the maximum distance of any "candidate" envelope parabola to $\hat{\hat{\theta}}$ at $\Delta \log(\lambda) = 0.5$:

$$\delta x_{\text{env}} = \operatorname{argmax}_i \frac{1}{2a_{\text{env},i}} = \operatorname{argmax}_i \frac{\tilde{\Delta}_i \log(\lambda)_{\text{corr}}}{(\tilde{x} - \hat{\hat{\theta}})^2} \ , \tag{4}$$

where

$$\begin{aligned} \tilde{\Delta}_i \log(\lambda)_{\text{corr}} &= a_i(\tilde{x} - \hat{\theta})^2 + \Delta_i \log(\lambda)_{\text{corr}} \text{ and} \\ \tilde{x} &= \hat{\theta} + \frac{\Delta_i \log(\lambda)_{\text{corr}}}{a_i(\hat{\theta} - \hat{\hat{\theta}})} \ . \end{aligned} \tag{5}$$

In case δx_{env} is smaller than the best fit uncertainty, this uncertainty would be assigned and the envelope on the lower or higher side of $\hat{\hat{\theta}}$ would coincide with the best fit likelihood approximation. The parameters of interest are the fit yields $N^{\Lambda_b^0 \to \Lambda_c^+ D_s^-}$ and $N^{\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}}$. Results of the discrete profiling methods are shown in Fig. 4.

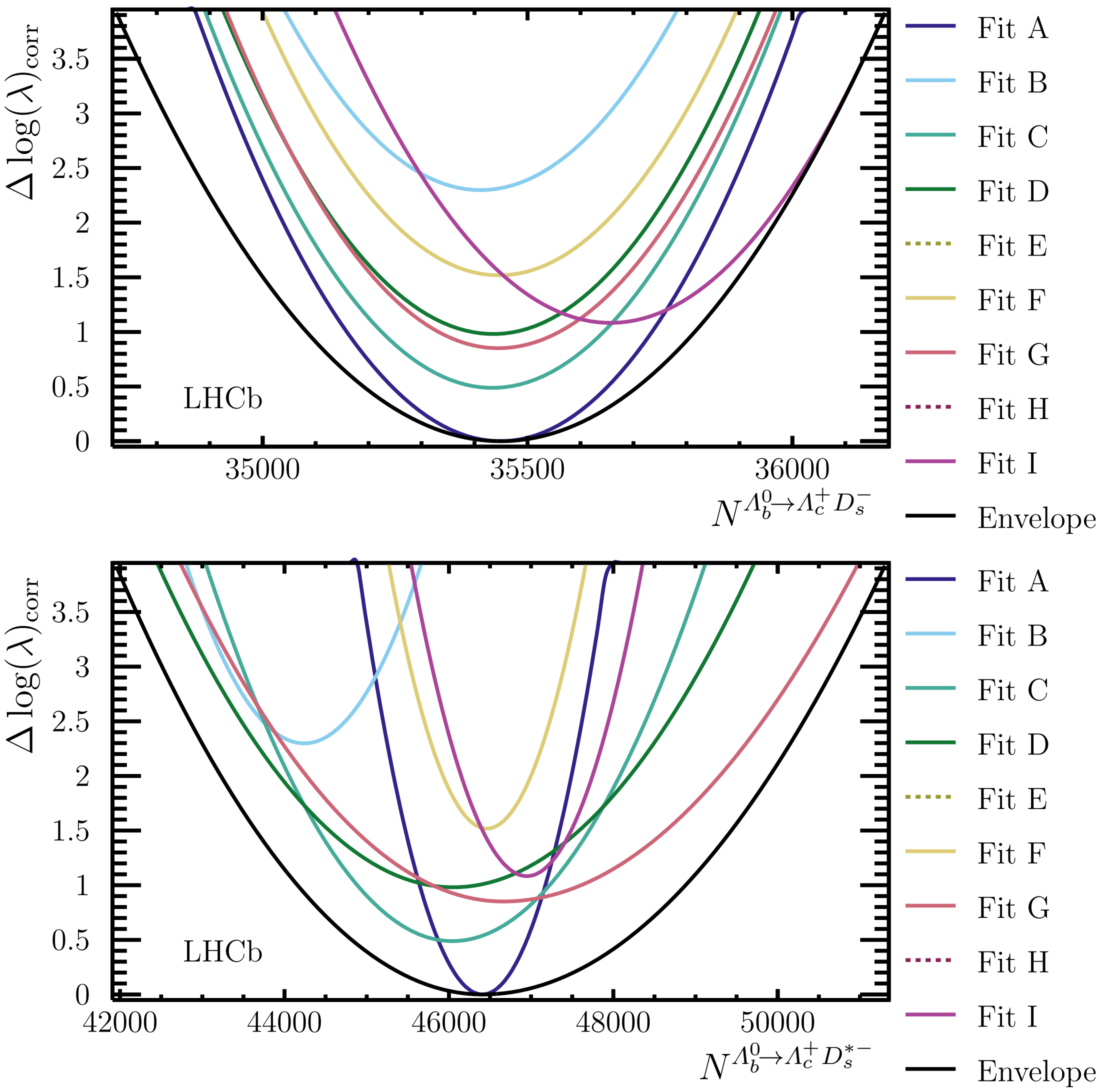

Figure 4: Results of the approximate discrete profiling method to obtain a systematic uncertainty on the fit model. Fits in which the minimal corrected likelihood is > 4 are out of the scope of the plots. These fits are indicated by a dashed line in the legend. Those fits are taken into account for the evaluation of the systematic uncertainty, but do not contribute in the cases shown here.