PTC

Library User Guide

Dan T. Abell

It doesn't matter how beautiful your theory is,
it doesn't matter how smart you are.
If it doesn't agree with experiment, it's wrong.

Richard P. Feynman

The purpose of science is not to lead us to everlasting wisdom,
but to place a limit on everlasting error.

Bertolt Brecht

Ignorance is no excuse, it's the real thing.

Irene Peter

PTC

Polymorphic Tracking Code

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Note to the Reader

This manual describes Étienne Forest's PTC, a software library of data structures and tools for both integrating and analysing the orbital and spin motion of particles in modern accelerators and storage rings. I believe that PTC is an important contribution that should be more widely known, understood, and used by the accelerator physics community, hence my efforts here. I will therefore greatly appreciate your reporting any difficulties you have with this document—errors, inconsistencies, infelicities of language, *etc.*—to dabell@txcorp.com.

Acknowledgements

First and foremost, I must thank Étienne Forest. Not only is he the author of PTC—hence *sine qua non*—he has also answered innumerable questions over the years and helped me to understand both the structure of PTC and the motivation behind many of the decisions that went into its design. Many of the examples in this manual are based on code he provided.

Al Kemp, of Impact Technical Publications, helped with the overall organisation and the initial draft. In addition, several of my colleagues at Tech-X commented on various parts of this document: George Bell, Andrey Sobol, and David Bruhwiler.

Very special thanks go to Desmond Barber, for his support, encouragement, and detailed reviews of the manuscript. His keen ear for the English language, broad knowledge of accelerator physics, and consistent willingness to help have improved this document in more ways than I can possibly note. I am deeply in debt to him.

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PTC

ONE

Introduction

1.1 PTC and FPP

The Polymorphic Tracking Code (PTC) is a library of Fortran90 data structures and subroutines for integrating the equations of orbital and spin motion for particles in modern accelerators and storage rings. The data structures hold the material that we will mould into physically correct and consistent three-dimensional computer models of the complex topologies found in machines such as colliders and recirculating linacs, including the effects of errors in both location and strength of the various machine elements. We use the subroutines to actually construct the computer model of a machine and then integrate through it—including the geometric transformations (translations and rotations) of machine elements, connecting elements to form trackable beamlines, and tracking the orbital and spin degrees of freedom. In a word, PTC is an *integrator*.

The Fully Polymorphic Package (FPP) is a package of polymorphic types and tools that provide PTC with facilities for analysis. In particular, FPP implements a polymorphic Taylor type (hence the *P* in PTC) that can change shape at execution time. This Taylor type makes it possible for FPP to extract a Poincaré map from PTC (or some other integrator). Moreover, FPP provides the tools to analyse the resulting map. The most common—and most important—tool is the normal form: with this at hand, one can compute tunes, lattice functions, and nonlinear extensions of these and all other standard quantities of accelerator theory. Indeed, the combination of PTC and FPP gives access to all of standard perturbation theory on complicated accelerator lattice designs.

In a nutshell, then, the three central features of PTC[1] are

- the Fortran90 types and code that facilitate fully three-dimensional placement of beamline elements and construction of non-simple accelerator topologies: colliders, recirculators, dog-bones, and combinations of these topologies;
- the polymorphic integration routines for orbit and spin;
- the use of maps derived from the integrator—via operator overloading and polymorphism—for computing the full range of accelerator properties, including the parameter dependence of these properties.

[1] When we say "PTC", we shall (almost always) mean PTC and FPP combined; but you keep in mind the distinction.

History of PTC

Latter half of 1980s, at SSC-CDG: Berz introduces automatic differentiation and develops DA package in Fortran77. Forest builds LieLib on top of DA, thus bringing the power of map methods to the analysis of maps produced by integrators.

Early 1990s: J. Bengtsson pioneers the use of run-time polymorphism to greatly simplify and improve the process of making run-time changes to, for example, computing parameter dependence. Forest and Bengtsson together develop the ideas from which derive the fundamental building blocks of PTC. Forest develops the dynamical Euclidean group. Middle to late 1990s: Forest develops LieLib into FPP (using LBNL versions of DA and LieLib). FPP replaces the real variable type `real(8)` with a new type called `real_8` to produce Taylor series for analysis. Forest also develops PTC-proper (the integrator). Together these tools compose PTC. Middle to late 2000s: Forest adds spin dynamics to PTC. L. Yang develops a C++ replacement for Berz's DA package.

Where to Obtain PTC

`http://www.takafumi.org/etienne/ptc/`

1.2 PTC Library User Guide

Where to Start Reading

The *Overview of PTC*, chapter 2, presents concepts important for understanding PTC. In-depth discussions of all these concepts can be found in several publications by Étienne Forest:

- Étienne Forest, "A Hamiltonian-free description of single particle dynamics for hopelessly complex periodic systems", *J. Math. Phys.*, 31(5):1133–1144, May 1990, DOI: 10.1063/1.528795;
- Étienne Forest and Kohji Hirata, "A contemporary guide to beam dynamics", Technical Report KEK-92-12, KEK, Tsukuba, Japan, August 1992;
- Étienne Forest, "Locally accurate dynamical Euclidean group", *Phys. Rev. E*, 55(4):4665–4674, April 1997, DOI: 10.1103/PhysRevE.55.4665;
- Étienne Forest, *Beam Dynamics: A New Attitude and Framework*, volume 8 of *The Physics and Technology of Particle and Photon Beams*, Harwood Academic Publishers, Amsterdam, 1998;
- Étienne Forest, "Geometric integration for particle accelerators", *J. Phys. A: Math. Gen.*, 39(19):5321–5377, May 2006, DOI: 10.1088/0305-4470/39/19/S03.

If you want to get started right away, begin with chapter 3, *Modeling an Accelerator with PTC*, and refer back to the overview as needed while working with the tutorial examples.

TWO

Overview of PTC

This overview describes the PTC approach to simulating and analyzing particle accelerators.

At the heart of PTC's accelerator simulation code is an integrator capable of tracking particles through various kinds of beamline elements. A particle going through a beamline element sees only the *local* magnetic field, and PTC uses a *local* reference frame for describing that magnetic field—the frame most appropriate for the geometry of that type of element. To track through a series of beamline elements, PTC locates the elements with respect to each other using geometric transformations that connect the reference frame of one element to the reference frame of neighboring elements. Those two pieces—the integrator and the geometric transformations—give PTC the capability to model an accelerator with arbitrarily placed beamline elements.

In tracking orbits through particle accelerators, we want the capability of modeling not only simple linear and ring accelerators, but also complex topologies, for example, the Continuous Electron Beam Accelerator Facility (CEBAF) at the Jefferson National Laboratory (JLab). To model a complex topology correctly[1], PTC uses a data structure that captures the location of the physical elements as well as the topology of the beam trajectory. It is that data structure that makes it possible for us to modify a *single* PTC beamline element in our simulation code—even when more than one distinct beam trajectory traverses that element. See the discussion of *Modeling Accelerator Topologies*, § 2.2.

[1] Here *correctly* means "in a manner that respects the physical reality".

PTC itself handles the integrator, the geometric transformations, and the associated data structures, which provide the capability to track particle orbits through complex topologies. Inside of PTC is a set of modules referred to collectively as the Full Polymorphic Package (FPP).[2] These routines extend the capabilities of PTC, making it possible not only to track particles, but also to propagate maps. PTC uses FPP to compute the maps one may analyze for accelerator properties of interest to us: lattice functions and the like. *Analyzing an Accelerator to Understand its Properties*, § 2.3, discusses this topic.

[2] É. Forest, Y. Nogiwa, and F. Schmidt. The FPP and PTC libraries. In Int. Conf. Accel. Phys. 2006, pages 17–21; and É. Forest, Y. Nogiwa, and F. Schmidt. The FPP documentation. In Int. Conf. Accel. Phys. 2006, pages 191–193

Finally, when we track particles in accelerators, we must sometimes account for particle interactions. *Modeling Particle Interactions*, § 2.4, discusses this topic.

For in-depth discussions of the topics introduced in this overview, see the *Bibliography*, page 135.

PTC and Reference Trajectories

A *reference trajectory* or reference *orbit* describes the ideal path of a particle through an ideal accelerator. Accelerator modeling codes that use a reference trajectory simulate particle orbits as deviations from that ideal. However, a particle traveling through a real accelerator "sees" only *local* magnetic and electric fields, and PTC respects this physical reality. When pushing a particle through, say, a quadrupole, PTC makes no distinction between that magnet sitting on a lab bench and that magnet in a beamline: it simply integrates the particle motion in a frame local to that quadrupole. As we shall see, an essential component—perhaps *the* essential component—of PTC is the collection of geometry routines that handle transformations between different frames of reference. This approach has the added benefit that even large displacements—*e.g.*, a quadrupole shifted to act as a combined-function bend—can be handled in the same consistent and physically correct manner.

2.1 Tracking Particles through an Accelerator

This section discusses four important concepts that enable PTC to perform accurate simulations of particles through an arbitrarily complex accelerator:

- blocks,
- geometric transformations,
- particle tracking,
- data structures for modeling accelerator topologies.

Blocks

PTC uses the longitudinal coördinate, generically denoted s, as the independent variable for integrating particle trajectories.[3] The particulars of PTC's s-based, or *element*-based, tracking serve to maintain the physical and mathematical integrity of each element in a beamline and, moreover, make it possible for PTC to provide information about particles inside an element, again without violating the element's physical and mathematical integrity. To explain how PTC does this, we use an analogy with LEGO® blocks.

[3] PTC also has the capability to perform first-order time-based tracking. See *Modeling Particle Interactions*, § 2.4.

Each different type of beamline element in an accelerator is analogous to a different type of LEGO block in the sense that LEGO blocks are self-contained objects. A magnet, for example, is a self-contained object that produces a local field that affects local particle trajectories. Local geometric considerations (*e.g.*, the shape and symmetry of the magnetic field) determine the coördinate system and frame of reference for a local Hamiltonian that describes the magnet. (We choose the coördinate system and frame of reference so as to push particles through the element as simply as possible.) If individual magnets have conflicting geometries, we patch them together. In other words, we use local coördinate transformations to connect particle trajectories between the differing geometries. The rest of the accelerator does not—and should not—affect our choice of reference frame for any given beamline element.

A LOCAL COÖRDINATE SYSTEM attached to each beamline element permits us to propagate physical quantities across the elements. That internal structure is the province of the integrator. In addition, each element includes three *reference frames*: a reference frame at the entrance, a reference frame at the midpoint (the *element* reference frame), and a reference frame at the exit. See figure 2.1. These extra frames, used by PTC when it hands a particle from one element to the next, allow PTC to

handle arbitrary changes in the placement of beamline elements. On entering an element, a particle has a certain phase-space location with respect to the entrance frame. PTC tracks the particle as it traverses the element, computing the particle's phase-space coördinates with respect to the element's exit frame. Knowing the relation between the exit frame of the current element and the entrance frame of the subsequent element, PTC uses the transformations of the dynamical Euclidean group[4] to hand the particle to the next element.

[4] É. Forest. Locally accurate dynamical Euclidean group. *Phys. Rev. E*, 55 (4):4665–4674, Apr. 1997. DOI: 10.1103/PhysRevE.55.4665

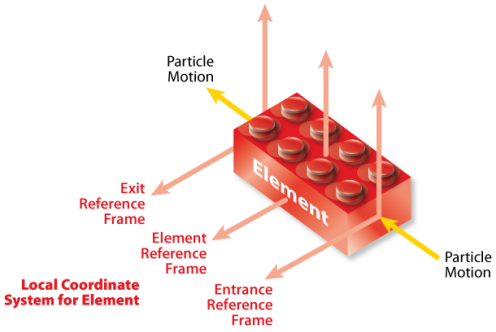


Figure 2.1: LEGO-block element, with reference frames for the entrance, element body, and exit.

The beamline element represented in figure 2.1 is a *drift*: a block with parallel entrance and exit faces to which local Cartesian reference frames are attached. The entrance and exit reference frames have the same unit vectors. The line that links the two frames has a length L, and it is perpendicular to both those frames. Note that for the purposes of this discussion, any beamline element (quadrupole, sextupole, solenoid, *etc.*) that does not bend the design orbit is a drift.

A LEGO-block element might also be a *bend*: a block with two faces that have parallel y-axes (vertical), and x-axes (horizontal) that meet at an angle θ. The two x-axes and the line joining the two origins form a plane perpendicular to the two faces. An arc of circle of length L passes perpendicularly through the origin of both faces. The purpose of this element is to bend an incoming particle trajectory by approximately angle θ. The internal details of the element—whether it is a sector bend, a bend with an irregular field, *etc.*—is, for this discussion, immaterial.

Note that calling the x and y axes "horizontal" and "vertical" reflects the bias of accelerator physicists towards flat rings with vertical dipole fields. What we locally (*i.e.*, within the element) call the x axis need not be horizontal. See, for example, the discussion of the vertical bend indicated in figure 2.5.

Figure 2.2 illustrates these two fundamental types of LEGO blocks with the entrance and exit reference frames for their *local* coördinate systems. The solid arrows show the direction of the local y-axes and the dashed arrows show the direction of the local x-axes. In summary, the bend is characterized by a bending radius ρ, a length L, and two faces, each with a reference frame. The drift is characterized by a length L and also two faces, each with a reference frame. The bend is the fundamental block used in the composition of complex blocks; the drift, of course, is just a special case of the bend.

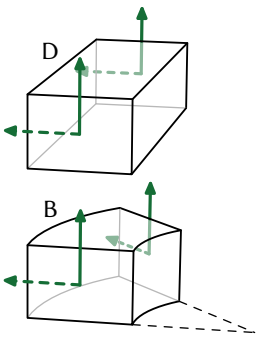


Figure 2.2: Two LEGO blocks: drift (D) and bend (B).

We are interested in the deterministic motion through each LEGO block, represented by the transformation

$$z_{\text{in}} \mapsto z_{\text{out}} = f(z_{\text{in}}), \tag{2.1}$$

where f denotes a vector of generally nonlinear functions connecting the local phase-space coördinate z_{in} defined in the entrance frame, to the local phase-space coördinate z_{out} in the exit frame. The construction of such a transformation depends on internal considerations that are not needed for an understanding of the LEGO-block approach. The LEGO block for a single element might, in fact, comprise a

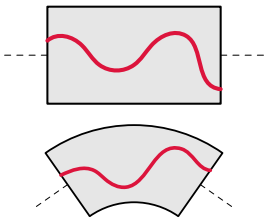


Figure 2.3: Particle trajectories through "drift" and "bend" LEGO blocks.

sequence of simpler blocks. This structure, dictated by the internal geometry of the element, may be hidden.

In figure 2.3 we show a pair of particle trajectories, in drift and bend LEGO blocks, passing from the entrance face through to the exit face. As suggested by the trajectories illustrated, the internal structure may be very complicated (think of a wiggler); but seen from the outside, our LEGO blocks are, quite simply, *blocks* with two faces that define local coördinates with respect to which one may define z_{in} and z_{out}—*nothing more*.

A beamline element is defined by a block with three reference frames, together with a model, or integrator, which describes how to propagate a particle from the face with the entrance reference frame to the face with the exit reference frame:

$$\text{element} = \text{block} + \text{model}.$$

The model transforms the phase-space variables from entrance to exit of the block according to (2.1). In other words, it represents the *physics* of the device. Note that the model also incorporates any approximations we make.

As an example, a block with a bend geometry might actually be a simple drift, keeping the transverse momenta invariant and changing only the positions. Or it might be a composition of blocks describing the body of a magnet and its fringe field regions. An element might also have non-Hamiltonian effects such as radiation.[5] The list of possible models is endless. In general, a model is defined by

- one or more blocks internal to the model;
- a model for each internal block;
- the equation of motion in each internal block together with its integration method and associated number of integration steps.

[5] Appendix A discusses PTC's internal state variables, which describe the characteristic dynamics you can choose for your accelerator model.

In summary, we define a beamline element locally: the definition, whether the element is on a work bench or in an accelerator, is determined solely by the characteristics of that element. And once we have defined a model for our element, we stick with it: the LEGO block is inviolate. Like a physical magnet, it exhibits identical properties under identical conditions. A significant virtue of our computer-based element, however, is that it is free to report about what is happening inside.

Geometric Transformations

Now that we have defined our two basic types of beamline elements (drift and bend), the next step is to fit them together. We need geometric transformations that connect the exit reference frame of one beamline element to the entrance reference frame of a subsequent element.

Continuing the LEGO-block analogy, we put the individual LEGO blocks on a base, which represents the accelerator model's *global frame*. Once we know the location of each LEGO block with respect to the global frame, we know their locations with respect to to one another. Figure 2.4 shows two such LEGO blocks and their reference frames on a base, which represents the layout of beamline elements in the global frame of an accelerator. With the LEGO blocks now on the global frame, we require transformations that translate the phase-space coördinates in the exit reference frame of one LEGO block to the phase-space coördinates in the entrance reference frame of the next LEGO block.

To construct, for example, a recirculating accelerator out of LEGO blocks, we connect bends and drifts one after another to model the desired machine. On reaching the last LEGO block, we require that the last block's exit face coincide with the first entrance face. This means that

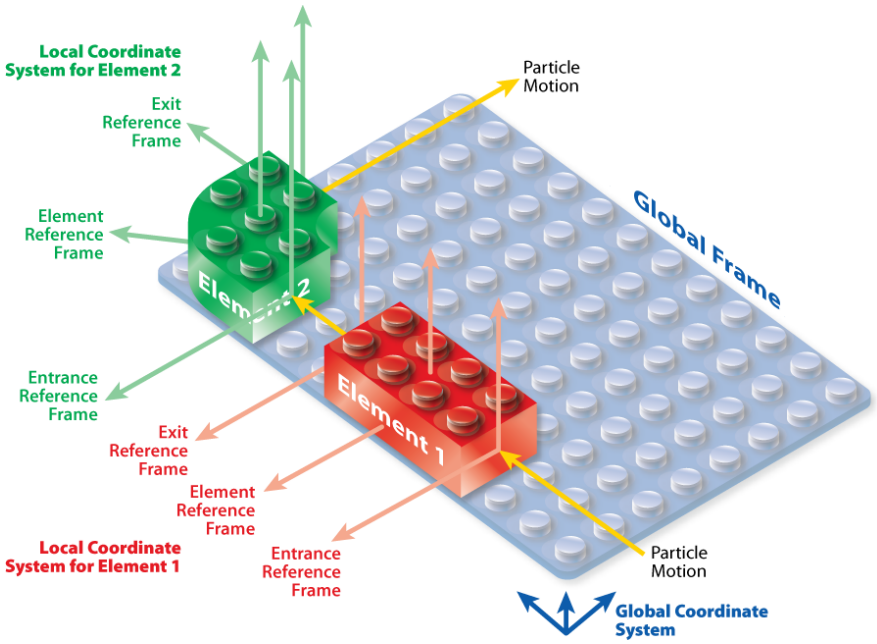


Figure 2.4: Two LEGO blocks (elements) on a base (global frame).

- the blocks' faces must be parallel;
- and the frames on the two faces must line up.

Connecting a horizontal bend with a horizontal drift, as for elements B_H and D in figure 2.5, is easy because the x- and y-axes of the two elements match. Connecting a horizontal drift with a vertical bend, as for elements D and B_V in figure 2.5, is not as straight-forward: their local x- and y-axes do not match. We need, in essence, a new type of LEGO block—one with an x-y rotation of angle ϕ. (In figure 2.5, of course, $\phi = 90°$.) To build an arbitrary accelerator, we shall require a full complement of such additional LEGO blocks.[6] This is the subject of *patching*, which uses geometric transformations to connect the exit frame of one beamline element to the entrance frame of a subsequent element. Most significantly, patching enables us to position beamline elements wherever we want them. We discuss this in more detail later in *Chart and Patch*, page 15.

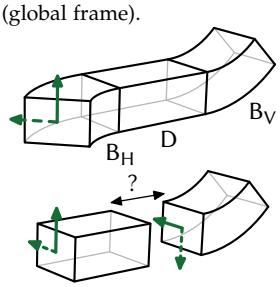


Figure 2.5: Connecting two horizontal LEGO blocks and a vertical LEGO block.

[6] É. Forest. Locally accurate dynamical Euclidean group. *Phys. Rev. E*, 55 (4):4665–4674, Apr. 1997. DOI: 10.1103/PhysRevE.55.4665

Particle Tracking

Now that we have the basic building blocks and the geometric transformations to fit them together, we can begin to think about how to track particles. Here we present some essential information about particle tracking in PTC.

In keeping with its LEGO-block philosophy, PTC tracks particles with respect to *local* frames of reference. The local phase-space coördinates together with PTC's knowledge of the local frames allow the interested user to reconstruct full 3D particle trajectories with respect to the global frame. In fact, doing exactly that is useful for checking that one has the constructed the correct lattice topology in PTC.

UNITS: PTC measures all lengths in meters, and all angles in radians. (It provides the constant `twopi` to simplify the conversion from degrees.[7]) Particle momenta are scaled by a value p_0. This scale momentum is usually set when defining the lattice (see, for example, the discussion of `set_mad` on page 27). A subtle point involves the

[7] Many other constants are defined in the module *precision_constants*.

fact that the scale set for one element may differ from that in the preceding element, *e.g.* in the case of acceleration. In such cases, the patching mentioned above must include not only geometric transformations, but also energy transformations.

TRACKING STATE: PTC defines a variable type called *internal_state*. Objects of this type may be used to modify certain assumptions PTC makes when it performs tracking. One may, for example, ask PTC to track only the 4D transverse phase-space variables; or one may turn on or off radiation. A variable of type *internal_state* is essentially a list of flags that one may pass to PTC tracking routines to modify the algorithms used. For more information, see *Internal States*, appendix A.

PHASE-SPACE COÖRDINATES: With the default tracking state, PTC uses the six phase-space variables[8]

$$(x, p_x, y, p_y, \delta, \ell). \tag{2.2a}$$

[8] PTC can turn all six of the variables into Taylor series, or it can omit two of them. Setting an *internal_state* flag to "only four dimensions" implements the latter behavior.

Here x and y denote the *local* transverse coördinates, and p_x and p_y denote the corresponding canonical momenta (divided by a scale momentum p_o). The fifth variable, δ, denotes the relative momentum deviation

$$\delta = \frac{p - p_o}{p_o};$$

and the sixth variable, ℓ, denotes the path-length deviation.[9]

[9] These last two variables are kept in this order for a technical reason. This ordering allows us to retain the natural meanings: positive δ implies higher energy, and positive ℓ implies longer path length. Because of the Hamiltonian structure, reversing their order would require us to reverse the sign on one of them.

One may modify the tracking state so as to change the phase-space variables. If desired, one may set a flag that causes PTC to compute flight time rather than path length. In this case, the phase-space coördinates are

$$(x, p_x, y, p_y, \varepsilon, ct). \tag{2.2b}$$

Here the fifth variable, ε, denotes the scaled energy deviation

$$\varepsilon = \frac{E - E_o}{p_o c},$$

where E_o is the energy associated with p_o. (In other words, if $p_o = mc^2\beta_o\gamma_o$, then $E_o = mc^2\gamma_o$.) And the sixth variable, ct, is the time-of-flight deviation multiplied by the speed of light.

One may set a separate flag that causes PTC to compute the total path length (or flight time) rather than the deviation. In this case the phase-space variables will be either

$$(x, p_x, y, p_y, \delta, L), \tag{2.3a}$$

or

$$(x, p_x, y, p_y, \varepsilon, cT), \tag{2.3b}$$

where L and T denote respectively the *total* path length and *total* flight time.

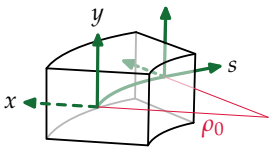


Figure 2.6: Geometry and local coördinates, (x, y, s), for a generic block in PTC.

HAMILTONIANS: In accord with its LEGO-block philosophy, PTC tracks a particle across an element using a Hamiltonian that is *local* to that element. The Hamiltonian used by PTC in the body of an element (no fringe field) then has the simple form

$$-(1+\kappa_o x)\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} + (1+\kappa_o x)\frac{q}{p_o}A_s(x,y) \tag{2.4a}$$

if one uses the variables (2.3a), or

$$-(1+\kappa_o x)\sqrt{1 + \frac{2}{\beta_o}\varepsilon + \varepsilon^2 - p_x^2 - p_y^2} + (1+\kappa_o x)\frac{q}{p_o}A_s(x,y) \tag{2.4b}$$

if one uses the variables (2.3b). Here $\kappa_o = 1/\rho_o$ denotes the curvature defined by the *geometry* of the element, and A_s denotes the longitudinal component of the vector potential. Figure 2.6 indicates the variables, including the longitudinal coördinate s. For straight elements, of course, $\kappa_o = 0$. If one uses the variables (2.2), then there is, effectively, an additional term that subtracts the default path length or flight time across that element.[10]

One may, if desired, have PTC track using an approximation of one of the Hamiltonians (2.4) (*i.e.*, using truncated expansions for the square-root and the vector potential). One might, for example, do this to speed the computation, but of course the validity of the expansion for your particular problem should be verified with careful testing.

When integrating across an element described by one of the Hamiltonians (2.4), one typically splits the Hamiltonian into two integrable pieces.[11] If those pieces are, for example, the two terms of (2.4a), then this is referred to as a *drift-kick* split. Since the motion of a particle in a uniform dipole is integrable, an alternative split involves writing $A_s(x, y)$ as a sum of two pieces: the part that produces a uniform dipole field, and everything else. Then the dipole field part is added to the drift term of the Hamiltonian, and we obtain a *bend-kick* split.[12] One may ask PTC to use one or the other of these splitting methods.

STRUCTURES FOR PARTICLES AND BEAMS: Because PTC tracks particles with respect to local frames of reference, a set of values for (as an example) the six phase-space variables (2.2a) will not mean too much unless one knows the context—in particular, where in the lattice the particle is located. When you need PTC to keep track of such information for you, it provides some additional structures. The type *beam* holds an $N \times 6$ array for the phase-space variables, as well as a pointer that tells you where in the lattice that beam is located. There is also a type *probe* that holds not only phase-space data, but also data about a particle's spin state.

Data Structures for Modeling Accelerator Topologies

PTC data structures fully account for the three-dimensional structure of a lattice and potential topological complexities such as those found in colliders and recirculators. In this section, we discuss a simple lattice and then two complex lattices.

A very simple lattice is shown in Figure 2.7, which illustrates basic forward propagation of particles through a sequence of magnets that never varies. Each magnet appears once in the sequence, and particles go magnet-to-magnet from M1 through M8. At that point, they reënter the magnet M1. A simple linked list of magnets suffices to model this basic topology:

$$\text{ring} = \text{linked-list}(\text{M1}, \text{M2}, \text{M3}, \text{M4}, \text{M5}, \text{M6}, \text{M7}, \text{M8}).$$

One might instead use an array of magnets, but that approach would require us to recreate and reallocate the array whenever we wish to add or insert a new element. A virtue of the linked list is that one may easily add or rearrange elements in the lattice. However, such a simple approach does not suffice when an accelerator topology reuses the magnets in different sequences, different directions, or both.

To see the difficulties associated with a recirculating beam, consider Figure 2.8, which shows a machine similar to the Continuous Electron Beam Accelerator Facility (CEBAF) at Thomas Jefferson National Laboratory (JLAB). It illustrates the concept of particles circulating through a varying sequence of magnets. The particles start out traveling through an injector from magnet M1 through magnet M4. During the

[10] D.P. Barber, K.A. Heinemann, and G. Ripken. A canonical 8-dimensional formalism for classical spin-orbit motion in storage rings: I. A new pair of canonical spin variables. *Z. Phys. C*, 64 (1):117–142, Mar. 1994. DOI: 10.1007/BF01557243

[11] R.I. McLachlan and G.R.W. Quispel. Splitting methods. *Acta Numer.*, 11:341–434, Jan. 2002. DOI: 10.1017/S0962492902000053

[12] É. Forest. *Beam Dynamics: A New Attitude and Framework*, volume 8 of *The Physics and Technology of Particle and Photon Beams*. Harwood Academic Publishers, Amsterdam, 1998; and É. Forest. Geometric integration for particle accelerators. *J. Phys. A: Math. Gen.*, 39(19):5321–5377, May 2006. DOI: 10.1088/0305-4470/39/19/S03

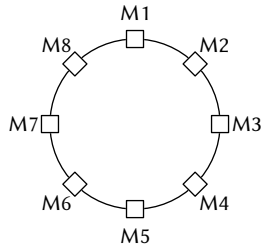


Figure 2.7: Forward propagation of particles in a circular accelerator.

In this discussion, we use the word "magnet" whether or not it is actually a magnet, a drift, or any other beamline element.

first trip circulating around the accelerator, the particles go through the sequence of magnets M5, M6, M7, M8, M9, M10, M11, and M12. During their second trip around the accelerator, the particles go through the sequence of magnets M5, M6, M13, M14, M9, M10, M15, and M16. And during their last trip, the particles go through the sequence of magnets M5, M6, M17, M18, M9, and M10. The particles are then dumped. Note that magnets M5, M6, M9, and M10 appear in all three circuits.

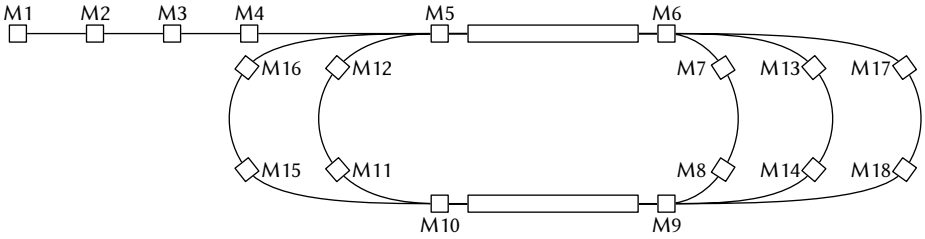


Figure 2.8: A recirculator illustrates particles traveling through a varying sequence of magnets.

A linked list of *magnets* cannot properly represent the physics of this situation. In a linked list, magnet M6, for example, must point to only one magnet. It cannot point to M7, M13, *and* M17 unless we create two clones of M6. Creating clones, however, is a bad idea: If we wish to adjust the position or strength of magnet M6, such adjustments would have be made three times: once in the magnet itself, and once in each of its two clones.

A better solution separates the magnets from the linked list that tracks the path of particles through the magnets. PTC does this with a linked list of containers called *fibres*, each with a pointer to the appropriate magnet. Adjustments to the position or strength of a magnet are made once and are automatically taken into account each time a container in the linked list points to that magnet. We discuss the linked list of containers with pointers to magnets in *Modeling Accelerator Topologies*, § 2.2.

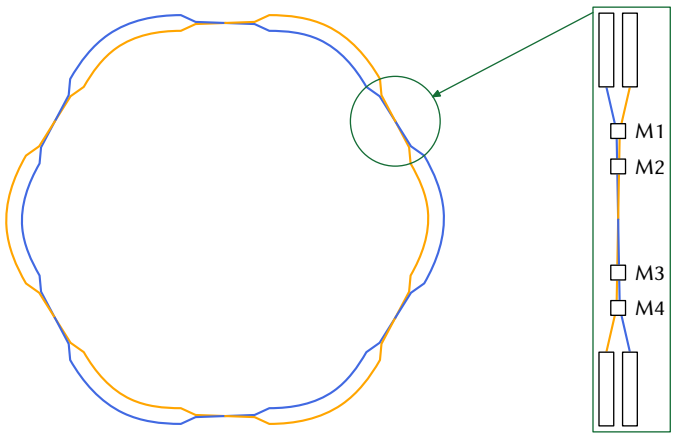


Figure 2.9: Particles circulating in different directions through an accelerator.

What about a collider? Figure 2.9, based on the intersecting rings of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL), shows particles circulating in both directions (forward and backward propagation) through some of the same magnets. For example, the particles following the blue path go though one set of magnets in the sequence M1, M2, M3, M4. Meanwhile, the

particles following the yellow path traverse the same set of magnets in the reverse order: M4, M3, M2, M1.

Using an array or a linked list with clones of magnets creates the same problems as before, and PTC alleviates the difficulties by using doubly-linked lists of containers that follow the particle paths—containers which have inside them pointers to the appropriate magnets. Changes to the strength or position of, say, magnet M3 are made once in the object for magnet M3 and are immediately reflected in the two linked lists—one linked list for each of the two particle beams.

The use of a *doubly*-linked list enables both forward and reverse propagation through a given sequence of beamline elements.

2.2 Modeling Accelerator Topologies

To set up a linked list of containers with pointers to the elements in a beamline, PTC employs three different data types: *element*, *fibre*, and *layout*. PTC also provides what it refers to as a *DNA database*, which one may populate with *layout*s that one may use to construct complex accelerator topologies. Finally, for the proper positioning of elements in the accelerator, PTC uses two additional data types: *chart* and *patch*. We discuss all these concepts in this section.

Element

The data type *element* represents a beamline element in a machine. It contains information about the element type (dipole, quadrupole, rf cavity, *etc.*), its physical properties (length, strength, *etc.*), and its location and orientation in space. In PTC, it is the fundamental object. PTC preserves each element's physical and mathematical integrity to ensure that it can be positioned, repositioned, or misaligned *as a whole*. At the same time, PTC provides the capability for one to look inside the element to see, for example, details of a particle trajectory. One should therefore *never* feel compelled to split an element in half.

In addition to information about the element itself, the data type *element* includes information about which *fibre*s point to it. This information is used by PTC to ensure that if an element is moved, then all beamlines containing it know about the move.

Fibre

The data type *fibre* contains a pointer to a beamline element as well as pointers to the fibres that precede and follow it along a particle path. When strung together in a linked list, a sequence of fibres defines a beamline. The fact that different fibres can point to the same element means that separate beamlines can share common elements; and this gives PTC the capability to model complex accelerator topologies.

A *fibre* contains, among other items,

- a pointer to an *element*;
- pointers to the previous and next *fibre*s along a beamline;
- a pointer to a *chart* that locates the element within the global reference frame;
- a pointer to a *patch* that connects the elements of successive fibres geometrically, energetically, and temporally;
- the *direction of propagation* through the element.

Layout

The data type *layout* represents a beamline as a doubly-linked list of *fibre*s. It follows the particle path by specifying the order of the fibres, which each point to an actual

element in the beamline.[13] In addition, a *layout* defines the direction of propagation through the sequence of fibres and whether the particles recirculate.

[13] Note that a *layout* need not represent an entire machine: it may represent just a piece of an machine. The full accelerator would then be built from several layouts.

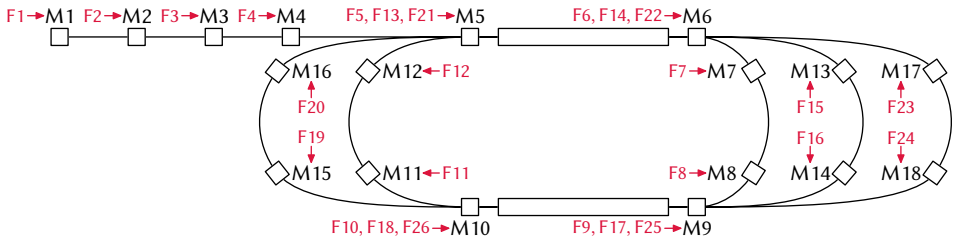


Figure 2.10: A layout with a linked list of fibres pointing to magnets.

As an example, consider the accelerator illustrated in figure 2.8. We could construct it as indicated in figure 2.10, which shows a PTC *layout* with a linked list of 26 fibres (F#) pointing to the 18 magnets (M#).

As before, we say "magnet", but it may mean any type of beamline element.

L1: M1 ↔ M2 ↔ M3 ↔ M4
L2: M5 ↔ M6
L3: M7 ↔ M8
L4: M9 ↔ M10
L5: M11 ↔ M12
L6: M13 ↔ M14
L7: M15 ↔ M16
L8: M17 ↔ M18

Figure 2.11: DNA database with eight DNA sequences (L1 through L8). The arrows represent the links in the doubly-linked list of fibres that constitutes a layout. Each fibre points to (contains) the indicated magnet (M1–M18).

MOST ACCELERATOR MODELING CODES do not, as PTC does, enforce a dichotomy between fibres and elements. As a consequence, it can be difficult to import lattice descriptions from other codes into PTC in a manner consistent with PTC's approach to modeling accelerators. To overcome this difficulty, PTC provides what it refers to as a *DNA database*. This database is first populated with a set of *layout*s whose fibres point to unique elements. In other words, no element appears more than once—via the fibres—in the set of DNA layouts. (This much may, if desired, be done by importing from an external lattice description.) These simple layouts—we shall refer to them as *DNA sequences*—may then be used to construct complex layouts of the form shown in figure 2.10.

As a concrete example, consider how we might do this for the accelerator shown in figure 2.8 or 2.10: We first create a set of layouts in which no magnet appears twice, and which have no magnets in common. The logical course in this case is to create the eight layouts L1 through L8 shown in figure 2.11. Note that each of the eighteen magnets appears just once in this database. Here we show each of the DNA sequences in a different color; and these colors match those used in figure 2.12, which shows the accelerator with the different DNA sequences indicated by the corresponding colors.

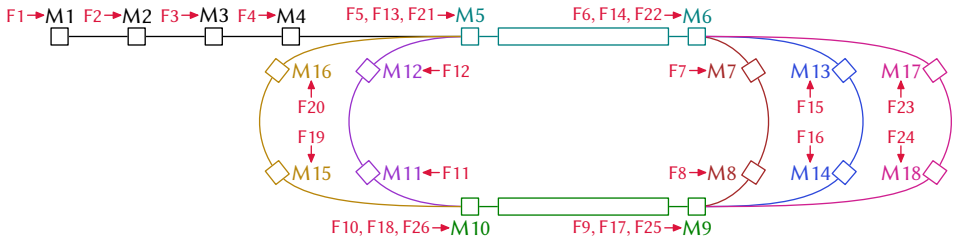


Figure 2.12: Eight layouts or DNA sequences.

Having created the eight DNA sequences in our DNA database, we can now use PTC to string these basic layouts together to create the *trackable layout* that represents the full machine illustrated in figure 2.8, or 2.10 or 2.12. Evidently, a trackable layout may be constructed from one or more DNA sequences.[14]

In general, once we have populated the DNA database (either from within PTC or by importing external lattice descriptions) we can use PTC to string together these basic layouts to create the additional layouts we need for tracking. These latter—T1

[14] By contrast, we may sometimes refer to the DNA sequences—high-level building blocks of our accelerator model—as *non-trackable layouts*.

through TN, say—contain fibres that point to elements in the DNA database. For the recirculator shown in figure 2.8, we would create one beamline layout T1 with 26 fibres, each pointing to an element in the DNA database. For the collider shown in figure 2.9, we would create two beamline layouts, T1 and T2, one layout for each ring.

Chart and Patch

PTC uses the new data types *chart* and *patch* to locate, connect, and move the elements of a beamline. The generic capability, which we term *patching*, connects the exit frame of one element to the entrance frame of the subsequent element.

The data type *chart* contains a pointer to a frame of reference (actually the collection of three frames illustrated in figure 2.1) that locates a fibre (really the element the fibre points to) within PTC's global three-dimensional reference frame. The chart also contains information that describes, if present, any misalignment of the fibre.

The data type *patch* contains information about the about the relation between the exit reference frame of one element and the entrance reference frame of the subsequent element. As a consequence, a *patch* is a property not of an *element* but of a *fibre*, and it must be computed *after* placing both elements in their final locations.

As a concrete example, consider the set of elements illustrated in figure 2.13. There we see three elements separated by a pair of drifts. We know the locations of these elements (in particular the reference frames attached to these elements) because of the information stored in the corresponding charts. The second drift is explicitly defined as the third beamline element. The first drift, however, is not explicitly defined. The exit frame of element 1 and the entrance frame of element 2 do not coincide, and this indicates the need for a patch. The remaining elements all have exit-entrance frame pairs that coincide, and hence no additional patches are required.

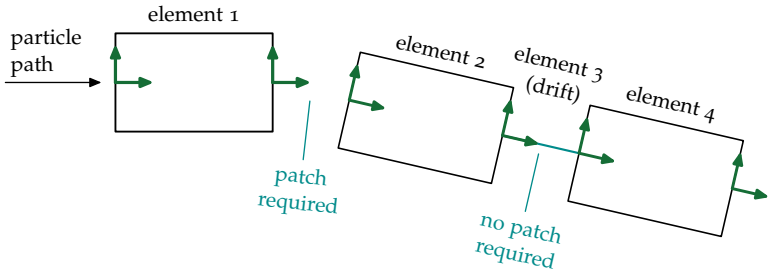


Figure 2.13: Patching elements in a single beamline.

As another example of patching, figure 2.14 shows three magnets in a recirculator. A high-energy particle passing through magnet 1 follows the red trajectory to magnet 2. After exiting magnet 2, the particle continues around the machine and returns with a lower energy (after deceleration by appropriately phased rf fields) to magnet 1. Because the particle has less energy, magnet 1, the separator, now bends it along the blue trajectory towards magnet 3, the first magnet in a new beamline. Magnet 1 of figure 2.14 must, of course, appear in two fibres: the first points to magnet 2 as the subsequent element, while the second points to magnet 3. What about the patches? Assuming the exit frame of magnet 1 and the entrance frame of

magnet 2 have the same unit vectors but different origins. then the patch between the fibres pointing to magnets 1 and 2 must correspond to a simple drift of the appropriate length—and that is exactly what PTC will compute. The patch between the fibres pointing to magnets 1 and 3 corresponds to three actions: a drift of length d, a coördinate frame rotation by angle α, and a translation of length h.

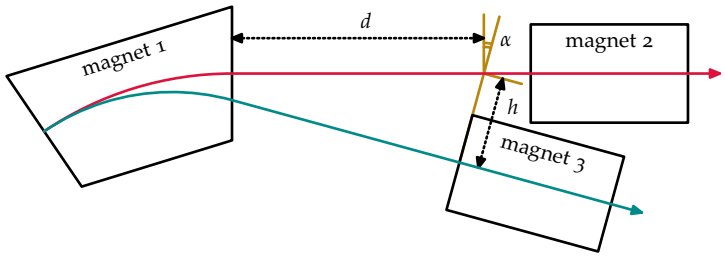


Figure 2.14: Patching elements in multiple beamlines.

Misalignments

The accelerator we design is never the one we build. This fact makes it essential to investigate the margin for error in any design. To simulate misalignment errors, PTC uses the approach illustrated in figure 2.15. There we show the original element 2 with a light gray outline, and the misaligned element 2 with a red outline. To push a particle from the exit of element 1 to the entrance of element 3, PTC first applies the patch from element 1 to the original element 2. Then PTC traverses element 2 in three steps: an entrance misalignment, the standard element 2, and an exit misalignment. PTC then applies the patch from element 2 to element 3. If we remove the misalignment (which translates and rotates element 2), the element returns to its design position in the fibre.

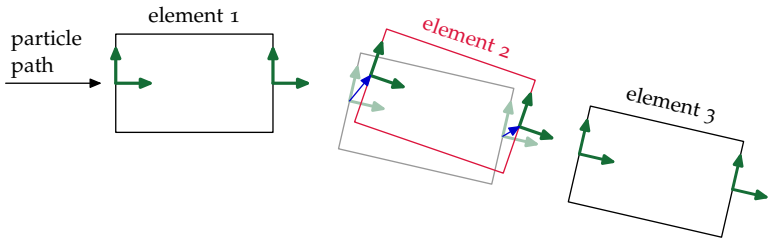


Figure 2.15: Misaligning an element.

2.3 Analyzing an Accelerator to Understand its Properties

In this overview chapter, the first two sections, *Tracking Particles through an Accelerator* and *Modeling Accelerator Topologies*, discuss the essential concepts and data structures used by PTC to construct a computer model of an accelerator and simulate particle trajectories through that model. Your next activity is likely to be that of asking questions of your simulated accelerator: Where is the particle in that magnet? What are the Twiss parameters at this location in the ring? What are the horizontal and vertical tunes of this machine? How much synchrotron radiation does my particle emit while traversing this magnet? Such questions are the domain of *analysis*.

At the heart of the software design decisions made for PTC lies the desire to analyze *real* accelerators. The fact that real accelerators are complicated beasts—with many individual magnets performing many individual functions—leads directly to PTC's simulating accelerators using the LEGO-block approach, which emphasizes a *local* view of beamline elements and particle tracking. Similarly, the desire to analyze leads directly to PTC's use of map-based methods, which emphasize a *global* view of the accelerator.

Map-based methods are fundamental to the analysis of real accelerators. Think of a one-turn map as a piece of sophisticated diagnostic hardware that you install at some point in the ring. It enables you to observe the beam turn after turn. It follows that any *sensible* question you might ask about the beam at that point in the ring must be contained within that one-turn map. Indeed, all of standard perturbation theory may be expressed entirely in terms of the one-turn map.[15]

[15] É. Forest. A Hamiltonian-free description of single particle dynamics for hopelessly complex periodic systems. *J. Math. Phys.*, 31 (5):1133–1144, May 1990. DOI: 10.1063/1.528795

If you desire information at some other point in the ring, then you must, of course, install another sophisticated beam monitor at that new location. Using map-based methods, this will be equivalent to a proper combination of the one-turn map at your original location and the transfer map that connects the two locations of interest. As a consequence, a one-turn map plus partial-turn transfer maps contain the answers to all sensible questions you might ask of an accelerator model.

The analogy of a one-turn map as a sophisticated beam monitor also implies—quite correctly—that no connection exists between the *construction* of a one-turn map and its *analysis*. The construction is done using a strictly local approach, which is our only hope of getting the physics right in this complicated and messy world. Once we've obtained the one-turn map, we may forget where it came from and what various frames of reference were used to compute it. We now concentrate on asking sensible questions of our "beam monitor".

Local versus Global Information

PTC provides you with two kinds of information: local and global.

Information about an element, or a particle in an element, is called *local* if it can be obtained independently of any knowledge of the element's position in an accelerator. It could be obtained even if the element were a prototype sitting on a test bench in your lab. The field strength at a particular location in the element is an obvious example of local information. Another example is a particle's trajectory through a magnet: When, for example, an electron appears at the entrance of a quadrupole, we can predict its (local) trajectory without any knowledge of where the electron came from, or where it is going. Yet another example of local information is the amount of synchrotron radiation emitted by a particle as it traverses a magnet.

Information is called *global* if it can be obtained only after constructing an accelerator. The dynamic aperture is an example of global information, because it makes sense only in the context of circulating particles around an accelerator ring. Other examples include

tune	closed orbit	normal mode decomposition
tune shift	resonance	damping partition number
chromaticity	linear lattice function	nonlinear distortion function
anharmonicity	equilibrium emittance	short-, mid-, and long-term stability

and much more.

Note that all local quantities are governed solely by the particulars of an element and the underlying equations of motion. Particle information, local field strengths, and the Lorentz equation are all you need to know for the computation of local

quantities. Global quantities, on the other hand, are governed by the fact that an accelerator ring is designed to circulate particles in stable orbits for many turns. They result from our efforts to *interpret* the one-turn map.

Polymorphs and Normal Form

The simplest simulation of particle trajectories in an accelerator represents a particle as an array z of real numbers, which constitutes the phase-space coördinates and any other quantities (spin, for example) that need to be tracked. One then tracks the particle through the accelerator by integrating the appropriate equations of motion with initial conditions given by z^i. If we wish to know the behavior of nearby particles, then we could, of course, launch other particles with initial conditions $z^i + \varepsilon$ and study how the results vary with ε. However, the polymorphic aspect of PTC enables us to do this in a way that gives us access to all of standard perturbation theory, including the kinds of global information mentioned above.

Given a software library that implements a truncated power series algebra (TPSA), one can use polymorphism and operator overloading to turn any particle tracking code into a map generation code. Rather than propagating a particle through the accelerator, one may instead propagate an array of truncated power series (TPS) that represents the particle and its phase-space neighborhood. The result of such tracking is a one turn map for that region of phase space. The polymorphism of PTC makes it possible to do this in a transparent manner.

THE CONCEPT OF A NORMAL FORM is absolutely central to the modern view of accelerator analysis. In the very simplest case, a one-turn map reduces to a matrix M, and we factor it in the form of a similarity transformation:

$$M = A \cdot N \cdot A^{-1}.$$

Here N denotes a *normal form*, and A denotes the matrix that transforms between N and M. Continuing our simplest example, N generates rotations in each of the separate phase planes, and A converts the circles of normal form coördinates to generic phase-space ellipses. If we go to another location in the accelerator, the one-turn map will differ, but it will have the same normal form:

$$M' = A' \cdot N \cdot A'^{-1}.$$

In other words, N is an invariant of the ring. Global scalars, by which we mean global quantities that do not depend on location around the accelerator, may be derived from N. The s-dependent global quantities (SDGQs) may be derived from the transformations A, A', ... Thus, for example, one extracts tunes from N and linear lattice functions from the As.

Two very important benefits derive from a normal form factorization of the one-turn map: The first benefit is that this factorization generalizes in a straightforward manner to nonlinear maps. We may thus write the map $M(z)$ as the composition

$$M(z) = A \circ N \circ A^{-1}(z), \tag{2.5}$$

where now M, A, and N all denote nonlinear functions on phase space. As in the matrix case, the normal form $N(z)$ yields the global scalars, and the transformation $A(z)$ yields the SDGQs. N, for example, contains not only the tunes, but also the nonlinear information of how the tunes vary with amplitude. The second benefit is

that we may use the *same code* that produced the one-turn map $M(z)$ to propagate the normalizing transformation $A(z)$. Doing exactly this—using the polymorphic capabilities of PTC, of course—we can compute SDGQs at many locations around the ring.

As with all things wonderful, some caveats attach to the normal form decomposition (2.5). The first caveat is associated with the approximate nature of the work we do. Unless our system is extremely special, we can know $M(z)$ only up through some finite order. Moreover, because of the complicated nature of particle orbits in accelerators, the expansion is generally asymptotic. As a consequence, we should examine all analyses in the light of common sense and actual particle tracking.[16]

[16] This should *already* be part of your routine.

The second caveat is associated with the fact that the transformation $A(z)$ is not unique: If B denotes any transformation that commutes with N, then replacing $A(z)$ in (2.5) with $A \circ B(z)$ yields the same result. In other words, the particular A we choose to work with depends on certain conventions. For physical quantities associated with $A(z)$, this freedom in the definition of A necessarily has no effect. But some quantities[17] that people discuss *do*, in fact, depend on the particular choice for the normalizing transformation A. Such quantities, needless to say, cannot be physical and should be used with great care, or not at all.[18]

[17] One example is the so-called phase advance. At matched locations in a ring, the normalizing transformations (chosen with common conventions!) are identical; as a consequence, the phase advance between such locations is well-defined. Between *un*matched locations, however, the definition of phase advance can depend on the conventions used in extracting the As.

[18] A.W. Chao. SLIM—An early work revisited. In *Proceedings of the 11th European Particle Acelerator Conference, Genoa, Italy, 23–27 June 2008*, pages 2963–2967, Geneva, 2008. European Physical Society

2.4 Modeling Particle Interactions

This overview chapter has, until now, discussed concepts related to single-particle dynamics. In this section we introduce some concepts related to the inclusion of particle interactions in PTC simulations. Some of these concepts, however, apply also to single-particle dynamics, and hence we urge even readers not interested in space-charge to at least skim this section.

We begin with a caveat: *The inclusion of particle interactions violates the philosophy of PTC.* To see this, consider a beam of self-interacting particles at a moment when the head has entered a magnet and the tail has not. Particles in the head and tail communicate via the self interactions, and hence the dynamics of particles in the head and tail affect one another. If we place the magnet in a different location, particles in the head will follow different trajectories. The particle interactions communicate this information to the tail, and now particles in the tail also follow different trajectories—their behavior differs because of a change in a magnet they have yet to see. This scenario tells us that if our beam is spatially extended and self-interacting, we cannot define an isolated propagator for each element. In other words, particle interactions violate the LEGO-block approach to modeling accelerators.

To fit within the philosophy of PTC, beamline elements must be independent of one another. This idealization—upon which most tracking codes rely—should be borne in mind when using PTC—or any of those other codes—to simulate beams with self-interacting particles.[19]

[19] Overlapping fringe fields also violate the principle of magnet independence; and in that case, too, one must exercise care.

In the rest of this section, we discuss some of the data types useful for including particle interactions: *integration_node*, *node_layout*, *probe*, and *temporal_probe*. We also describe the basic idea behind the time-based tracking capability of PTC.

Integration Node

The data type *integration_node* represents a step of integration in PTC. In particular, this data type includes entrance and exit reference frames, pointers to the previous and next integration nodes along a particle path (see *Node Layout*), and a pointer

to the parent *fibre* that contains it. Integration nodes allow us to examine data inside a beamline element without violating the element as a fundamental unit—a self-contained LEGO block. They allow us to resist the temptation to "slice" an element by hand: they *are* the slices. The data type *integration_node* also contains an integer that determines the *type* of integration node, of which there are five. See figure 2.16. On traversing the integration nodes that represent an element, one encounters, in order,

- an entrance patch (and misalignment, if any), which connects the geometry described by the fibre to that of the preceding fibre in the linked list;
- an entrance fringe field, which contains approximate fringe effects (if any);
- N body integration nodes representing the body of element;[20]
- an exit fringe field, which contains approximate fringe effects (if any);
- an exit patch, which connects the geometry described by the fibre to that of the following fibre in the linked list.

[20] The choice of N, which you are free to modify, obviously affects the accuracy of your simulation. Chapter 9 discusses how to split elements into integration nodes.

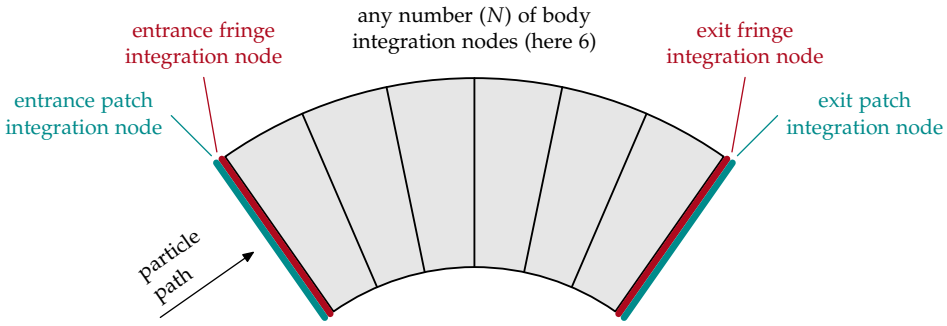


Figure 2.16: $N + 4$ integration nodes cover an element.

Using s-based tracking, we can obtain particle data at the entrance and exit of each integration node in the body of a magnet, that is, at the seven black lines in figure 2.16. In addition, we can obtain first-order information about the location within a node of a particle at some fixed time (see *Time-based Tracking*).

The integration nodes *contain no duplicate data about the elements*; they have no existence apart from the beamline elements they represent. Because the data in the integration nodes reside on the fibres (which point to the elements), PTC is able to carry over changes affecting elements (*e.g.*, a change in the magnetic field) and changes affecting the fibres (*e.g.*, misalignments) to the integration nodes.

Node Layout

The data type *node_layout* represents a beamline as a linked list of *integration_node*s. It includes pointers to the first and last integration nodes in the beamline, as well as a pointer to the parent *layout*. To access the integration nodes of a particular fibre, one may step through them, making use the fact that the data type *fibre* includes pointers to the first, last, and middle integration nodes for the corresponding element.

A *node_layout* is not created automatically when you create a *layout*: You must ask PTC to create it and populate the associated reference frames.

Probe and Temporal Probe

The data type *probe* represents a particle. In particular, it contains, for a given particle, both the orbital phase-space data and a *spinor* for the spin data. Should the

particle become lost, the *probe* contains a *logical* in which to record this fact, as well a pointer to the *integration_node* in which the particle loss occured.

The data type *temporal_probe* contains a *probe* together with information relevant to the time-based tracking capability of PTC. This includes a pointer to the *integration_node* that currently contains the particle.

Time-based Tracking

Any high-order s-based integrator may be converted to a first-order time-based integrator *provided information is available about the three-dimensional environment.* When we ask PTC to perform time-based tracking, PTC augments its s-based information with temporal information. PTC's tracking remains fundamentally s-based, but with the additional information, it can determine which integration node a particle is in at a given time. PTC can then compute a first-order accurate location inside that node for the particle at that time.

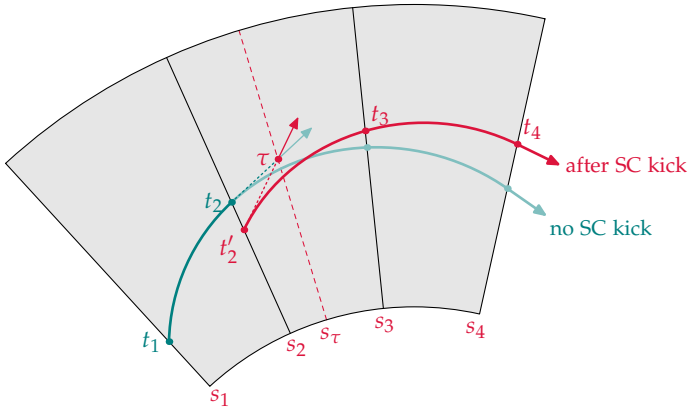


Figure 2.17: Applying a space-charge kick at time τ.

PTC's time-based tracking capability allows us to obtain a snapshot of the beam at a fixed time. Figure 2.17 illustrates this approach to applying a space-charge kick at time τ:

1. PTC checks node by node the entrance and exit times of a particle to determine the integration node for which $\tau \in [t_{\text{entrance}}, t_{\text{exit}})$. After locating the appropriate node, PTC returns the particle to the node entrance. At this point in figure 2.17, because τ falls between times t_2 and t_3, the particle is on the blue curve at the point labeled with time t_2.
2. PTC determines the time difference $\delta t = \tau - t_{\text{entrance}}$. Then, using the particle's position and momentum at the node entrance as an initial condition, and assuming the current integration node to be a drift, PTC computes a position and momentum for the particle at time τ. It records the shift δs in the *temporal_probe*. If the element *is* a drift, the position of the particle at time τ is exact. If the element is *not* a drift, the position of the particle at time τ is a close approximation. At this point in figure 2.17, the particle is at the point labeled with time τ.
3. After completing the above two steps for all particles in the beam, you can apply space-charge kicks to your particles. At this point in figure 2.17, the particle remains at the point labeled with time τ, but it has a new momentum, as indicated by the angle between the light blue arrow and the red arrow.

4. PTC now drifts the particle—with its new momentum—back to the entrance of the integration node. At this point in figure 2.17, the particle is back in the s_2 plane, but now on the red curve at the point labeled with time t_2'.
5. PTC now continues tracking the particle—with its new momentum—seeking the integration node for which t_{entrance} and t_{exit} bracket the time for the next space-charge kick.

THREE

Modeling an Accelerator with PTC

This chapter explains how to use PTC to model the geometry for a range of accelerators. To that end, we introduce three accelerator topologies and show how to define their geometries using PTC. After going through these models in detail, you should have a good understanding of how to use PTC to model your own accelerator designs. If some of your elements move together as units—because they're tied to a girder, for example—then you will also want to absorb the information presented in chapter 4, *Linking Magnets Together and Moving Them as a Group*.

In § 3.1 we describe briefly the accelerator topologies we shall model. In § 3.2 we introduce the PTC source code for our examples. The three accelerator topologies use some common structures, and we describe the subroutines for these in § 3.3. Then, in §§ 3.4 and 3.5, we construct our DNA sequences and show in detail how to construct our three accelerator designs. We conclude, in § 3.6, with some housekeeping for our DNA database.

3.1 Accelerator Models

Figure 3.2 illustrates the three accelerator topologies we shall model:

- figure-eight,
- ring with both forward and reverse propagation,
- collider.

All three machines are based on a ten-cell ring, with each cell composed of seven elements: a drift, a focusing quadrupole, a short drift, a dipole, a short drift, a defocusing quadrupole, and another drift.[1] In the figure-eight and collider examples, the dipole is a rectangular bend. In the ring with forward and reverse propagation, however, the dipole is what we shall call, for want of a better term, a *"straight" bend* (with the quotation marks!). By this we shall mean a rectangular bending magnet whose entrance and exit reference frames are parallel to one another and orthogonal to the main axis of the magnet. This implies—see *Chart and Patch*, page 15—that any cell containing a "straight" bend will require patching.

[1] When you see the particular numbers, in a few pages, you may recognize this cell as that of the Los Alamos Proton Storage Ring.

Figure 3.1 illustrates the two basic cells. We outline the quadrupoles in blue, the rectangular bend in red, and the "straight" bend in green. The black lines indicate the drifts. The cell with the rectangular bend does not require patching (unless the direction or charge change), because the reference frames of adjacent elements lie on top of one another. The cell with the "straight" bend, however, requires patching (regardless of direction or charge), because the adjacent drifts have frames that are rotated with repect to those of the "straight" bend.

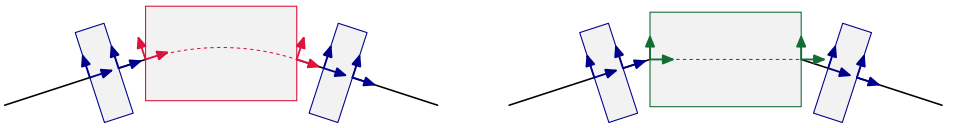


Figure 3.1: Basic cells for the three accelerator models.

The basic ring (see the `build_PSR` example in figure 3.3 on page 26) has ten cells. It has 70 fibres pointing to 70 elements. The ring with forward and reverse propagation has 140 fibres pointing to 140 elements.

The figure-eight and the collider each have 140 fibres pointing to 134 elements. In each of those models the upper and lower rings share the elements at the start of one cell (long drift, quadrupole, short drift) and the elements at the end of another cell (short drift, quadrupole, long drift).

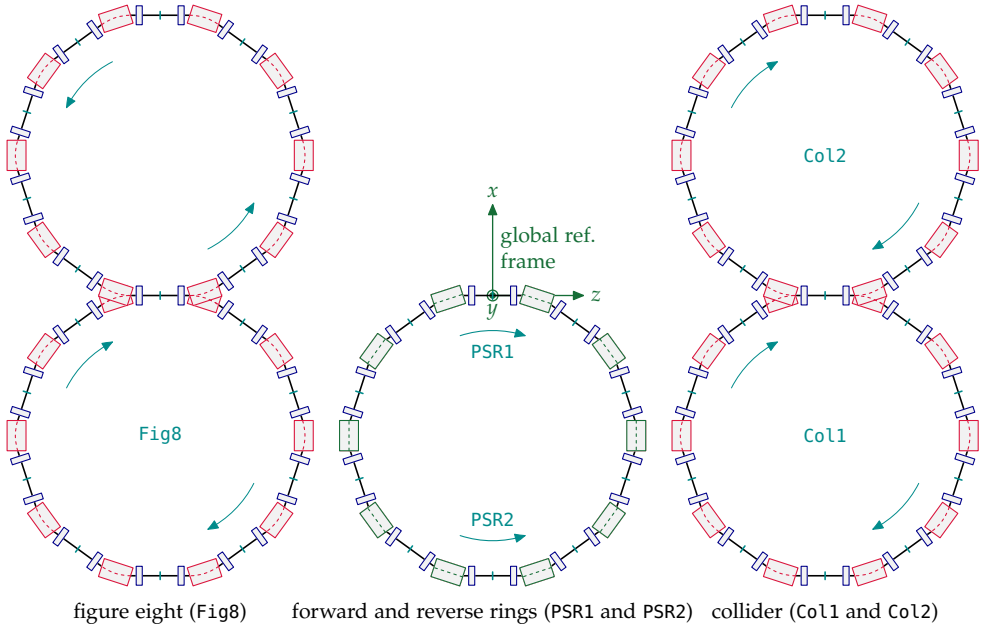


Figure 3.2: Accelerator models. Arrows indicate the direction of motion of particle beams in the constituent layouts.

Our three tutorial examples are not real accelerators: They do not have particle injectors or dumps; and two of the examples are not physically possible, because some of their elements overlap. The goal of these examples is to introduce the principal PTC concepts involved in modeling the geometry of an accelerator. After learning these concepts, we can use PTC to model complex real-world accelerators.

3.2 Geometry Tutorial Source File

The example code in this chapter is from the PTC geometry tutorial source file, `ptc_geometry.f90`, which is given in appendix C. The line numbers of the code in the examples refer to the line numbers of the code in that appendix.

Initial Code

The initial code in the `ptc_geometry.f90` source file includes type declarations and performs some initialization.

`ptc_geometry.f90` This program describes the geometry of several PTC lattices.

```
program ptc_geometry
use run_madx
use pointer_lattice
implicit none

character*48 :: command_gino
logical(lp) :: doit
integer :: i, j, mf, pos, example
real(dp) :: b0
real(dp), dimension(3) :: a, d
real(dp), dimension(6) :: fix1, fix2, mis, x
type(real_8), dimension(6) :: y1, y2
type(layout), pointer :: L1, L2, L3, L4, L5, L6
type(layout), pointer :: PSR1, PSR2, Fig8, Col1, Col2
type(fibre), pointer :: p1, p2, b, f
type(internal_state) :: state

type(pol_block) :: qf(2), qd(2)
type(normalform) :: n1, n2
type(damap) :: id
type(taylor) :: eq(4)
type(gmap) :: g
!----------------------------------

Lmax = 100.d0
use_info = .true.

!== user stuff : one layout necessary before starting GUI
call ptc_ini_no_append
```

The module `run_madx` tells PTC to use the `madx_ptc_module`, which defines the global variable `m_u`.[2] This variable, which we shall use frequently, denotes a linked list of layouts[3] that will constitute our DNA database. The `gino` mentioned on line 6 refers to a Windows®-based graphical user interface developed for PTC by Étienne Forest. Its use is not required.

PTC uses the following type declarations for modeling accelerator topologies:

- `real(dp)` for double precision real numbers,
- `type(layout)` for *layout*s,
- `type(fibre)` for *fibre*s,
- `type(internal_state)` for setting the characteristic dynamics desired of your accelerator (see *Internal States*, appendix A).

These data types will be discussed in this chapter.

PTC uses the following type declarations for analyzing accelerator properties:

- `type(real_8)` for polymorphs,
- `type(pol_block)` for polymorphic blocks,

[2] Think "*Mad Universe*".

[3] We use the linked-list aspect of `m_u` only briefly in § 3.6, *DNA Arrays*,.

- `type(normalform)` for normal forms,
- `type(damap)` for differential algebra maps,
- `type(taylor)` for Taylor maps,
- `type(gmap)` for a vector of Taylor maps.

These latter data types will be described in chapter 5, *Taylor Polymorphism and Knobs*.

The global variable `Lmax` defines the maximum length (here 100 meters!) of an integration node. For more information about splitting elements into integration nodes, see *Symplectic Integration and Splitting*, chapter 9.

This initial code also, on line 29, initializes a layout; this must be done before starting PTC's Gino-based graphical user interface for Windows.

3.3 Subroutines

The end of the `ptc_geometry.f90` source file contains three subroutines we use to create the DNA sequences—non-trackable layouts for our DNA database. Figure 3.3 shows the layouts that the three subroutines create. In subsequent sections of this chapter, we describe how to use the layouts from this database, or pieces of these layouts, to form the accelerator models of figure 3.2.

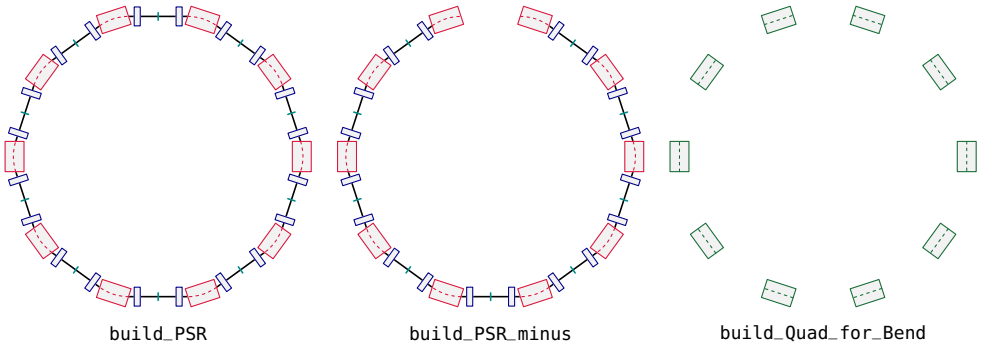


Figure 3.3: Subroutines for creating DNA sequences.

build_PSR

The subroutine `build_PSR` creates the basic ten-cell ring lattice shown on the left in figure 3.3.

```
490 subroutine  build_PSR(PSR)
    use run_madx
    use pointer_lattice
    implicit none

495 type(layout), target :: PSR

    real(dp) :: ang, brho, kd, kf, Larc
    type(fibre) :: b, d1, d2, qd, qf
    type(layout) :: cell
500 !----------------------------------
```

```
    call make_states(.false.)
    exact_model = .true.
    default = default + nocavity + exactmis
505 call update_states
    madlength = .false.

    ang = (twopi * 36.d0 / 360.d0)
    Larc = 2.54948d0
510 brho = 1.2d0 * (Larc / ang)
    call set_mad(brho = brho, method = 2, step = 10)
    madkind2 = drift_kick_drift

    kf =  2.72d0 / brho
515 kd = -1.92d0 / brho

    d1 = drift("D1", 2.28646d0)
    d2 = drift("D2", 0.45d0)
    qf = quadrupole("QF", 0.5d0, kf)
520 qd = quadrupole("QD", 0.5d0, kd)
    b  = rbend("B", Larc, ang)
    cell = d1 + qd + d2 + b + d2 + qf + d1

    PSR = 10 * cell
525 PSR = .ring.PSR

    call survey(PSR)
    end subroutine build_PSR
```

The lines 502–506 set important internal state variables for tracking. In the call to `make_states`, the boolean argument `.false.` means we are modeling a proton lattice. Use `.true.` for electrons.[4] To use the full "square-root" Hamiltonian, we set the global parameter `exact_model` to `.true.` The global variable `default`, of type *internal_state*, is here modified from its default value by adding the following two flags:

- `nocavity`, which tells PTC to ignore RF cavities;
- `exactmis`, which tells PTC to treat misalignments exactly.

(For more on internal-state variables, see appendix A.) Finally, we set the flag `madlength` to `.false.` This means that PTC will use the arc length, rather than the chord length, to define the geometry of a rectangular bending magnet. Use `.true.` to make PTC use the chord length.

The call, in line 511, to `set_mad` defines (via `brho`) the scale momentum p_o for the fibres in the layout returned by this subroutine. It also specifies the type (`method = 2`) and number (`STEP = 10`) of integration steps in the body of each element. We then define the five fibres needed for our basic cell:

- Fibres `d1` and `d2` denote the long and short drifts.
- Fibres `qf` and `qd` denote the focusing and defocusing quadrupoles.
- And fibre `b` denotes the rectangular bending magnet. Note that it is defined by its arc length and bend angle—here `Larc` and `ang`, respectively. See figure 3.4.

The symbols in quotes (*e.g.*, `"QF"`) are the *names* given to these elements.[5] We may now define the variable `cell`, of type *layout*, as the appropriate sequence of fibres.

[4] For other particles, use the ratio of particle mass to electron mass as the argument of `make_states`.

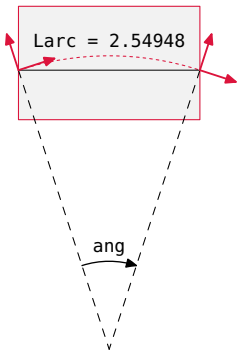


Figure 3.4: Geometry of the rectangular bend.

[5] Because a layout may have any number of these elements, the names actually define *classes* of elements.

Finally, the layout PSR returned by this subroutine is defined as 10 cells, and line 525 modifies PSR to make it a ring, *i.e.*, closed.

At this point, line 525, the elements of the PSR lattice are present, and in the correct sequence of fibres, in layout PSR. But should we ask for the location of those elements,[6] we would discover that all of them have their entrance frames at the global origin. In other words, they are all stacked on top of one another at the global origin. The call to survey causes PTC to loop through the fibres in PSR, moving each element so that its entrance frame coincides with the exit frame of the previous element. For the PSR lattice, the reference frames for the bends and quadrupoles line up exactly with those of their adjacent drifts. As a consequence, this lattice requires no patching. We have therefore finished building the PSR lattice in PTC.

[6] How to ask this of PTC will become clear later in this chapter. See, for example, page 33—especially the discussion of moving and rotating elements.

A FEW READERS WILL COMPLAIN that the set of lines 517 through 522 violates the philosophy of PTC—and they have a point. This may seem subtle, but we urge all other readers to invest the time required to comprehend this point. Look, for example, at the definition of the bend given in line 521. After careful consideration of this definition and what happens on the succeeding lines, the reader should recognize that this definition actually serves a dual purpose. On the one hand, the setting of arc length and bend angle define the *geometry* of the element. On the other hand, this line also serves to define the *physics* of this element. To verify this claim, query PTC for the magnetic field of this element:

```
write(6,'(a,f7.4)') "PSR B-field = ", b%mag%bn(1) * brho
```

will do the trick. You will learn that PTC already knows the magnetic field has a value of 1.2 T! This information, of course, requires a knowledge of not only the geometry, but also the particle mass and energy. *Here* is the apparent violation of the PTC philospophy, which aims to separate the geometric description of beamline elements (location, reference frames, *etc.*) from their physical content (magnetic field strength, *etc.*). The resolution of this conflict lies in the fact that once we have nailed down the geometry, we are then free to modify the lattice with misalignments, changes to the magnetic field, and the like.

build_PSR_minus

The subroutine build_PSR_minus defines the partial ring shown in the center of figure 3.3. It begins with a bend, a short drift, a quadrupole, and a long drift; it continues with eight PSR cells; and it concludes with a long drift, a quadrupole, a short drift, and a bend. Relative to the PSR lattice, this partial ring is missing a short drift, a quadrupole, two long drifts, a quadrupole, and a short drift. The subroutine is a straightforward modification of that for build_PSR.

```
    subroutine  build_PSR_minus(PSR)
    use run_madx
    use pointer_lattice
535 implicit none

    type(layout), target :: PSR

    real(dp) :: ang, brho, kd, kf, Larc
540 type(fibre) :: b, d1, d2, qd, qf
    type(layout) :: cell
    !----------------------------------
```

```
    call make_states(.false.)
    exact_model = .true.
    default = default + nocavity + exactmis
    call update_states
    madlength = .false.

    ang = (twopi * 36.d0 / 360.d0)
    Larc = 2.54948d0
    brho = 1.2d0 * (Larc / ang)
    call set_mad(brho = brho, method = 6, step = 10)
    madkind2 = drift_kick_drift

    kf =  2.72d0 / brho
    kd = -1.92d0 / brho

    d1 = drift("D1", 2.28646d0)
    d2 = drift("D2", 0.45d0)
    qf = quadrupole("QF", 0.5d0, kf)
    qd = quadrupole("QD", 0.5d0, kd)
    b  = rbend("B", Larc, ang)
    cell = d1 + qd + d2 + b + d2 + qf + d1

    PSR = b + d2 + qf + d1 + 8 * cell + d1 + qd + d2 + b
    PSR = .ring.PSR

    call survey(PSR)
    end subroutine build_PSR_minus
```

build_Quad_for_Bend

The subroutine `build_Quad_for_Bend` defines the layout shown on the right in figure 3.3. It has ten "straight" bends, which have the same length as the rectangular bends in the PSR lattice. Because this lattice contains no drifts, it will require patching. In addition, because it contains no focusing elements, the elements of this layout will actually be used only as entries in other layouts.

```
    subroutine  build_Quad_for_Bend(PSR)
    use run_madx
    use pointer_lattice
    implicit none

    type(layout),target :: PSR

    real(dp) :: ang, ang2, brho, b1, Larc, Lq
    type(fibre) :: b
    !-----------------------------------

    call make_states(.false.)
    exact_model = .true.
```

```
    default = default + nocavity + exactmis
    call update_states
    madlength = .false.
590
    ang = (twopi * 36.d0 / 360.d0)
    Larc = 2.54948d0
    brho = 1.2d0 * (Larc / ang)
    call set_mad(brho = brho, method = 6, step = 10)
595 madkind2 = drift_kick_drift

    ang2 = ang / two
    b1 = ang / Larc
    Lq = Larc * sin(ang2) / ang2
600
    b = quadrupole("B_QUAD", Lq, 0.d0);
    call add(b, 1, 0, b1)
    b%mag%permfringe = .true.
    b%magp%permfringe = .true.
605 b%mag%p%bend_fringe = .true.
    b%magp%p%bend_fringe = .true.

    PSR = 10 * b
    PSR = .ring.PSR
610
    call survey(PSR)
    end subroutine build_Quad_for_Bend
```

Except for some changes in the type declarations, the code through line 595 in this subroutine is identical to that in the previous two subroutines. The "straight" bend is created in lines 597 through 606. We first compute the length `Lq` as the chord length of the PSR's rectangular bend. To achieve the desired "straight" reference frames, we define the "straight" bend as a zero-strength quadrupole. Then, to make this a dipole, we set, in line 602, the normalized dipole strength of `b` to the computed value `b1`. Finally, we set some flags to give this magnet the fringe fields appropriate to a rectangular bend.

Query: After the call to `survey` in line 611, where in the global frame are the magnets that this subroutine defines?[7]

[7] They lie all in a straight line, one after the other, starting at the global origin and extending to a point 10 `Lq` out along the z axis. Making this layout look like the right-hand layout of figure 3.3 will require more work.

3.4 Populating the DNA Database

Using the subroutines described in the previous section, we shall populate our DNA database with six DNA sequences (non-trackable layouts): `L1`, `L2`, `L3`, `L4`, `L5`, and `L6`. Note that we shall want each element in our three accelerator models to appear once and only once in the DNA sequence portion of the DNA database. This uniqueness will reflect the uniqueness of the individual beamline elements in our accelerators.

Layouts `L1` and `L2` will provide the elements for the concentric rings accelerator: trackable layouts `PSR1` and `PSR2`. (See figure 3.2.) Layouts `L3` and `L4` will provide the elements for the figure-eight accelerator: trackable layout `Fig8`. And layouts `L5` and `L6` will provide the elements for the collider: trackable layouts `COL1` and

Non-trackable layouts	Trackable layouts
L1: 70 elements (build_PSR)	PSR1: created from L1 and L2
L2: 10 elements (build_Quad_for_Bend)	PSR2: created from L1 and L2
L3: 64 elements (build_PSR_minus)	Fig8: created from L3 and L4
L4: 70 elements (build_PSR)	Col1: created from L6
L5: 64 elements (build_PSR_minus)	Col2: created from L5 and L6
L6: 70 elements (build_PSR)	

Table 3.1: DNA database for the PTC geometry tutorial

COL2. Table 3.1 summarizes the layouts we plan to create: six DNA sequences, or non-trackable layouts, and five trackable layouts.

Query: As initially constructed (by build_PSR), layouts L1, L4, and L6 are identical. So also are layouts L3 and L5. Why don't we avoid this duplication?[8]

[8] Because we want to be able to modify—move, mispower, *etc.*— the individual elements. We do not want changes made to COL1, for example, to appear in Fig8.

The following four lines create the first DNA sequence:

```
call append_empty_layout(m_u) ! DNA sequence 1
call set_up(m_u%end)
L1 => m_u%end
call build_PSR(L1)
```

The call to append_empty_layout appends an empty layout to the global linked list of layouts m_u. The layout pointer m_u%end points to this empty layout. The second line initializes this new layout; and the third line makes L1 point to it. Finally, the call to build_PSR populates layout L1 with the 70 elements shown for the PSR in figure 3.3.

We repeat this process five times for layouts L2, L3, L4, L5, and L6. For L2, we call build_Quad_for_Bend, which populates that layout with 10 elements (the "straight" bends). For L3, we call build_PSR_minus, which populates that layout with 64 elements. And for L4, L5, and L6, we repeat the calls to build_PSR and build_PSR_minus.

```
call append_empty_layout(m_u) ! DNA sequence 2
call set_up(m_u%end)
L2 => m_u%end
call build_Quad_for_Bend(L2)

call append_empty_layout(m_u) ! DNA sequence 3
call set_up(m_u%end)
L3 => m_u%end
call build_PSR_minus(L3)

call append_empty_layout(m_u) ! DNA sequence 4
call set_up(m_u%end)
L4 => m_u%end
call build_PSR(L4)

call append_empty_layout(m_u) ! DNA sequence 5
call set_up(m_u%end)
L5 => m_u%end
```

```
60 call build_PSR_minus(L5)

   call append_empty_layout(m_u) ! DNA sequence 6
   call set_up(m_u%end)
   L6 => m_u%end
65 call build_PSR(L6)
```

We now have six DNA sequences, or non-trackable layouts, in our DNA database. Moreover, each beamline element appears just once in that database.

The PTC type *element* contains an object, `doko` (from the Japanese word for "where"), that enables PTC to keep track of which fibres point to a given beamline element. This object `doko` allows PTC to use a given element multiple times in the database of trackable layouts. We shall see several examples of this in the next section.

3.5 Modeling Complex Accelerator Topologies

In the previous section, we created the layouts in the left-hand column of table 3.1. We now set about creating the layouts (right-hand column of table 3.1) for the three accelerator models shown in figure 3.2: the ring with forward and reverse propagation, the figure-eight lattice, and the collider. Complex accelerator topologies typically have a one-to-many correspondence between some of the beamline elements (stored in the DNA sequences of the DNA database) and the fibres in the trackable layouts. By the end of this section, you should understand how to set up such correspondences.

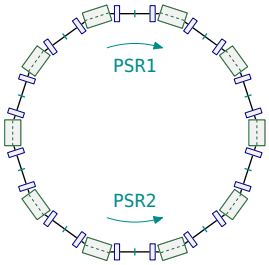


Figure 3.5: Ring with forward and reverse propagation.

Ring with Forward and Reverse Propagation

Here we model an accelerator that carries a pair of counter-propagating beams (figure 3.5, also middle lattice in figure 3.2). The two beams will propagate through different layouts—`PRS1` for the forward-propagating beam, and `PRS2` for the backward-propagating beam—but the two trackable layouts must, of course, share the same ring with the same beamline elements. They will just have different directions of propagation and oppositely charged particles.

There is a further complication: this ring uses "straight" bends instead of rectangular bends (see figure 3.1). In order to place these elements in their proper locations, we will use the lattice of layout `L1` as a guide, taking the "straight" bends from layout `L2`.[9] The final result will be a pair of layouts, `PSR1` and `PSR2`, each of which refer to fibres in two different DNA sequences, `L1` and `L2`.

[9] The point of this complication is to illustrate some of PTC's geometry operations and the process of patching. It should also, we hope, emphasize the LEGO-block concepts of PTC (*cf.* figure 2.3 and the associated discussion).

We begin by calling `append_empty_layout` on `m_u` and then setting the layout pointer `PSR1` to point to this new layout:

```
72 !== PSR1 : forward ring  (layout 7)
   call append_empty_layout(m_u)
   PSR1 => m_u%end
```

We shall populate this layout with a linked list of fibres taken appropriately from layouts `L1` and `L2`. Two fibre pointers `p1` and `p2` will step through these two layouts, and a third fibre pointer, `f`, will keep track of our location in `PSR1`. Elements taken from `L1` have the correct geometric relations, but the "straight" bends taken from `L2` will have to be moved to their correct locations.

```
   p1 => L1%start
   p2 => L2%start
   do i = 1, L1%n
     if(p1%mag%name == "B") then
80     ! read bends from L2
       call append_point(PSR1, p2)
       f => PSR1%end
       d = p1%chart%f%o - f%chart%f%o
       call translate(f, d)
85     call compute_entrance_angle(f%chart%f%mid, p1%chart%f%mid, a)
       call rotate(f, f%chart%f%o, a, basis = f%chart%f%mid)
       p2 => p2%next
     else
       call append_point(PSR1, p1)
90   end if
     p1 => p1%next
   end do ! elements in PSR1 now in correct locations
```

In the above block of code, we point `p1` and `p2` respectively to the starts of DNA sequences `L1` and `L2`. The `do` loop over the `L1%n` fibres in `L1` appends to `PSR1` the desired fibres from `L1` or `L2`—bends from `L2`, everything else from `L1`:

- If `p1` points to a bend, then in line 81 we append the current fibre of `L2` to `PSR1`. This, of course, will always be a "straight" bend—a `B_QUAD`. Later, in line 87, we advance `p2` to the next fibre in `L2`.
- If `p1` does *not* point to a bend, then in line 89 we append the current fibre of `L1` to `PSR1`.
- In either case, we advance, in line 91, `p1` to the next fibre in `L1`.

After we append a "straight" bend to `PSR1`, we must do some extra work to place it in the correct location. Since we want a given "straight" bend to have the same location and orientation as the corresponding rectangular bend, we simply compute and perform the required translations and rotations. This happens in lines 82 through 86:

1. Point `f` to the fibre most recently appended to `PSR1`.
2. Compute, in line 83, the vector `d` from the center of the newly appended "straight" bend (`f%chart%f%o`)[10] to the center of the corresponding rectangular bend in `L1` (`p1%chart%f%o`).
3. Translate the newly appended fibre (line 84).
4. Compute, in line 85, the rotation `a` from the frame attached to the middle of the newly appended "straight" bend (`f%chart%f%mid`) to the frame attached to the middle of the corresponding rectangular bend in `L1` (`p1%chart%f%mid`).[11]
5. Rotate the newly appended fibre about its center (`f%chart%f%o`) by angles `a`, which are given with respect to the frame attached to the middle of that element (`f%chart%f%mid`).

When the above `do` loop terminates, the beamline elements of `PSR1` are all in their correct locations. However (recall the discussion concerning figure 3.1) the "straight" bends of `PSR1` all require patching. This happens in the following block of code:

[10] Read `f%chart%f%o` as "f-chart-frame-origin".

[11] Because rotations do not commute, their order is important. PTC performs coördinate rotations in the order x-axis, $-y$-axis, z-axis.

```
   f => PSR1%start
95 do i = 1, PSR1%n
     if(f%mag%name == "B_QUAD") then
```

```
      call find_patch(f%previous, f, next = .true.)
      call find_patch(f, f%next, next = .false.)
    end if
100 f => f%next
  end do ! PSR1 now patched
```

Within the `do` loop over all elements in `PSR1`, we apply the patches required between each `B_QUAD` (or "straight" bend) and its preceding and trailing elements. No other elements require patching.

Finally, in the following three lines of code, we give layout `PSR1` a formal name, and we ensure that it forms a closed topological ring, so that particles can circulate. Note that the second two lines must *both* be executed to make the layout `PSR1` form a closed ring. The call to `ring_L` in line 105 sets some pointers inside the layout `PSR1` that connect the end of `PSR1` to the start, and vice versa.

```
  PSR1%name = "PSR 1"
  PSR1%closed = .true.
105 call ring_L(PSR1, .true.) ! make it a ring topologically
```

Query: At this point, where in the global frame are the magnets, the "straight" bends, of layout `L2`?[12]

To construct layout `PSR2` for the backward-propagating beam, we follow essentially the same steps as for `PSR1`. (See the next block of code.) There are three significant differences: (i) In this case, all the elements are already in their proper physical locations,[12] so here we may omit the geometric computations and operations performed during the construction of `PSR1`. (ii) Because of the backwards propagation, we initialize the pointers `p1` and `p2` respectively to the *ends* of layouts `L1` and `L2`; and we advance those pointers not to the next but to the *previous* fibres in their respective layouts. (See lines 117 and 124.) (iii) We must add the information that the beam described by this layout traverses its elements in the reverse direction (line 122) and has particles with negative charge (line 123).[13]

[12] The geometry operations performed on the bend *fibre*s of `PSR1` applied, ultimately, to the *element*s in `L2`. Hence those elements are now oriented as in the right-hand layout of figure 3.3, with the global origin centered between the adjacent ends of the top two magnets.

[13] Both direction and charge are properties of fibres, and both default to +1.

```
  !== PSR2 : backward ring (layout 8)
  call append_empty_layout(m_u)
110 PSR2 => m_u%end

  p1 => L1%end
  p2 => L2%end
  do i = 1, L1%n
115 if(p1%mag%name == "B") then
      call append_point(PSR2, p2)
      p2 => p2%previous
    else
      call append_point(PSR2, p1)
120 end if
    f => PSR2%end
    f%dir = -1
    f%charge = -1
    p1 => p1%previous
125 end do
```

```
    f => PSR2%start
    do i = 1, PSR2%n
      if(f%mag%name == "B_QUAD") then
130     call find_patch(f%previous, f, next = .true.)
        call find_patch(f, f%next, next = .false.)
      end if
      f => f%next
    end do
135
    PSR2%name = "PSR 2"
    PSR2%%closed = .true.
    call ring_L(PSR2, .true.) ! make it a ring topologically
```

Trackable layouts `PSR1` and `PSR2` are now complete. Both contain 70 fibres pointing to the same set of 70 elements in DNA sequences `L1` and `L2`.

Figure-Eight

Here we model a figure-eight lattice—the left-hand lattice in figure 3.2. It carries a single beam clockwise (forward) around the lower ring, then counter-clockwise (backward) around the upper ring. The two rings share several elements in one straight section. See figure 3.6. The single trackable layout `Fig8` we shall construct from DNA sequences `L3` and `L4` (see table 3.1). For the lower ring, we use the full ten-cell PSR lattice in layout `L4`. For the upper ring, we use all of layout `L3` (`PRS_minus`), and we re-use six of the fibres in layout `L4`. In addition, so that this layout does not overlap our previous layouts, `PSR1` and `PSR2`, we shall place it 40 m distant in the negative z direction.

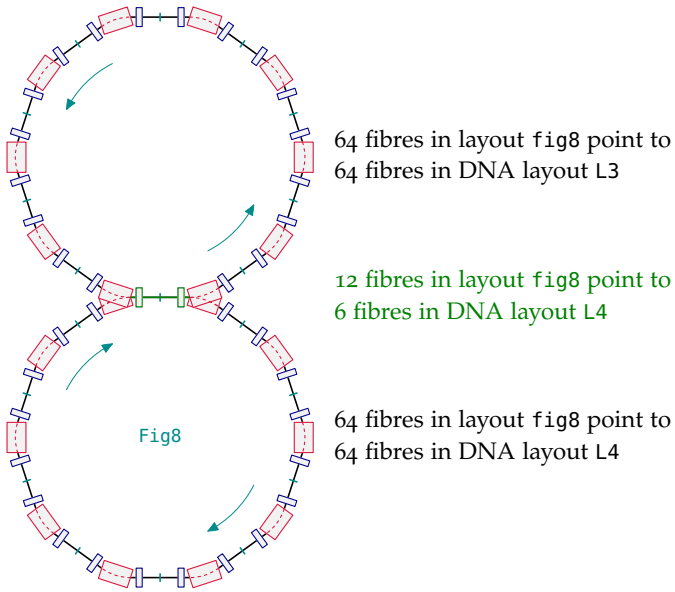


Figure 3.6: `Fig8` fibres pointing to elements in `L3` and `L4`.

The construction of layout `Fig8` takes place in the following six blocks of code. We describe each in detail.

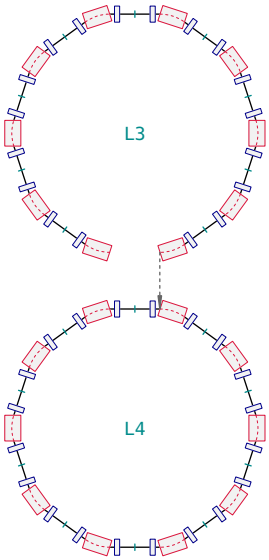


Figure 3.7: Matching L3 to L4.

In this first block of code, we translate layout L4 a distance 40 m in the negative z direction (lines 142–144). We then (lines 145–147) rotate layout L3 180° about the entrance of its first element (L3%start%chart%f%a). This rotates the gap in L3 down to the bottom. Now note that the bend to the right of this gap (after the rotation) belongs to the last fibre in L3. To finish placing L3 in the correct location, we want the *end* of its last bend to coincide with the entrance of the first bend in the lower ring, L4. See figure 3.7. We therefore point, in line 148, the fibre pointer p1 to the first bend ("B") in L4; compute, in line 149, the vector d from the exit of the last bend in L3 (L3%end%chart%f%b) to the entrance of the first bend in L4 (p1%chart%f%a); and then translate, in line 150, layout L3 by vector d.

```
   !== Fig8 : figure-eight lattice (layout 9)
   d = zero
   d(3) = -40.d0
   call translate(L4, d)
145 a = zero
   a(2) = pi
   call rotate(L3, L3%start%chart%f%a, a)
   call move_to(L4, p1, "B", pos)
   d = p1%chart%f%a - L3%end%chart%f%b
150 call translate(L3, d)
```

At this point, the beamline elements for the figure-eight lattice are all correctly located and oriented.

In the next block of code, we first call append_empty_layout on m_u and set the layout pointer Fig8 to point to this new layout. Then, in lines 154–158, we append in order all the fibres of L4 to Fig8.

```
   call append_empty_layout(m_u)
   Fig8 => m_u%end
   p1 => L4%start
155 do i = 1, L4%n
     call append_point(Fig8, p1)
     p1 => p1%next
   end do
```

We now have the complete lower ring.

In the next block of code, we start the upper ring by appending to Fig8 the first three fibres of the lower ring. These three new fibres in Fig8 will, of course, point to the same physical elements as do the first three fibres in Fig8: a long drift, a defocusing quadrupole, and a short drift. During the calls to append_point, PTC will add this information to the dokos of the corresponding elements in L4. As a consequence, each of those three elements in DNA sequence L4 knows that it is pointed to by two different fibres in trackable layout Fig8.[14]

[14] Each of those elements already knows it belongs to a fibre in layout L4.

Note that we need not initialize the pointer p1 to L4%start: the last execution, in line 157, of p1 => p1%next returns p1 to the beginning of the layout.

```
160 write(6,*) p1%mag%name
   call append_point(Fig8, p1)
   p1 => p1%next
   write(6,*) p1%mag%name
```

```
    call append_point(Fig8, p1)
165 p1 => p1%next
    write(6,*) p1%mag%name
    call append_point(Fig8, p1)
```

In the next block of code, we append the fibres of layout `L3` to `Fig8`. Since our beam traverses `L3` in the reverse direction, we initialize the fibre pointer `p1` to `L3%end` (line 169), and we advance that pointer to the *previous* fibre (line 174). During this process, we take care of two other tasks: We tell PTC that these elements are traversed in the reverse direction (line 172); and, if the element is a bend, we reverse the sign of the magnetic field (line 173). The particle charge remains the same (default value +1).

```
    p1 => L3%end
170 do i = 1, L3%n
      call append_point(Fig8, p1)
      Fig8%end%dir = -1
      if(p1%mag%name == "B") p1%mag%bn(1) = -p1%mag%bn(1)
      p1 => p1%previous
175 end do
```

To complete the upper ring, and hence our trackable layout `Fig8`, three fibres remain to be appended: the last three fibres of the lower ring, `L4`. This is accomplished in the next block of code. First, in line 177, we point `p1` to the fibre containing the short drift near the end of layout `L4`. We then append to `Fig8` that fibre and the next two. During the calls to `append_point`, PTC will add the appropriate information to the `dokos` of the corresponding elements.

```
    p1 => L4%end%previous%previous
    write(6,*) p1%mag%name
    call append_point(Fig8, p1)
180 p1 => p1%next
    write(6,*) p1%mag%name
    call append_point(Fig8, p1)
    p1 => p1%next
    write(6,*) p1%mag%name
185 call append_point(Fig8, p1)
```

Finally, in the last block of code for `Fig8`, we give our new layout a formal name (`"Figure-Eight"`), ensure that it is topologically closed, and apply any necessary patches. The call to `check_need_patch`, line 194, returns the integer `pos` equal to zero if no patch is needed. A non-zero value indicates the type of patch required. By using `check_need_patch`, we can apply patches only where necessary. Our figure-eight lattice requires patching because some of the constituent elements are traversed in the reverse direction. In particular, there are adjacent fibres `f` which have opposite values of `f%dir`. This happens between the short drift at end of the common straight section and the first bend of the upper ring; and also between the last bend of the upper ring and the subsequent short drift.

```
    write(6,*) "Fig8 has ", Fig8%n, " fibres"
    Fig8%name = "Figure-Eight"
    Fig8%closed = .true.
```

```
call ring_L(Fig8, .true.) ! make it topologically closed

p1 => Fig8%start
do i = 1, Fig8%n
  call check_need_patch(p1, p1%next, 1.d-10, pos)
  if(pos /= 0) call find_patch(p1, p1%next, next = .false.)
  p1 => p1%next
end do
```

Trackable layout `Fig8` is now complete. It has 140 fibres pointing to 134 elements in DNA sequences `L4` and `L3`. Twelve of the fibres in `Fig8` point to the six elements (four drifts and two quadrupoles at the top of `L4`) which are common to the upper and lower rings. See figure 3.6.

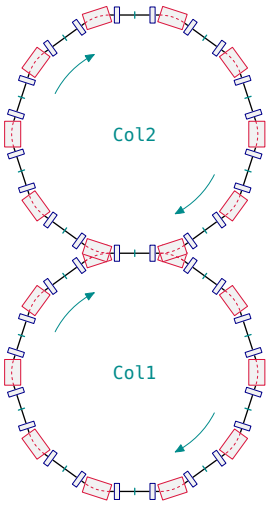


Figure 3.8: Collider.

[15] Yes, this sounds much like the description for layout `Fig8`. Be sure to note the *differences*.

Collider

Here we model a collider—figure 3.8, also the right-hand lattice in figure 3.2—which has clockwise propagating beams in each of its two rings. This model comprises two layouts, `Col1` and `Col2`, which we shall construct from DNA sequences `L5` and `L6` (see table 3.1). For the lower ring, we use the full ten-cell PSR lattice in layout `L6`. For the upper ring, we use all of layout `L5` (`PRS_minus`), and we re-use six of the fibres in layout `L6`.[15] In addition, so that this layout does not overlap our previous layouts, `PSR1`, `PSR2`, and `Fig8`, we shall place it 40 m distant in the positive z direction.

The construction of layouts `Col1` and `Col2` takes place in the following four blocks of code. We describe each in detail.

In this first block of code, we place all the beamline elements of layouts `Col1` and `Col2` in their correct locations: We translate layout `L6` a distance 40 m in the positive z direction. We then rotate layout `L5` 180° about the entrance of its first element (`L5%start%chart%f%a`). This rotates the gap in `L5` down to the bottom. To finish placing `L5` in the correct location, we want the *end* of its last bend to coincide with the entrance of the first bend in the lower ring, `L6`. We therefore point layout pointer `p1` to the first bend (`"B"`) in `L6`; compute the vector `d` from the exit of the last bend in `L5` (`L5%end%chart%f%b`) to the entrance of the first bend in `L6` (`p1%chart%f%a`); and then translate layout `L5` by that vector `d`.

```
!== Col1 : lower collider ring (layout 10)
!== Col2 : upper collider ring (layout 11)
d = zero
d(3) = 40.d0
call translate(L6, d)
a = zero
a(2) = pi
call rotate(L5, L5%start%chart%f%a, a)
call move_to(L6, p1, "B", pos)
d = p1%chart%f%a - L5%end%chart%f%b
call translate(L5, d)
```

In the next block of code, we construct layout `Col1`: We call `append_empty_layout` on `m_u` and set the layout pointer `Col1` to point to this new layout. Then we append in order all the fibres of `L6` to `Col1`. Finally, we give our new layout a formal name (`"Collider 1"`) and ensure that it is topologically closed. This layout does not require patching.

```
    call append_empty_layout(m_u)
    Col1 => m_u%end
    p1 => L6%start
215 do i = 1, L6%n
      call append_point(Col1, p1)
      p1 => p1%next
    end do

220 write(6,*) "Collider 1 has ", Col1%n, " fibres"
    Col1%name = "Collider 1"
    Col1%closed = .true.
    call ring_L(Col1, .true.) ! make it a ring topologically
```

In the next block of code, we construct layout `Col2`: We call `append_empty_layout` again on `m_u`, set the layout pointer `Col2` to point to this new layout, and then locate, in line 227, the first short drift in layout `L6`. To populate `Col2`, we now, in lines 228–233, march backwards appending the six elements in the straight section at the top of layout `L6` (short drift, defocusing quadrupole, two long drifts, focusing quadrupole, and short drift). While doing this, we inform PTC, in line 231, that this layout traverses those elements in the opposite direction. Finally, in lines 234–238, we append in order all the fibres of `L5` to `Col2`.

```
225 call append_empty_layout(m_u)
    Col2 => m_u%end
    p1 => L6%start%next%next
    do i = 1, 6
      write(6,*) p1%mag%name
230   call append_point(Col2, p1)
      Col2%end%dir = -1
      p1 => p1%previous
    end do
    p1 => L5%start
235 do i = 1, L5%n
      call append_point(Col2, p1)
      p1 => p1%next
    end do
```

In this last block of code for `Col2`, we give our new layout a formal name (`"Collider 2"`), ensure that it is topologically closed, and apply any necessary patches. This layout requires patches only where *fibre*`%dir` switches sign. As in our earlier layout `Fig8`, this happens at the two ends of the common straight section, where they join the bends of the upper ring.

```
240 write(6,*) "Collider 2 has ", Col2%n, " fibres"
    Col2%name = "Collider 2"
    Col2%closed = .true.
    call ring_L(Col2, .true.) ! make it a ring topologically

245 p1 => Col2%start
    do i = 1, Col2%n
```

```
    call check_need_patch(p1, p1%next, 1.d-10, pos)
    if(pos /= 0) call find_patch(p1, p1%next, next = .false.)
    p1 => p1%next
end do
```

The trackable layouts `Col1` and `Col2` are now complete. They have 140 fibres pointing to 134 elements in DNA sequences `L6` and `L5`. Twelve of the fibres in `Col1` and `Col2` point to the six elements which are common to the upper and lower rings.

3.6 DNA Arrays

The following code constitutes a bit of housekeeping. We have created a set of six layouts—`L1`–`L6`—that form the core of our DNA database. What we do here is tell each of the trackable layouts we have created—`PSR1`, `PSR2`, `Fig8`, `Col1`, and `Col2`—which DNA sequences they use. This information is stored in the array `DNA` that is part of the data held in each PTC *layout*.

In line 258 we record in `PSR1` the fact that DNA sequence `L1` is used by `PSR1`. Then line 260 records the fact that `PSR1` also uses DNA sequence `L2`. Note that this latter line makes use of the linked-list character of our DNA database. (Recall, see page 25, that `m_u` is a linked list of layouts.) Since `PSR1%DNA(1)%L` already points to `L1` (see line 258), and since `L2` is the next layout in the DNA database, then `PSR1%DNA(1)%L%next` points to `L2`. (In this very simple case you could, of course, replace lines 259–261 with the single statement `PSR1%DNA(2)%L => L2`.) The remaining lines in this block of code populate the DNA arrays for the other trackable layouts.

```
allocate(PSR1%DNA(2))
PSR1%DNA(1)%L => L1
do i = 2, 2
  PSR1%DNA(i)%L => PSR1%DNA(i-1)%L%next ! L2
end do

allocate(PSR2%DNA(2))
PSR2%DNA(1)%L => L1
do i = 2, 2
  PSR2%DNA(i)%L => PSR2%DNA(i-1)%L%next ! L2
end do

allocate(Fig8%DNA(2))
Fig8%DNA(1)%L => L3
do i = 2, 2
  Fig8%DNA(i)%L => Fig8%DNA(i-1)%L%next ! L4
end do

allocate(Col1%DNA(2))
Col1%DNA(1)%L => L5
do i = 2, 2
  Col1%DNA(i)%L => Col1%DNA(i-1)%L%next ! L6
end do

allocate(Col2%DNA(2))
```

```
    Col2%DNA(1)%L => L5
    do i = 2, 2
      Col2%DNA(i)%L => Col2%DNA(i-1)%L%next ! L6
285 end do
```

FOUR

Linking Magnets Together and Moving Them as a Group

In many accelerator designs, there are groups of elements that move together as a unit. The Large Hadron Collider, for example, has dual-bore magnets that carry the two counter-propagating beams through the cryostats. A more common example is a group of elements assembled onto a single substrate. When misalignments are applied to such units, our accelerator modeling code should respect the internal geometric constraints.

PTC allows us to link elements together to model groups of elements that move as units. In this chapter, we describe the tools PTC provides, and we illustrate their use by applying them to the collider (layouts `Col1` and `Col2`) of the previous chapter.

4.1 Siamese and Girders

A *siamese* consists of elements that are linked together so that one can move them as a group. Those elements may be in the same or different layouts. The elements in a siamese are often parallel, as in a collider; but they may be in different arcs of a recirculator—arcs, for example, that share common cryogenics.

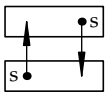


Figure 4.1: A pair of elements linked together as a siamese.

To create a siamese, we build a circular linked list containing the several elements we wish to tie together. See figure 4.1. When moving a siamese, PTC traverses the linked list, moving all the siamese elements in concert. Note that doing this properly—*i.e.* preserving the geometric relations between the siamese elements—requires the use of a common reference frame. Figure 4.2, for example, illustrates incorrect and correct rotations of a pair of elements linked together as a siamese. In the left-hand graphic of that figure, the same rotation applied to the *separate* elements breaks the geometry of the siamese. In the right-hand graphic, the use of a common reference frame when rotating the elements preserves the geometry. When we link the elements together as a siamese and then ask PTC to rotate the siamese—as opposed to the individual elements—PTC takes care of the details and preserves the internal geometric constraints. We discuss geometric operations applied to siamese later in this chapter. See also *Operations on Siamese*, page 75.

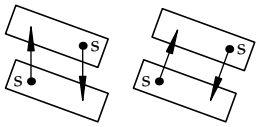


Figure 4.2: Incorrect and correct rotations of a siamese.

A siamese does not have an independent reference frame; instead, a siamese frame is defined in terms of translations and rotations with respect to the frame of one of its constituent elements. Misalignments of a siamese are then specified in a similar way with respect to the siamese frame. If we zero the misalignments, the siamese returns to its original location.

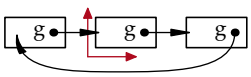


Figure 4.3: A trio of elements linked together as a girder that has its own reference frame.

A *girder* is a collection of siamese and regular elements tied to a substrate so that one can move them as a unit. Like a siamese, we construct a girder as a circular linked list containing the elements belonging to the common substrate. See figure 4.3. Unlike a siamese, a girder typically has its own reference frame, independent of any element on the girder. This is actually a *pair* of reference frames—stored in the PTC data type *affine_frame*. The two frames specify location and orientation for the original and misaligned girder. This structure simplifies the process of misaligning a girder: If applying a *different* misalignment to the girder, PTC starts with the original frame. If *adding* to an existing misalignment, PTC uses the misaligned frame as its starting point. If we remove the misalignment, PTC easily returns the girder to its original position in the lattice. We discuss geometric operations applied to girders later in this chapter. See also *Operations on Girders*, page 76.

Do note that linking together a group of elements on a girder does not mean those elements may no longer move with respect to one another. It is only the geometric operations that apply specifically to girders that will preserve the geometric relations between the girder elements. A similar comment applies for the siamese.

In this chapter we show how to create a pair of siamese and a girder in the collider—trackable layouts `Col1` and `Col2` from chapter 3—as well as how to misalign them. The code in this chapter is from the PTC geometry tutorial source file, `ptc_geometry.f90`, which is given in appendix C. The line numbers of the code shown here refer to the line numbers of the code in that appendix.

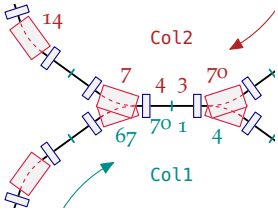


Figure 4.4: Collider interaction region. The numbers show the indices of a few of the fibres within the corresponding layout.

4.2 Building Siamese, Girders, and their Reference Frames

At each end of the straight section shared by layouts `Col1` and `Col2`, see figure 4.4, is a pair of overlapping bend magnets. In the first block of code below, we group each pair of bends as a siamese. For the pair at the left-hand end of the straight, we set, in the first two lines, pointers `p1` and `p2` respectively to the 67^{th} and 7^{th} fibres of layouts `Col1` and `Col2`. Those two fibres contain the pair of elements we wish to group together as a siamese. Each of those elements—`p1%mag` and `p2%mag`—contains an element pointer called `siamese`. In lines 388 and 389 we now set each element's `siamese` pointing to the other element, thus creating a circular linked list that ties these two elements together.

In a similar fashion, the next four lines link together as a siamese the two bends at the right-hand end of the common straight. Those elements belong to the 4^{th} and 70^{th} fibres respectively of layouts `Col1` and `Col2`.

```
   call move_to(Col1, p1, 67)
   call move_to(Col2, p2, 7)
   p1%mag%siamese => p2%mag
   p2%mag%siamese => p1%mag
390 call move_to(Col1, p1, 4)
   call move_to(Col2, p2, 70)
   p1%mag%siamese => p2%mag
   p2%mag%siamese => p1%mag
```

Our next goal is to group together onto a girder the above two siamese and all the intervening elements shared by layouts `Col1` and `Col2`. In addition, we will add one more element to our girder: the bend in the 14^{th} fibre of layout `Col2` (see upper left of figure 4.4). We do this not because this example seems likely from an engineer's perspective, but because we want to illustrate the flexibility of PTC's approach. In

particular, we wish to emphasize the fact that elements tied to a common girder need not be adjacent. Nevertheless, tied to a common substrate, they will move as a unit.

The next block of code links together the elements we want on our girder. As for the case of siamese, every PTC *element* contains an element pointer called `girders`, and we use this to construct the linked list that ties our girder elements together. Pointing `p1` to the short drift at the left-hand end of the common straight section (in fibre 68 of layout `Col1`), we deal first with the girder elements common to layouts `Col1` and `Col2`. In lines 396–400, we march along the straight section, pointing `p2` to the next fibre, linking the elements in `p1` and `p2` (line 398), and then advancing `p1`. After that loop terminates, both `p1` and `p2` point to fibre 4 in layout `Col1`, and we have linked together the elements of the common straight section plus that last bend.

```
    call move_to(Col1, p1, 68)
395 f => p1  ! remember start of girder linked-list
    do i = 2, 7
      p2 => p1%next
      p1%mag%girders => p2%mag
      p1 => p1%next
400 end do
    call move_to(Col2, p2,  7)
    p1%mag%girders => p1%mag%siamese
    p1%mag%siamese%girders => p2%mag
    p2%mag%girders => p2%mag%siamese
405 call move_to(Col1, p1, 67)
    call move_to(Col2, p2, 14)
    p1%mag%girders => p2%mag
    p2%mag%girders => f%mag
```

In the rest of this block of code, we link in the remaining four bends we want on our girder: 7, 14, and 70 from `Col2`, and 67 from `Col1`. (Because these four indices are distinct, we simplify the following description: instead of saying, for example, "the bend in fibre 4 of `Col1`", we shall say simply "bend 4".) First, we move `p2` to bend 7. Line 402 then links bend 4 to bend 70, because `p1%mag%siamese` already points to the latter. Line 403 links bend 70 to bend 7; and line 404 links bend 7 to bend 67, because `p2%mag%siamese` already points to the latter. It now remains for us to link in bend 14 and then close our linked list of girder elements. To do this, we first point `p1` to bend 67 and `p2` to bend 14. Then line 407 adds the girder link from one to the other of those two elements. Finally, line 408 closes the linked list.

We now have a pair of siamese and a girder defined in our collider. Our next task is to define appropriate siamese and girder frames. Though we may locate those frames wherever we wish, a sensible choice for the girder might be to make it coincide with the entrance frame of the first fibre in layout `Col1`, at the center of our collider's common straight section. We accomplish this in the next block of code.

After moving `p1` to the first fibre in layout `Col1`, we then, in line 410, allocate memory for the special dual reference frame needed by our girder, `p1%mag%girder_frame`.[1] This has four pieces we need to define: `a` and `b` for the origins of the original and current (or misaligned) frames, and `ent` and `exi` for the bases of those two frames.[2] In the remaining four lines of this block of code, we set these components equal to the corresponding parts (origin or basis) of the entrance reference frame of the ele-

[1] Read `alloc_af` as "allocate affine frame".

[2] Here the *origin* of a reference frame refers to a three-dimensional vector that stores the coördinates of the local frame's origin with respect to the global frame. The *basis* refers to a 3×3 matrix whose rows contain the orthogonal unit vectors of the local frame, written with respect to the global frame. See chapter 8 for more details.

ment in fibre p1. This information is held in p1%mag%parent_fibre%chart%f, which you might read as "the magnet frame attached to the chart associated with this element's parent fibre.".

```
    call move_to(Col1, p1, 1)
410 call alloc_af(p1%mag%girder_frame, girder = .true.)
    p1%mag%girder_frame%a   = p1%mag%parent_fibre%chart%f%a
    p1%mag%girder_frame%ent = p1%mag%parent_fibre%chart%f%ent
    p1%mag%girder_frame%b   = p1%mag%parent_fibre%chart%f%a
    p1%mag%girder_frame%exi = p1%mag%parent_fibre%chart%f%ent
```

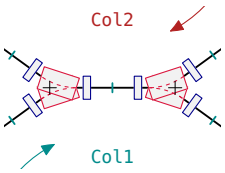


Figure 4.5: Collider interaction region. The small cross-hairs indicate the frame location for each siamese.

As set here, the original and current frames are identical, so this girder is in its design location. Later, if we ask PTC to misalign this girder, it will modify girder_frame%b and girder_frame%exi, leaving girder_frame%a and girder_frame%ent to record the design location and orientation of our girder.

In the following block of code, we also define reference frames for the two siamese. In this case, we choose to locate the siamese frames along the axis of the common straight section, five meters from the center of that straight, and oriented parallel to the girder's frame of reference. (See the cross-hairs in figure 4.5.) Consider first the left-hand siamese: We compute, in lines 415–416, the desired origin of our siamese frame. Then, in the following two lines, we move fibre pointer b to bend 67 and allocate the necessary memory.[3] Here is where a siamese and girder differ. When PTC allocates a *girder* frame (by setting girder = .true., as in line 410 above), it allocates memory for the data a, ent, b, and exi. For the siamese frame (the default, as in line 418 below, is girder = .false.) PTC allocates memory for the data d and angle, which specify not the frame itself, but rather how to get there (translation and angle) from the entrance frame of the local element. Our next step is therefore to compute d and angle. This we do in line 419 with a call to find_patch. The first two arguments are respectively the origin and basis of bend 67's entrance frame. The second two arguments are the desired origin (a) and basis (same as for the girder). And the last two arguments are the computed translation and angle, which we store in our siamese frame: b%mag%siamese_frame%d and b%mag%siamese_frame%angle.

In a similar fashion, the remaining lines in this block of code define the reference frame for the other siamese.

[3] This means the siamese frame will be attached to bend 67, but one could equally well attach it to bend 7, the other magnet in this siamese.

```
415 a = p1%mag%girder_frame%a
    a(3) = a(3) - 5.d0
    call move_to(Col1, b, 67)
    call alloc_af(b%mag%siamese_frame)
    call find_patch(b%mag%p%f%a, b%mag%p%f%ent, &
420                 a, p1%mag%girder_frame%ent, &
                    b%mag%siamese_frame%d, b%mag%siamese_frame%angle)
    a = p1%mag%girder_frame%a
    a(3) = a(3) + 5.d0
    call move_to(Col1, b, 4)
425 call alloc_af(b%mag%siamese_frame)
    call find_patch(b%mag%p%f%a, b%mag%p%f%ent, &
                    a, p1%mag%girder_frame%ent, &
                    b%mag%siamese_frame%d, b%mag%siamese_frame%angle)
```

4.3 Examples of Misalignments

The remaining blocks of code in this chapter show some examples of misalignment operations on the girder and siamese—with variations in their order and options. In the margin are corresponding figures that illustrate the effect of the different misalignments. (For comparison, figure 4.6 shows the same portion of the collider with no misalignments.) To make the effects clear, we have made very exaggerated "misalignments": $\pi/8\,\mathrm{rad} = 22.5°$ for rotations, and 2 m for displacements.

The various misalignments are specified by the six-component vector `mis`: the first three components describe translation, while the last three describe rotation in PTC order. This code is probably only somewhat self-explanatory. Nevertheless, we present it here for you to look at and mull over, with only a little in the way of comments. A detailed explanation is given in chapter 8, *Geometric Routines*.

Concerning line 431, note that bend 7 (*i.e.* the bend contained in fibre 7 of layout `Col2`) is present in both the girder and the left-hand siamese, which are the parts we misalign in the examples given here. That bend is not *directly* associated with either the girder frame or the siamese frame. But it is *in*directly associated: via the linked lists that define the girder and the siamese, PTC can always find the appropriate girder or siamese frame.

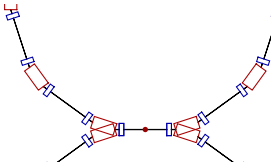


Figure 4.6: No misalignment.

```
    write(6,*) "Example # (from the manual) 1--11 ?"
430 read(5,*) example
    call move_to(Col2, p2, 7)
    if(example == 1) then            ! Example 1
      mis = 0.d0
      mis(5) = pi / 8.d0
435   call misalign_girder(p2, mis)
    elseif(example == 2) then        ! Example 2
      mis = 0.d0
      mis(1) = 2.0d0
      call misalign_girder(p2, mis)
440 elseif(example == 3) then        ! Example 3
      mis = 0.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
      mis = 0.d0
445   mis(1) = 2.d0
      call misalign_girder(p2, mis, add = .false.)
    elseif(example == 4) then        ! Example 4
      mis = 0.d0
      mis(5) = pi / 8.d0
450   call misalign_girder(p2, mis)
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_girder(p2, mis, add = .true.)
    elseif(example == 5) then        ! Example 5
455   mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
```

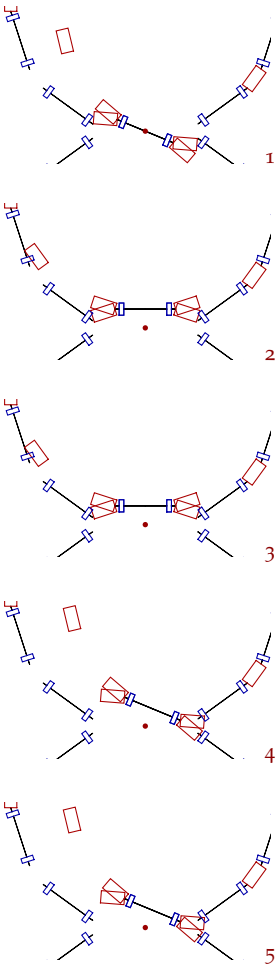
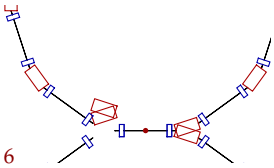
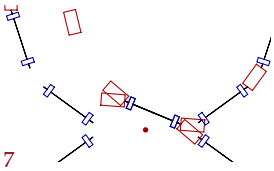
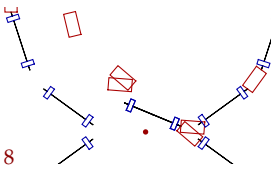


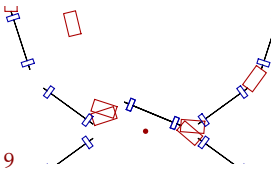
Figure 4.7: Examples 1 (top) through 5 (bottom).

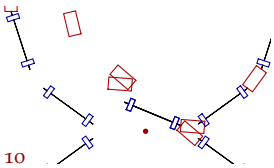
The examples above all misalign only the girder. In the following examples, we apply a misalignment also to one of the siamese. Note that the difference between examples 7 and 8 is solely to the optional argument `add` in lines 470 and 478.











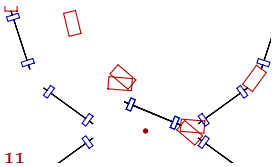


Figure 4.8: Examples 6 (top) through 11 (bottom).

```
    elseif(example == 6) then        ! Example 6
460   mis = 0.d0
      mis(1) = 2.d0
      call misalign_siamese(p2, mis)
    elseif(example == 7) then        ! Example 7
      mis = 0.d0
465   mis(1) = 2.d0
      call misalign_siamese(p2, mis)
      mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
470   call misalign_girder(p2, mis, add = .false.)
    elseif(example == 8) then        ! Example 8
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_siamese(p2, mis)
475   mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis, add = .true.)
    elseif(example == 9) then        ! Example 9
480   mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
      mis = 0.d0
485   mis(1) = 2.d0
      call misalign_siamese(p2, mis, add = .false.)
    elseif(example == 10) then       ! Example 10
      mis = 0.d0
      mis(1) = 2.d0
490   mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_siamese(p2, mis, add = .true.)
495 elseif(example == 11) then       ! Example 11
      mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
500   mis = 0.d0
      mis(1) = 2.d0
      call misalign_siamese(p2, mis, add = .false., &
                            preserve_girder = .true.)
    end if
```

FIVE

Taylor Polymorphism and Knobs

PTC supports the full usage of Taylor maps derived from the integrator for the computation of lattice functions. PTC uses FPP, a package of polymorphic types and tools to extract and analyze the Taylor maps. Information explaining FPP[1] is at http://mad.web.cern.ch/mad/PTC_proper/

[1] The CERN folder is called PTC_proper to distinguish it from the information on MAD-X. Most of the information in the PTC_proper folder is about FPP.

5.1 Polymorphs

A *polymorph* is the FORTRAN90 type *real_8*:

```
type real_8
  type (taylor) :: T   ! active if Taylor
  real(dp) :: r        ! active if real or knob
  integer :: kind      ! 1=real,2=Taylor,3=Taylor knob,0=special
  integer :: i         ! used for knobs
  real(dp) :: s        ! scaling for knobs
  logical(lp) :: alloc ! is Taylor is allocated in DA-package?
end type real_8
```

Polymorphs allow the computation of parameter-dependent maps.

States of a Polymorph

A polymorph `Y` can be real (`Y%kind = 1`), Taylor (`Y%kind = 2`), or a special Taylor called a *knob* (`Y%kind = 3`).

Computing a Taylor Map

Suppose we have a closed orbit at position 1 given by six real numbers `fix(1:6)`. We can construct the following array of six polymorphs:

```
type(real_8) :: Y(6)
type(damap) :: id

call init(state, NO, 0)
call alloc(Y)
```

```
call alloc(id)

id = 1
Y = fix + id
```

The variable `state` describes internal states of PTC.

The variable `id` is a differential algebra map (`damap`), and `id = 1` constructs the identity map.

Then `Y = fix + id` creates the polymorphic component `Y(i)` given by `Y(i) = fix(i) + `x_i, where x_i denotes the i-th component of the identity map `id`.

To compute a one-turn map, for example, we track `Y` around the machine (here `R`) as if it were six real numbers:

```
call track(R, Y, 1, state)
```

At the end, `Y` contains the Taylor map, to order `NO`, about the closed orbit `fix`. The internal state variable `state` determines the exact nature of the map:

1. `state = default0` specifies a 6-D map with cavities;
2. `state = default0 + nocavity0` specifies a 6-D map with cavities skipped;
3. `state = default0 + only_4d0` specifies a 4-D phase space (X, P_X, Y, P_Y);
4. `state = default0 + delta0` specifies a 4-D phase space (X, P_X, Y, P_Y), plus energy as the fifth variable; cavities are skipped.

For more information about states, see appendix A.

5.2 Knobs

A *knob* is a polymorph that turns itself into a simple Taylor series when used. A knob cannot be on the left side of an equal sign.

Knobs let users set up parameters that can be changed without having to recompile.

Using Knobs

Example: The `bn(2)` (quadrupole component) of a magnet is made into the first knob:

```
bn(2)%kind = 3
bn(2)%i = 1
bn(2)%s = 1
```

At execution time, during an operation involving `bn(2)`, the knob becomes the following Taylor map:

```
bn(2)= bn(2)%r + bn(2)%s * X_j
```

In the map above, `j= npara_fpp + bn(2)%i`.

The integer `npara_fpp` depends on the state: It equals the minimal number of variables compatible with the state selected. In the four `state` examples above:

1. npara_fpp = 6 ⇒ j = 7
2. npara_fpp = 6 ⇒ j = 7
3. npara_fpp = 4 ⇒ j = 5
4. npara_fpp = 5 ⇒ j = 6

Creating Knobs

While FPP has a routine to help users make a knob, PTC has tailored routines to put knobs into a layout and remove knobs from a layout. The types, subroutines, and routines are

- type pol_block;
- subroutine scan_for_polymorphs(R,B) or R = B;
- unary +, as in +state, to activate knobs in a track routine;
- TPSAfit(1:lnv) array;
- set_TPSAfit and set_element logicals;
- subroutine kill_para_L(R).

For more information about the unary + used to activate knobs in a track routine, see *Internal States*, appendix A.

Polymorphic Blocks

This section discusses type pol_block, setting values for polymorphic blocks, and removing polymorphic blocks from layouts.

Type pol_block

This data type creates an object to be compared with an actual layout. It identities families or single elements using the name, part of the name, or the vorname (first name) of an element to make certain variables knobs. (The last name is the family, for example: QF.)

```
type pol_block
  character(nlp) name
  integer :: n_name
  character(vp) vorname

  ! types for setting magnet using global array TPSAfit
  real(dp), dimension(:), pointer :: TPSAfit
  logical(lp), pointer :: set_TPSAfit
  logical(lp), pointer :: set_element

  ! types for parameter dependence
  integer :: npara ! should not be used anymore

  ! knob index
  integer ::  ian(nmax), ibn(nmax)
  real(dp) :: san(nmax), sbn(nmax)
  integer :: ivolt, ifreq, iphas
  integer :: ib_sol

  ! scales for knobs
```

```
  real(dp) :: svolt, sfreq, sphas
  real(dp) :: sb_sol

  ! user defined functions
  type(pol_block_sagan) :: SAGAN
end type pol_block
```

Consider the following `pol_block qf`:

```
qf = 0         ! initialize the pol_block qf
qf%name = 'QF' ! specify a family name
qf%ibn(2) = 1  ! set normal quad strength as first parameter
```

If we call the routine `scan_for_polymorphs(R, qf)` or `R = qf`, then the DNA layout `R` is scanned. If a polymorphic magnet on any fibre of the layout `R` is named `'QF'`, then `bn(2)` becomes a knob. In our example:

```
bn(2) = bn(2%r + qf%sbn(2) * X(fpp_npara) + qf%ibn(2)
bn(2) = bn(2)%r + X(fpp_npara) + 1
```

The index `fpp_npara` depends on the state as explained above.

One may also specify an integer `qf%n_name`. If, for example, `qf%n_name = 2`, then a polymorph is set if the magnet name matches `'QFxxxxxxxxxxxxxx'`, where `x` denotes any character.

Once the knobs are set on the lattice using the routine `scan_for_polymorphs`, tracking routines can be invoked after the DA-package has been initialized.

Setting Values using Polymorphic Blocks

Polymorphs allow for the computation of parameter-dependent maps. These maps can be analyzed by various methods including normal forms. From these maps one may attempt to fit certain computed quantities by modifying the parameters of the polymorphs on the ring.

This is done as follows with `scan_for_polymorphs` or the = sign assignment. The global parameter `set_TPSAfit` turns the `scan_for_polymorphs` routine into a routine that inputs the array `TPSAfit(1:C_%np_pol)` into variables that the `pol_blocks` make into knobs. Note that the knobs exist only in the polymorphic version of the magnet located at `fibre%magp`. The polymorphic version is copied into the real magnet `fibre%mag` if `set_element` is true.

```
set_TPSAfit = .true.
set_element = .true.
Col1%DNA(1)%L = qf(1)
Col1%DNA(1)%L = qd(1)
Col1%DNA(2)%L = qf(2)
Col1%DNA(2)%L = qd(2)
set_element = .false.
set_TPSAfit = .false.
```

Removing Polymorphic Blocks from Layouts

To remove a polymorphic block from a layout use the subroutines

```
call kill_para(Col1%DNA(1)%L)
call kill_para(Col1%DNA(2)%L)
```

5.3 Tutorial Example

The example code in this chapter is from the PTC geometry tutorial source file, `ptc_geometry.f90`, which is given in appendix C. The line numbers of the code in the examples refer to the line numbers of the code in the appendix.

This tutorial example shows how to create a map for the collider with polymorphs and knobs.

The first six lines of code initialize the polymorphic block for the focusing quadrupoles (`qf`) in `Col1` and `Col2`, give the quadrupoles the family name `'QF'`, and set their normal strength as the first parameter in the Taylor series. The following six lines do the same for the defocusing quadrupoles (`'QD'`).

```
qf(1) = 0
qf(1)%name = 'qf'
qf(1)%ibn(2) = 1
qf(2) = 0
qf(2)%name = 'qf'
qf(2)%ibn(2) = 3
qd(1) = 0
qd(1)%name = 'qd'
qd(1)%ibn(2) = 2
qd(2) = 0
qd(2)%name = 'qd'
qd(2)%ibn(2) = 4
```

The next four lines of code declare `qf` and `qd` as independent in DNA layouts `L5` and `L6`. They perform the same function as calls to the subroutine `scan_for_polymorphs`. If a polymorphic magnet on any fibre of the DNA layouts `L5` and `L6` is named `'QF'` or `'QD'`, then `ibn(2)` becomes a knob.

```
Col1%dna(1)%L = qf(1)
Col1%dna(1)%L = qd(1)
CoL1%dna(2)%L = qf(2)
CoL1%dna(2)%L = qd(2)
```

Once the knobs are set on the lattice using the `scan_for_polymorphs` routine (or equivalent), we can invoke tracking routines after the DA-package has been initialized.

The following lines of code define the closed orbit if not 0.

```
101 continue
state = default0 + only_4d0

fix1 = 0.d0
```

```
   fix2 = 0.d0;
335 call init(state, 2, c_%np_pol) ! c_%np_pol is automatically computed
```

The 2 is automatically computed above—counting the number of DNA variables (1-4). This means the Taylor series now has eight variables.

```
   call find_orbit(CoL1, fix1, 1, state, 1.d-6)
   call find_orbit(Col2, fix2, 1, state, 1.d-6)
   call alloc(y1)
   call alloc(y2)
340 call alloc(id)
   call alloc(n1)
   call alloc(n2)
   call alloc(eq);
   id=1 ! identity damap
345 y1 = id + fix1 ! this is permitted in ptc only (not fpp)
   y2 = id + fix2 ! closed orbit added to map
```

The plus sign in the next two lines of code activates the knobs. If we remove the plus sign, PTC will ignore the knobs.

```
   call track(Col1, y1, 1, +state) ! unary + activates knobs
   call track(Col2, y2, 1, +state)
```

After accounting for knobs, the code computes the tunes (with and without knobs). Equations 1 and 2 compute the tunes for `col1`; equations 3 and 4 compute the tunes for `col2`.

The first number is the difference between the goal and what we have obtained, which should be as close to 0 as possible.

```
   n1 = y1 ! normal forms: abused of language permitted by ptc
350 n2 = y2 ! normally one should do => damap=y; normalform=damap
   write(6,*) " tunes 1 "
   write(6,*) n1%tune(1:2)
   write(6,*) " tunes 2 "
   write(6,*) n2%tune(1:2)
355 eq(1) = n1%dhdj%v(1) - 0.254d0
   eq(2) = n1%dhdj%v(2) - 0.255d0
   eq(3) = n2%dhdj%v(1) - 0.130d0
   eq(4) = n2%dhdj%v(2) - 0.360d0
   do i = 1, 4
360   eq(i) = eq(i) <= c_%npara
   end do

   call kanalnummer(mf,"eq.txt")
   do i=1,4
365   call daprint(eq(i), mf)
   end do
   close(mf)
```

```
    call kill(y1)
    call kill(y2)
    call kill(id)
    call kill(n1)
    call kill(n2)
    call kill(eq)
    call init(1,4)
    call alloc(g,4)
    call kanalnummer(mf,"eq.txt")
    do i = 1, 4
      call read(g%v(i), mf)
    end do
    close(mf)

    g = g.oo.(-1)
    tpsafit(1:4) = g
    set_tpsafit = .true.
    set_element = .true.
    Col1%dna(1)%L = qf(1)
    Col1%dna(1)%L = qd(1)
    Col1%dna(2)%L = qf(2)
    Col1%dna(2)%L = qd(2)
    set_tpsafit = .false.
    set_element = .false.
    call kill(g)
```

We need to kill the knobs after we compute them: the two calls to `kill_para` kill the knobs in DNA layouts `L5` and `L6`.

```
    write(6,*) " more "
    read(5,*) i
    if(i == 1) goto 101
    call kill_para(Col1%dna(1)%l)
    call kill_para(Col1%dna(2)%l)
```

SIX

Computing Accelerator Properties

This chapter explains how to compute global and local accelerator properties and provides examples of the code required for the computations.

6.1 Global Scalars

Global scalars apply to the accelerator as a whole. They can be derived only after the complete accelerator has been modeled, and they are the same at any point on the closed orbit.

Tunes

```
x = zero
x(5) = delta
call find_orbit(lattice, x, 1, istate, 1.d-7)
call init(istate, 1, 0)
call alloc(y)
call alloc(normal)
call alloc(id)
id = 1
y = x + id
call track(lattice, y, 1, istate)
normal = y
write(6,'(a,3(2x,f9.6))') 'tunes:', normal%tune
```

Chromaticity

Anharmonicity

6.2 s-Dependent Global Quantities

Betatron Amplitude

```
x = zero
x(5) = delta
```

```
call find_orbit(lattice, x, 1, istate, 1.d-7)
call init(istate, 1, 0)
call alloc(y)
call alloc(normal)
call alloc(id)
id = 1
y = x + id
call track(lattice, y, 1, istate)
normal = y
y = x + normal%a_t  ! track this---normalizing transformation
p => lattice%start
beta_x = (y(1).sub.'10') ** 2   + (y(1).sub.'01') ** 2
beta_y = (y(3).sub.'0010') ** 2 + (y(3).sub.'0001') ** 2
write(6,'(a,2(2x,f9.6))') 'beta_x, beta_y: ', beta_x, beta_y
do j = 1, lattice%n
  call track(lattice, y ,j, j+1, istate)
  beta_x = (y(1).sub.'10') ** 2   + (y(1).sub.'01') ** 2
  beta_y = (y(3).sub.'0010') ** 2 + (y(3).sub.'0001') ** 2
  write(6,'(a,2(2x,f9.6))') 'beta_x, beta_y: ', beta_x, beta_y
  p => p%next
end do
```

Dispersion

```
x = zero
x(5) = delta
call find_orbit(lattice, x, 1, istate, 1.d-7)
call alloc(id)      ! type(damap)
call alloc(disp)    ! type(damap)
call alloc(xt)      ! type(damap)
call alloc(eta)     ! type(real_8), dimension(6)
call alloc(y)       ! type(real_8), dimension(6)
call alloc(normal)  ! type(normalform)
id = 1
y = x + id
call track(lattice, y, 1, default)
normal = y
y = x + normal%A_t
p => lattice%start
do j = 1, lattice%n
  call track(lattice, y ,j, j+1, default)
  id = 0
  xt = y
  disp = xt * id
  x = xt
  id = 1
  disp = id - disp
  xt = disp * xt
  eta = x + xt
```

```
  dispx = (eta(1).sub.'00001')
  dispy = (eta(3).sub.'00001')
  write(6,'(a,2(2x,f9.6))') 'disp_x, disp_y: ', dispx, dispy
end do
```

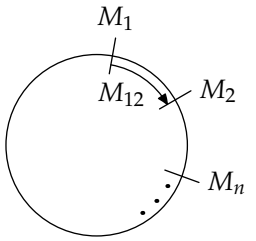


Figure 6.1: One-turn and partial-turn transfer maps.

Phase Advance

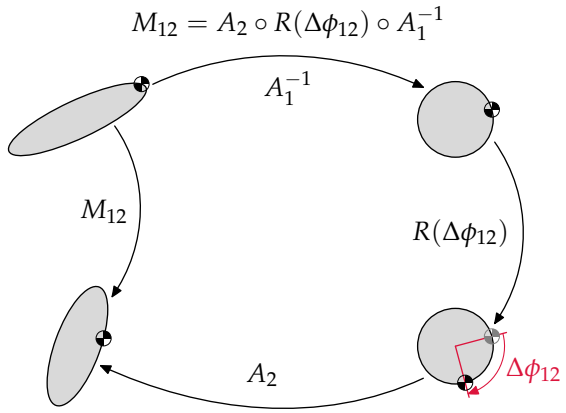


Figure 6.2: This graphic illustrates the essential relationships between the one-turn map and the normal form at two different locations in a ring lattice.

See caveat at end of § 2.3!

```
x = zero
x(5) = delta
call find_orbit(lattice, x, 1, istate, 1.d-7)
call init(istate, 1, 0)
call alloc(y)
call alloc(normal)
call alloc(id)
id = 1
y = x + id
call track(lattice, y, 1, istate)
normal = y
y = x + normal%a_t
theta_prev = zero
phi = zero
write(6,'(a,2(2x,f9.6))') ' CS phase adv:', phi(1:2)
p => lattice%start
do j = 1, lattice%n
  call track(lattice, y ,j, j+1, istate)
  theta(1) = atan2((y(1).sub.'01'), (y(1).sub.'10')) / twopi
  theta(2) = atan2((y(3).sub.'0001'), (y(3).sub.'0010')) / twopi
  do k = 1, 2
    if(theta(k) < zero .and. abs(theta(k)) > tiny) then
      theta(k) = theta(k) + one
    end if
    dphi(k) = theta(k) - theta_prev(k)
```

```
      if(dphi(k) < zero .and. abs(dphi(k)) > tiny) then
        dphi(k) = dphi(k) + one
      end if
      phi(k) = phi(k) + dphi(k)
    end do
    theta_prev = theta
    write(6,'(a,2(2x,f9.6))') ' CS phase adv:', phi(1:2)
    p => p%next
  end do
```

Beam Envelope

Text and example code here.

6.3 Local Quantities

Local properties do not apply to the accelerator as a whole. They are derived from individual magnets, and they differ at different points on the closed orbit. The trajectory of a particle through a magnet is local; it is derivable from the individual magnet irrespective of the magnet's position in the accelerator.

SEVEN

Tracking Routines

PTC's tracking routines are divided into four categories:

- standard tracking routines on fibres,
- tracking routines on integration nodes,
- tracking routines on 3-D information through an integration node,
- time-based tracking routines.

A fifth section documents the closed-orbit routine.

Mandatory arguments for the tracking routines are in `regular black` type. Optional arguments are in *`red italic`* type.

Positions are normally specified by `I1`, `I2`, `fibre1`, `fibre2`, `node1`, or `node2`. Generally, if only `x1` is present (`x=I`, `fibre`, or `node`), this produces a one-turn map from position `x1` back to position `x1` if the layout is closed. If the layout is opened, then it goes to the end of the line.

If `x1` and `x2` are present, the routine tracks through `x1` all the way to the front of `x2` (`x2` not included).

The only exception to all of this is the time-based tracking routine.

7.1 Standard Tracking Routines on Fibres

These routines do not support spin and radiation.

Track

```
track (R, X, I1, I2, K)
```

`X` is an array of six `real(dp)` or `REAL_8`.
`I1` and `I2` are the position of the fibres.
`Result=TRACK_FLAG (R,X,I1,I2,k)`
`Result=logical`; true indicates stable; false indicates unstable.

```
track (C, X, K, CHARGE)
```

This routine tracks through the fibre `C`.

Find_orbit

```
find_orbit(R, FIX, LOC, STATE, eps, TURNS)
```

The `find_orbit` routine works in the same way as the subroutine `find_orbit_x` but on the fibre structure without radiation. For more information about the subroutine `find_orbit_x`, see *Find_orbit_x*.

`LOC` is the integer location in the layout `R`.

7.2 Tracking Routines on Integration Nodes

These routines support spin and radiation.

Routines for Tracking either Probe or Probe_8

For the type definitions of `probe` and `probe_8`, see *Probe*.

TRACK_PROBE2

```
TRACK_PROBE2(R,XS,K,I1,I2)
```

```
I1,I2 = NODE POSITION
I1 only implies a one-turn map.
K=INTERNAL STATE
```

TRACK_PROBE

```
TRACK_PROBE (R,XS,K,FIBRE1,FIBRE2,NODE1,NODE2)
```

`FIBRE1,FIBRE2,NODE1,NODE2` are all integer positions of either the fibre or the integration node.

TRACK_NODE_PROBE

```
TRACK_NODE_PROBE(T,XS,K)
```

`T` is an integration node.

Object-Oriented Routines

```
TRACK_PROBE2 (XS,K,FIBRE1,FIBRE2,NODE1,NODE2)
TRACK_PROBE (XS,K,FIBRE1,FIBRE2,NODE1,NODE2)
TRACK_NODE_PROBE (XS,K,FIBRE1,FIBRE2,NODE1,NODE2)
```

These are all calls to the same routine. The fibres and the nodes are actual pointers to the objects.

One turn can be done as follows:

```
TRACK_PROBE (XS,K,NODE1=T,NODE2=T%PREVIOUS)
```

In the TRACK_PROBE_X routine, the fibres and the nodes are pointers to the objects. The routine wraps the TRACK_PROBE routine shown above.

```
TRACK_PROBE_X (R,X,K,U,T,FIBRE1,FIBRE2,NODE1,NODE2)
```

Routines for Tracking either Real or Real_8

All these routines wrap the above routines and therefore support radiation, beam-beam and s-dependent apertures.

TRACK_NODE_X

```
TRACK_NODE_X(T,X,K)
```

TRACK_PROBE_X

```
TRACK_PROBE_X(R,X,K,U,T,FIBRE1,FIBRE2,NODE1,NODE2)
```

U=LOGICAL where TRUE indicates UNSTABLE (optional).
T is a pointer to the fibre where the particle is lost (optional).

TRACK_BEAM

This routine tracks a beam of particles.

```
TRACK_BEAM(R,B,K,T,FIBRE1,FIBRE2,NODE1,NODE2)
```

For the type definition of BEAM, see Section B.2.

7.3 Tracking Routines on 3-D Information through an Integration Node

These routines track three-dimensional information through an integration node.

Track_node_v

The TRACK_NODE_V routine tracks a trajectory and records its three-dimensional position for plotting at the beginning and the end of a node.

```
TRACK_NODE_V (T,V,K,REF)
```

T is an integration node.

V is of type THREE_D_INFO. The type definition is given below.

REF=TRUE or FALSE. If REF=TRUE, then the results of the TRACK_FILL_REF routine are used. The ray is magnified by V%SCALE (see type THREE_D_INFO below) with respect to a trajectory computed and stored by the TRACK_FILL_REF routine, which tracks the ray FIX from fibre I1 back to fibre I1.

```
TRACK_FILL_REF(R,FIX,I1,K))
```

Here is the data type definition for three-dimensional information.

```
TYPE THREE_D_INFO
  REAL(DP) A(3),B(3)          ! CENTRE OF ENTRANCE AND EXIT FACES
  REAL(DP) ENT(3,3),EXI(3,3) ! ENTRANCE AND EXIT FRAMES FOR DRAWING MAGNET FACES
  REAL(DP) WX,WY              ! WIDTH OF BOX FOR PLOTTING PURPOSES
  REAL(DP) O(3),MID(3,3)      ! FRAMES AT THE POINT OF TRACKING
  REAL(DP) REFERENCE_RAY(6)   !
  REAL(DP) X(6)               ! RAY TRACKED WITH REFERENCE_RAY USING A TYPE(BEAM)
  REAL(DP) R0(3),R(3)         ! RAY POSITION GLOBAL RETURNED
  REAL(DP) SCALE              ! MAGNIFICATION USING REFERENCE_RAY
  LOGICAL(LP) U(2)            ! UNSTABLE FLAG FOR BOTH RAY AND REFERENCE_RAY
END TYPE THREE_D_INFO
```

7.4 Time-based Tracking Routines

These routines provide time-based tracking of temporal probes and temporal beams.

Track_time

```
TRACK_TIME(XT,DT,K)
```

XT is a temporal probe. For the type definition of TEMPORAL_PROBE, see Section B.2.

Track_temporal_beam

```
TRACK_TEMPORAL_BEAM(B,DT,STATE)
```

B is a temporal beam. For the type definition of TEMPORAL_BEAM, see Section B.2.

7.5 Closed-Orbit Routine

This routine finds the closed orbit.

Find_orbit_x

```
FIND_ORBIT_X(R,FIX,STATE,eps,TURNS,fibre1,node1)
```

Here `fibre1` and `node1` are integer positions. The routine finds the fixed point for `TURNS` turns; one turn if not specified.

The argument `eps= real(dp)` number is used to do numerical differentiation—typically `1.d-6` works.

For a no-cavity fixed point, `fix(5)` must contain the energy variable.

EIGHT

Geometric Routines

PTC's geometric routines are divided into three categories:

- affine routines on pure geometry,
- affine routines on computer objects,
- dynamical routines.

Mandatory arguments for the geometric routines are in `regular black` type. Optional arguments are in *`red italic`* type.

8.1 Affine Routines on Pure Geometry

Affine routines on pure geometry act on a pure geometrical affine basis: `A(3)` and `V(3,3)` where `A` represents the coordinates of a point in the global frame, and `V(3,3)` the coordinates of a triad of vectors.

Theory

Consider a point a and vector basis (v_1, v_2, v_3) attached to a solid (a magnet, for example). See figure 8.1.

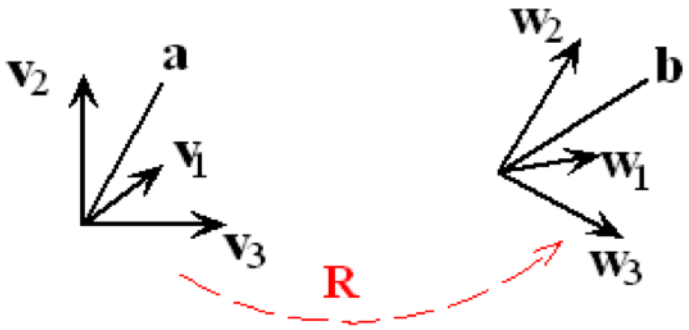


Figure 8.1: Rotating point a and vector basis (v_1, v_2, v_3) by R.

The vector a can be expressed as follows:

$$a = \sum_i a_i v_i.$$

The basis vectors v_i can be expressed in terms of a global basis, that is, PTC's global frame:

$$v_i = \sum_j V_{ij} e_j.$$

Suppose we rotate the solid by a rotation R defined by its action on the basis (v_1, v_2, v_3):

$$w_i = R v_i = \sum_j R_{ij} v_j.$$

Then we ask the following question: what are the components of w_i in the global basis (e_1, e_2, e_3) in terms of the component array V_{ij}?

Note: The components of b in the frame (w_1, w_2, w_3), which is the image of a upon rotation of the magnet, are also given a_i because this point is fixed in the magnet.

Solution:

$$w_i = R v_i = \sum_j R_{ij} v_j = \sum_{jk} R_{ij} V_{jk} e_k = \sum_k W_{ik} e_k.$$

Thus we have

$$W = RV.$$

The components of the vector b in the global frame are then

$$b_k = \sum_{ij} a_i R_{ij} V_{jk} e_k \to b = (RV)^t a.$$

We now address a slightly harder problem, which is essential in PTC. The rotation R, instead of being defined in the frame (v_1, v_2, v_3), might be defined on a totally different frame (u_1, u_2, u_3):

$$u_i = \sum_k U_{ik} e_k.$$

Therefore, prior to rotating the basis (v_1, v_2, v_3), we must express it in the frame (u_1, u_2, u_3):

$$a = \sum_{ik} a_i V_{ik} e_k = \sum_{ikn} a_i V_{ik} U^{-1}_{kn} u_n = \sum_{ikn} a_i V_{ik} U_{nk} u_n.$$

We can now apply the rotation R defined on the basis (u_1, u_2, u_3):

$$b = \sum_{iknm} a_i V_{ik} U_{nk} R_{nm} u_m = \sum_{iknm} a_i V_{ik} U_{nk} R_{nm} U_{mk} e_k.$$

Thus the final result of W is:

$$W = V U^t R U$$

The point b in global coordinates is:

$$b_i = \sum_k a_i W_{ik} \to b = (V U^t R U)^t a.$$

Notice that if $V = U$, we regain the previous result.

PTC factors the rotation R in the form

$$R = R_z R_y R_x$$

Using local variables results in the need to go back and forth between frames. The magnets must be placed within the global frame. However, the tracking is in local variables. We need routines that are able to connect geometrically both points of view.

Descriptions of the Routines

This section describes the affine routines on pure geometry.

GEO_ROT

```
GEO_ROT(V(3, 3),W(3, 3),A(3),B(3),ANG(3),BASIS(3,3))
```

This routine exactly reproduces the theory in the previous section.
`BASIS` is an optional variable, which is set equal to the U described in Section 8.1.
The real array `ANG(3)` is used to define R:

$$R = R_z(ang(3))R_y(ang(2))R_x(ang(1))$$

```
GEO_ROT(V(3, 3),W(3, 3),ANG(3),BASIS(3,3))
```

This routine is the same as the previous routine except that `A` and `B` are not needed.

```
GEO_ROT(V(3, 3),A(3),ANG(3),I,BASIS(3,3))
```

Here the final W and B are copied back into V and A. The integer `I` can be ± 1 to produce the inverse rotation: $R^{\pm 1}$.

```
GEO_ROT(V(3, 3),ANG(3),I,BASIS(3,3))
```

This routine is the same as the previous routine without the `A` vector.

GEO_TRA

```
GEO_TRA(A(3),V(3, 3) (3),D,I)
```

`A` is translated by $\pm$ `D` expressed in the basis `V`; `I` = ± 1.

$$\text{result} = \sum_{ij}(a_j e_j \pm d_i V_{ij} e_j)$$

The result is put back in `A`, that is:

$$A \leftarrow A \pm V^t D$$

Rotating and Translating the Frames of a Magnet

```
ROTATE_FRAME(F,OMEGA,ANG,ORDER,BASIS(3,3))
```

`F` is of type `magnet_frame` and contains three affine frames tied to the magnet.

$$F = \{(F\%A(3), F\%ENT(3)), (F\%O(3), F\%MID(3)), (F\%B(3), F\%EXI(3))\}$$

The entire content of `F` is rotated as shown in figure 8.2.

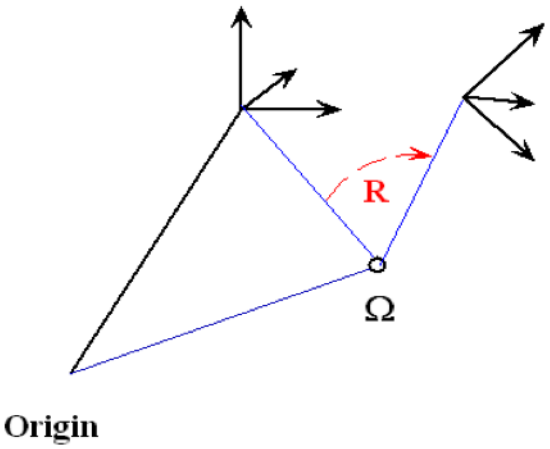


Figure 8.2: Rotating and translating the frames of a magnet.

```
TRANSLATE_FRAME(F,D,ORDER,BASIS(3,3))

CALL CHANGE_BASIS(D,BASIS,DD,GLOBAL_FRAME)

P%A = P%A + ORDER * DD
P%B = P%B + ORDER * DD
P%O = P%O + ORDER * DD
```

The frame is simply translated by `D`. The translation `D` is expressed in `BASIS` if present.

CHANGE_BASIS

```
CHANGE_BASIS(A(3),V(3,3),B(3),W(3,3))
```

The component vector `A`, expressed in the basis `V`, is re-expressed as `B` using basis `W`. `B` is the output of this subroutine.

$$\sum_{ik} a_i V_{ik} e_k = \sum_{ik} b_i W_{ik} e_k \rightarrow b = W V^t a.$$

COMPUTE_ENTRANCE_ANGLE

```
COMPUTE_ENTRANCE_ANGLE(V(3,3),W(3,3),ANG(3))
```

This is a crucial routine. Given two frames `V` and `W`, which most likely represent a magnet or a beam line, the routine computes a rotation R in the standard PTC order, that is,

$$R = R_z(\texttt{ang(3)}) R_y(\texttt{ang(2)}) R_x(\texttt{ang(1)})$$

such that `V` is transformed into `W`. It is the reverse routine from `GEO_ROT`.

FIND_PATCH_B

This routine connects the affine frame `A,V` to the affine frame `B,W`.

```
FIND_PATCH_B(A(3),V(3,3),B(3),W(3,3),D(3),ANG(3))
```

```
SUBROUTINE FIND_PATCH_B(A,V,B,W,D,ANG)
  ! FINDS PATCH BETWEEN V AND W : INTERFACED LATER FOR FIBRES
  IMPLICIT NONE
  REAL(DP), INTENT(INOUT) :: V(3,3),W(3,3)
  REAL(DP), INTENT(INOUT) :: A(3),B(3),D(3),ANG(3)
  CALL COMPUTE_ENTRANCE_ANGLE(V,W,ANG)
  D=B-A; CALL CHANGE_BASIS(D,GLOBAL_FRAME,D,W);
END SUBROUTINE FIND_PATCH_B
```

The above code is a simple example of the usage of the geometric routines. This geometric routine is very close to the dynamical set up of PTC. Patches are written as an "x" rotation, a "y" rotation, a "z" rotation, followed by a translation (transverse + drift). Note that the translation $D = B - A$ between is expressed in the final frame W. This is normal if dynamical rotations precede the translations.

FIND_INVERSE_PATCH

This routine is the precise inverse of the `FIND_PATCH_B` routine. The affine frame `B,W` is the output.

```
INVERSE_FIND_PATCH(A(3),V(3,3),D(3),ANG(3),B(3),W(3,3))
```

```
SUBROUTINE INVERSE_FIND_PATCH(A,V,D,ANG,B,W)
  ! USED IN MISALIGNMENTS OF SIAMESE
  IMPLICIT NONE
  REAL(DP), INTENT(INOUT):: V(3,3),W(3,3)
  REAL(DP), INTENT(INOUT):: A(3),B(3),D(3),ANG(3)
  REAL(DP) ::  DD(3)
  W=V
  CALL GEO_ROT(W,ANG,1,BASIS=V)
  CALL CHANGE_BASIS(D,W,DD,GLOBAL_FRAME)
  B=A+DD
END SUBROUTINE INVERSE_FIND_PATCH
```

8.2 Affine Routines on Computer Objects

Affine routines on PTC's computer objects act on the affine bases of magnets, siamese, girders, integration nodes, and fibres. These objects contain a myriad of affine bases to help us locate the trajectories in 3-D.

Within this category are two subcategories:

- *Affine Routines on Fibrous Structures:* Displacements that correspond to the design positioning and thus requiring patching.
- *Misalignment Routines:* Misalignments representing errors that do not require patching. The misalignments displace the magnet away from a fibre that contains it. The misalignments also displace girders away from their original position.

Affine Routines on Fibrous Structures

We discussed in the previous section PTC's geometric tools on the affine frame. This is useful in giving a pictorial representation of a ring in 3-D. One can imagine linking PTC with a CAD program equipped with a magnet widget containing at least one affine frame, say the cord frame `O(3), MID(3,3)` of a PTC magnet.

Of course, PTC is dynamically a more complex structure than just magnets: fibres, integration nodes, layouts, etc. All these objects have affine frames attached to them, and we must be able to move them.

We describe first the routines that displace PTC structures away from a standard "MAD8" configuration.[1] These are used in the generation of "non-standard" systems.

[1] The quotation marks indicate that there is really nothing standard about the so-called standard implicit geometry of MAD8. It is worth pointing out that the code MAD of CERN and the code SAD of KEK have different implicit geometry once vertical magnets are invoked: nothing is standard.

Patching Routines

The central power of PTC is its ability to place magnets in arbitrary positions. To do so, the concept of a fibre with a patch is necessary.

FIND_PATCH

```
FIND_PATCH(EL1,EL2_NEXT,NEXT,ENERGY_PATCH,PRECi)
```

`EL1` and `EL2_NEXT`

`PREC` is a small `real(dp)` number. If the norm of the geometric patch is smaller than `PREC`, then the patch is ignored. If `energy_patch` is true, then it compares the design momenta of the magnets in both fibres. If the magnitude of difference is greater than `PREC`, then an energy patch is put on.

CHECK_NEED_PATCH

This routine checks whether a patch is needed without actually applying it to the layout. The routine returns the integer `PATCH_NEEDED`. If zero, it is not needed.

```
CHECK_NEED_PATCH(EL1,EL2_NEXT,PREC,PATCH_NEEDED)
```

Fibre Content

Before going any further, we remind the reader of the frames contained within a fibre:

- the frames of the fibre itself located in type `chart`, that is, `fibre%chart%f`,
- the frames of the magnet `fibre%mag%p%f` and its polymorphic twin,
- the frames of the integration nodes associated with this fibre/magnet, if they are present,
- the frame of a girder that might be tagged on this magnet.

Subroutines Invoking the Magnet and Relying on the DNA

When complex structures are constructed in PTC, it is preferable to follow a strict discipline. First various layouts with no "cloning" of magnets are created in a "mad universe" of data type `MAD_UNIVERSE`. The code MAD-X calls this universe `M_U`.

These no-clone layouts contain the actual magnet database of the accelerator complex. Therefore we refer to these layouts as the DNA of the complex.

Then trackable structures are appended after the DNA. Since the magnets of these structures must be in the DNA, they are created with the `APPEND_POINT` routine rather than the standard `APPEND_EMPTY` or `APPEND_FIBRE` used for the DNA production.

When the `APPEND_POINT` routine is used, a magnet will automatically retain memory of its various fibre appearances though a data type called `FIBRE_APPEARANCE`.

```
TYPE FIBRE_APPEARANCE
  TYPE(FIBRE), POINTER :: PARENT_FIBRE
  TYPE(FIBRE_APPEARANCE), POINTER :: NEXT
END TYPE FIBRE_APPEARANCE
```

Each magnet of the DNA contains a pointer called:

```
TYPE(FIBRE_APPEARANCE), POINTER :: DOKO
```

which constitutes a linked list storing all the appearances of this magnet in the pointer `parent_fibre`. The linked list is terminated, or grounded, at the last fibre appearance. If more trackable structures are created, this linked list is extended for each magnet.

There are all sorts of reasons why we may have multiple appearances of a DNA magnet: recirculation, common section of colliders or even multiple trackable structures of the same physical object.

It is through the `DOKO` construct that the following routines know where all the magnets are located.

Translation Routines with No Automatic Patching

This section describes translation routines that do not automatically patch fibres together.

TRANSLATE_FIBRE

This routine uses the `DOKO` construct, if associated, to locate all the appearances of a magnet and rotate the frames on all integration nodes if present.

```
TRANSLATE_FIBRE(R,D(3),ORDER,BASIS(3,3),DOGIRDER)
```

Here `R` is a fibre to be translated by `D`. `Dogirder=true` forces the translation of the girder frame if present.

TRANSLATE_LAYOUT

This routine

- scans the layout R from position I1 to I2 inclusive if present. I1 and I2 are defaulted to 1 and $R\%N$ respectively.
- calls TRANSLATE_FIBRE with dogirder=true on each fibre.
- uses the DOKO construct, if associated, to locate all the appearances of a magnet and rotate the frames on all integration nodes if present.

```
TRANSLATE_LAYOUT(R,D,I1,I2,ORDER,BASIS)
```

Rotation Routines with No Automatic Patching

This section describes rotation routines that do not automatically patch fibres together.

ROTATE_FIBRE

The comments that apply to TRANSLATE_FIBRE also apply to ROTATE_FIBRE.

```
ROTATE_FIBRE(R,OMEGA,ANG,ORDER,BASIS,DOGIRDER)
```

ROTATE_LAYOUT

The comments that apply to TRANSLATE_LAYOUT also apply to ROTATE_LAYOUT.

```
ROTATE_LAYOUT(R,OMEGA,ANG,I1,I2,ORDER,BASIS)
```

DNA-Designed Rotation Routines with Automatic Patching

This section describes rotation routines that automatically patch fibres together. The routines have been designed to rotate magnets stored in the DNA database.

ROTATE_MAGNET

```
ROTATE_MAGNET(R,ANG,OMEGA,ORDER,BASIS,PATCH,PREC)
```

R is a magnet. The routine rotates parent_fibre, which rotates all the frames of the magnet.

If patch=true, then all the appearances of this magnet stored in DOKO are patched—provided the norm of the patches is greater than PREC. It is advisable to set PREC to a not too small number (for example, 10^{-10}) to avoid useless patches.

If there is no DNA, that is, if PTC runs in pure compatibility mode with standard codes, then patches are on the parent fibre of the magnet.

TRANSLATE_MAGNET

This routine operates like ROTATE_MAGNET described above.

```
TRANSLATE_MAGNET(R,D,ORDER,BASIS,PATCH,PREC)
```

Operations on Siamese

For a discussion of siamese, see § 4.1.

Siamese are tied together by the pointer `SIAMESE`, which sits on `fibre%mag%siamese`.

To create a siamese structure, we make a circular linked list of magnets. For example, suppose three DNA fibres `f1`, `f2`, and `f3` must be tied together. This can be done as follows:

```
f1%mag%siamese=>f2%mag
f2%mag%siamese=>f3%mag
f3%mag%siamese=>f1%mag
```

Siamese Frame of Reference

Because a siamese does not have its own frame of reference, it is advisable to set up a so-called `affine_frame` on the siamese. In the above example, one picks up any magnet of the siamese, for example `f1%mag`, and calls the routine:

```
CALL ALLOC_AF(F1%MAG%SIAMESE_FRAME)
CALL FIND_PATCH(F1%CHART%F%A,F1%CHART%F%ENT,A,ENT,&
                F1%MAG%SIAMESE_FRAME%D,F1%MAG%SIAMESE_FRAME%ANGLE)
```

The siamese frame is located in relative coordinates from the entrance of the fibre that contains the magnet on which `siamese_frame` is attached (green objects above). Generally we may know the desired frame of reference in absolute coordinates given by the red `A(3)` and `ENT(3,3)` above. The relative translation and rotation can be computed, and the result is stored in the blue variables.

ROTATE_SIAMESE

This routine rotates the siamese by a set of angles `ang(1:3)` in the usual PTC order.

```
ROTATE_SIAMESE(S2,ANG,OMEGA,ORDER,BASIS,PATCH,PREC)
```

`S2` is any fibre that contains an element of the siamese string. The intricate usage of the optional variables `OMEGA,ORDER,BASIS` is best explained by displaying the actual code:

```
CALL FIND_AFFINE_SIAMESE(S2,CN,FOUND)
IF(FOUND) CALL FIND_FRAME_SIAMESE(CN,B,EXI,ADD=MY_FALSE)

IF(PRESENT(BASIS)) THEN
  BASIST=BASIS
ELSE
  IF(FOUND) THEN
    BASIST=EXI
  ELSE
    BASIST=GLOBAL_FRAME
  ENDIF
```

```
ENDIF
IF(PRESENT(OMEGA)) THEN
  OMEGAT=OMEGA
ELSE
  IF(FOUND) THEN
    OMEGAT=B
  ELSE
    OMEGAT=GLOBAL_ORIGIN
  ENDIF
ENDIF
```

If `OMEGA` is present, then it is used. If `BASIS` is present, it is also used. Normally one would expect both to be present or both to be absent. PTC does not check for this.

If they are not present, PTC looks for a siamese frame which is then used if it exists. Otherwise the global frame is used.

This routine calls the equivalent "magnet" routines over the entire string of siamese, and this permits automatic patching.

TRANSLATE_SIAMESE

This routine functions like the above `rotate_siamese` with the same priorities concerning the optional variable `BASIS`.

```
TRANSLATE_SIAMESE(S2,D,ORDER,BASIS,PATCH,PREC)
```

Operations on Girders

For a discussion of girders, see § 4.1.

To create a girder structure, we make a circular linked list of magnets. Let us create a girder structure with the three siamese `f1`, `f2`, and `f3` above. In addition, let us put a single magnet `f0` on the girder.

```
f0%mag%girder => f1%mag
f1%mag%girder => f2%mag
f2%mag%girder => f3%mag
f3%mag%girder => f0%mag
```

Four magnets are on the girder; three are in a siamese as well as on the girder.

Girder Frame of Reference

Unlike a siamese, a girder has a frame of reference. For example, we may elect to put the frame of reference on `f0`:

```
CALL ALLOC_AF(F0%MAG%GIRDER_FRAME,GIRDER=.TRUE.)
```

Then we set the following two affine frames of the girder to the same value:

```
F0%MAG%GIRDER_FRAME%ENT = ENT
F0%MAG%GIRDER_FRAME%A = A
F0%MAG%GIRDER_FRAME%EXI = ENT
F0%MAG%GIRDER_FRAME%B = A
```

The affine frame `A,ENT` can be any convenient frame chosen by the people who align the girder on the floor of the machine.

If a girder is misaligned, the affine frame `B,EXI` contains the new position of the girder. If the misalignments are removed, then `B,EXI` coincides with `A,ENT`. This allows the girder to have an existence independent of the fibres themselves. One can move fibres within a girder during its creation while keeping this frame fixed.

ROTATE_GIRDER and TRANSLATE_GIRDER

The routines operate exactly as the siamese routines do and therefore do not require any special description. The routines describe "design" displacements of the girder and therefore the affine frames `A,ENT` and `B,EXI` are moved together.

```
ROTATE_GIRDER(S2,ANG,OMEGA,ORDER,BASIS,PATCH,PREC)
TRANSLATE_GIRDER(S2,D,ORDER,BASIS,PATCH,PREC)
```

Misalignment Routines

PTC has three fundamental misalignment routines related to a single magnet, a siamese, and a girder.

The single magnet and the siamese are defined with respect to their fibre position and are thus inherently similar. The girder has a special frame of reference independent of the fibre.

If one is not careful, a single magnet or a siamese misalignment may break the girder. To avoid this problem, we start with the girder misalignment.

MISALIGN_GIRDER

```
MISALIGN_GIRDER(S2,S1,OMEGA,BASIS,ADD)
```

To understand how the misalignments affect single magnets, siamese, and girders, we examine the following:

- example 0—girder,
- example 1—girder after rotation and additive misalignment,
- example 2—misalign siamese followed by misalign girder,
- example 3—misalign girder followed by misalign siamese,
- example 4—misalign siamese with `PRESERVE_GIRDER=.true.`.

In these examples, we assume that a girder frame of reference has been defined; otherwise the girder becomes simply a giant siamese.

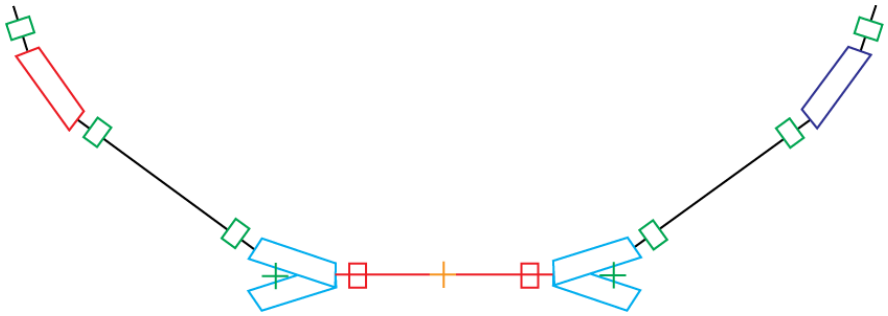


Figure 8.3: Example 0: girder.

Example 0

On the image in figure 8.3, the red and the cyan magnets are part of a single girder. The origin of the girder frame of reference is located in the middle of the red drift and displayed with an orange cross.

The cyan magnets form two siamese: one on the left and one on the right. The array `MIS(1:6)` contains the actual misalignments: the translations in `MIS(1:3)` and the rotations in `MIS(4:6)`. They are applied in the standard PTC order: rotation around the x, then the y, and finally the z-axis, followed by the translation.

Example 1

Now we rotate the girder by 22.5 degrees, as shown in figure 8.4.

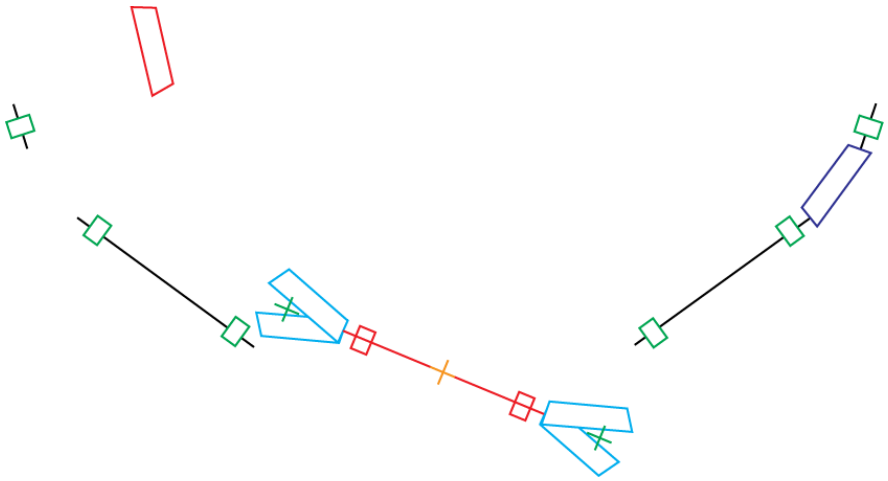


Figure 8.4: Example 1: girder after 22.5 degree rotation.

This was done with the command:

```
MIS=0.d0
MIS(5)=PI/8.d0
CALL MISALIGN_GIRDER(B,MIS,ADD=.FALSE.)
```

If we follow this command by:

```
MIS=0.d0
MIS(1)=2.d0
CALL MISALIGN_GIRDER(B,MIS,ADD=.TRUE.)
```

The `ADD=.TRUE.` indicates that the second misalignment of the girder is additive. The misalignment is in the direction of the rotated girder, as shown in figure 8.5.

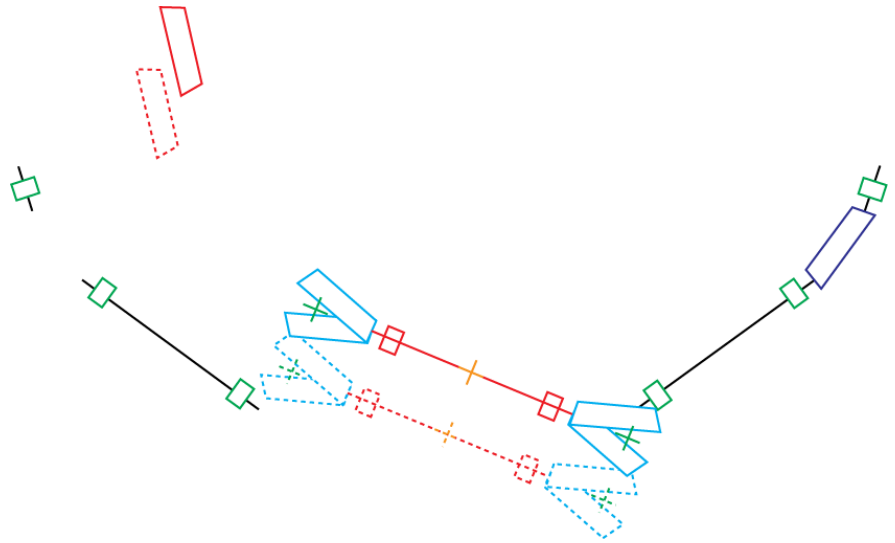


Figure 8.5: Example 1: girder after an additive misalignment.

MISALIGN_SIAMESE

```
MISALIGN_SIAMESE(S2,S1,OMEGA,BASIS,ADD,PRESERVE_GIRDER)
```

Example 2

Let us consider the following sequence of calls where the fibre pointer `B` is pointing to a member of the left siamese:

```
MIS=0.d0
MIS(1)=2.d0
CALL MISALIGN_SIAMESE(B,MIS,ADD=.FALSE.)
MIS=0.d0
MIS(1)=2.d0
MIS(5)=PI/8.d0
CALL MISALIGN_GIRDER(B,MIS,ADD=.TRUE.)
```

Figure 8.6 shows the result.

Example 3

Now let us switch the order.

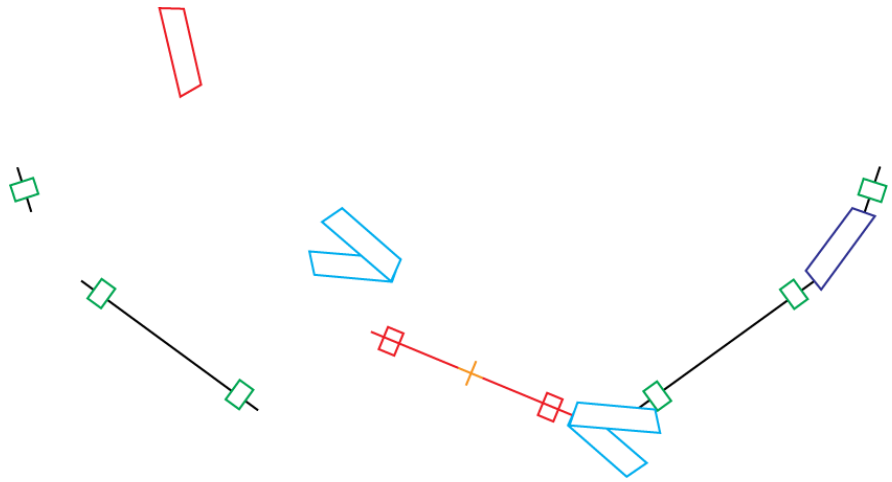


Figure 8.6: Example 2: misalign siamese followed by misalign girder.

```
MIS=0.d0
MIS(1)=2.d0
MIS(5)=PI/8.d0
CALL MISALIGN_GIRDER(B,MIS,ADD=.FALSE.)
MIS=0.d0
MIS(1)=2.d0
CALL MISALIGN_SIAMESE(B,MIS,ADD=.FALSE.)
```

Here we purposely made `ADD=.FALSE.` on the siamese call since naively one expects the position to be relative to the girder on which it is attached. The results should be the same, but they are not—as we see in figure 8.7.

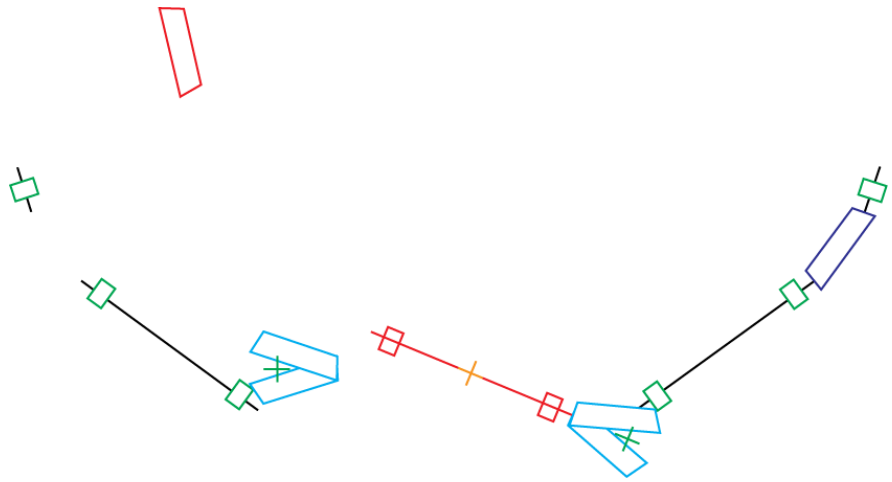


Figure 8.7: Example 3: misalign girder followed by misalign siamese.

All the magnets store their effective misalignments in one array. Therefore the siamese has no knowledge of being on a girder. `ADD=.FALSE.` sends the siamese back to its original fibre.

Example 4

This command avoids sending the siamese back to its original fibre prior to the misalignment:

```
MIS=0.d0
MIS(1)=2.d0
CALL MISALIGN_SIAMESE(B,MIS,ADD=.FALSE.,PRESERVE_GIRDER=.TRUE.)
```

Figure 8.8 shows the result.

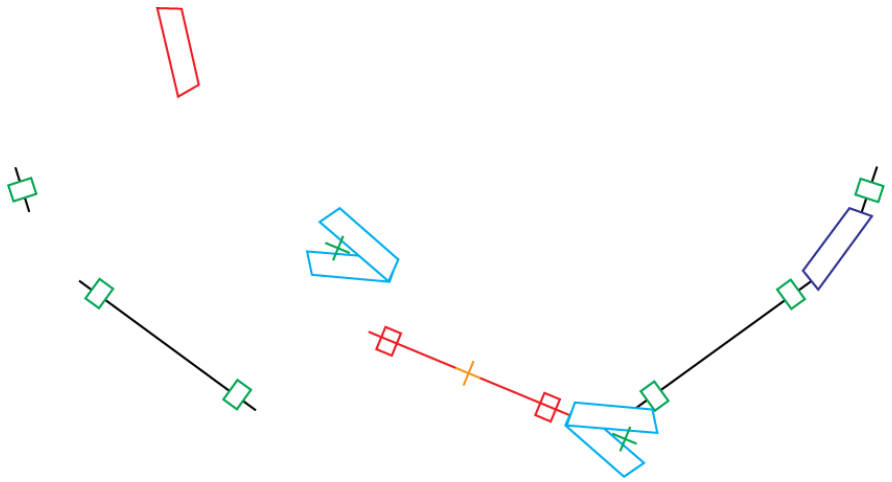


Figure 8.8: Example 4: misalign siamese with `preserve_girder=.true.`

The same command with `MIS=0.d0` in the siamese always returns the siamese to its girder position, not to fibre position.

MISALIGN_FIBRE

This routine behaves like the siamese routine but acts on a single fibre.

```
MISALIGN_FIBRE(S2,S1,OMEGA,BASIS,ADD,PRESERVE_GIRDER)
```

Note: If the `MISALIGN_FIBRE` routine is used on a siamese, it mildly breaks the siamese. One can imagine very tiny errors internal to a siamese and bigger errors on the siamese and yet bigger errors on a girder containing siamese and individual elements.

Therefore the following sequence of commands is acceptable if `B` is an element part that is of a siamese and part of a girder:

```
CALL MISALIGN_FIBRE(B,MIS1)
CALL MISALIGN_SIAMESE(B,MIS2,ADD=.TRUE.)
CALL MISALIGN_GIRDER(B,MIS3,ADD=.TRUE.)
```

Here we would imagine `MIS1` < `MIS2` < `MIS3`. Notice that `ADD=.TRUE.` on a siamese does not break a girder.

```
CALL MISALIGN_GIRDER(B,MIS3)
CALL MISALIGN_SIAMESE(B,MIS2,ADD=.FALSE. ,PRESERVE_GIRDER=.TRUE.)
CALL MISALIGN_FIBRE(B,MIS1,ADD=.TRUE.)
```

8.3 Dynamical Routines

Dynamical routines perform geometric rotations and translations, which act on the tracked object itself. PTC has an "exact" dynamical group and an "approximate" dynamical group. It is remarkable that the approximate dynamical rotations and translations also form a group—but it is not isomorphic to the affine Euclidean group.

The ultimate goal of PTC is to propagate particles and maps thanks to Taylor polymorphism. Placing geometric objects in three dimensions is worthless unless our rotations and translations can be translated into dynamical equivalents acting on rays (and on spin).

It is useful first to look at the Lie algebra acting on the t-based dynamics because it contains within it as a subgroup the affine part discussed above.

Exact Patching and Exact Misalignments: Dynamical Group

PTC first computes patching and misalignments geometrically. The connection between two affine frames is always expressed as follows through pure geometric computations:

$$\text{Connection} = T(d_x, d_y, d_z) \circ R_z \circ R_y \circ R_x$$

This product of operators is in the usual matrix ordering. Therefore the rotation in the x-axis acts first, followed by the y-axis, etc. The rotations are computed using `COMPUTE_ENTRANCE_ANGLE`.

We factor the rotation in that manner because the operators R_x and R_y are drifts in polar coordinates and are therefore nonlinear. For example, the rotation R_y is a pole face rotation, dubbed "prot" by Dragt in the code MARYLIE. The formula is given by:

$$\begin{aligned}
\overline{x} &= \frac{x}{\cos\alpha\left(1-\frac{p_x}{p_z}\tan\alpha\right)}, \text{ where } p_z = \sqrt{\left(1-\frac{2}{\beta_0}p_t+p_t^2\right)-p_x^2-p_y^2} \\
\overline{p}_x &= p_x\cos\alpha + p_z\sin\alpha \\
\overline{y} &= y + \frac{x p_y \tan\alpha}{p_z\left(1-\frac{p_x}{p_z}\tan\alpha\right)} \\
\overline{p}_y &= p_y \\
\overline{t} &= t + \frac{x\left(\frac{1}{\beta_0}-p_t\right)\tan\alpha}{p_z\left(1-\frac{p_x}{p_z}\tan\alpha\right)} \\
\overline{p}_t &= p_t
\end{aligned}$$

Here (t, p_t) form a canonical pair. In PTC $(-p_t, t)$ form a canonical pair.

We get the rotation R_x from R_y by interchanging x and y. Both rotations rotate the magnet towards the direction of propagation, that is, towards the z-direction.

The rotation R_z is the usual affine rotation along the z-axis. It is linear and transforms (x, y) and (p_x, p_y) identically, leaving (t, p_t) untouched. It rotates the x-axis of the magnet towards its y-axis.

The Lie operators for the three translations and rotations are given by:

$$\begin{aligned}
&:T_x: = :p_x:,\\
&:T_x: = :p_x:,\\
&:T_y: = :p_y:,\\
&:T_z: = :\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}:,\\
&:L_x: = :y\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}:,\\
&:L_y: = -:x\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}:,\\
&:L_z: = :xp_y - yp_x:.
\end{aligned}$$

It is remarkable that these Lie operators have the same Lie algebra as the ordinary affine (or time) Lie operators:

$$\begin{aligned}
&[L_x, L_y] = L_z, \qquad [L_x, p_y] = p_z, \qquad [L_x, p_z] = -p_y,\\
&[L_y, L_z] = L_x, \qquad [L_y, p_z] = p_x, \qquad [L_y, p_x] = -p_z,\\
&[L_z, L_x] = L_y, \qquad [L_z, p_x] = p_y, \qquad [L_z, p_y] = -p_x.
\end{aligned}$$

The Lie groups are therefore locally isomorphic. Of course one notices that "prot" (R_x and R_y) has a divergence at $\alpha = 90^\circ$. It is not possible in the "lens" or "s" paradigm to rotate a magnet map by 90° and get meaningful propagators. Therefore the dynamical group is locally isomorphic.

Inexact Patching and Exact Misalignments

PTC provides for an emasculated pseudo-Euclidean group. The Lie operators are obtained by expanding the dynamical maps keeping the energy dependence exact. This is in tune with the `exact_model=false` option.

$$\begin{aligned}
&:T_x: = :p_x:,\\
&:T_y: = :p_y:,\\
&:T_z(r_1, r_2): = :r_1 \underbrace{\left(-\frac{p_x^2 + p_y^2}{1(1+\delta)}\right)}_{D_z} + r_2\delta:,\\
&:L_x: = :y(1+\delta):,\\
&:L_y: = -:x(1+\delta):,\\
&:L_z: = :xp_y - yp_x:.
\end{aligned}$$

This group has seven generators for convenience. The Lie algebra differs from the original algebra of the Euclidean group as follows:

$$\begin{aligned}
&[L_x, L_y] = 0,\\
&[L_x, p_y] = T_z(0,1) = \text{emasculated} p_z,\\
&[L_y, p_x] = -T_z(0,1).
\end{aligned}$$

Why do we care about the approximate Euclidean group?

The reason is speed: if a fast post-processor to PTC is written. Let us assume that we have represented a (large) machine with `exact_model=false` and the drift-kick-drift option. Figure 8.9 schematically shows two magnets separated by a drift.

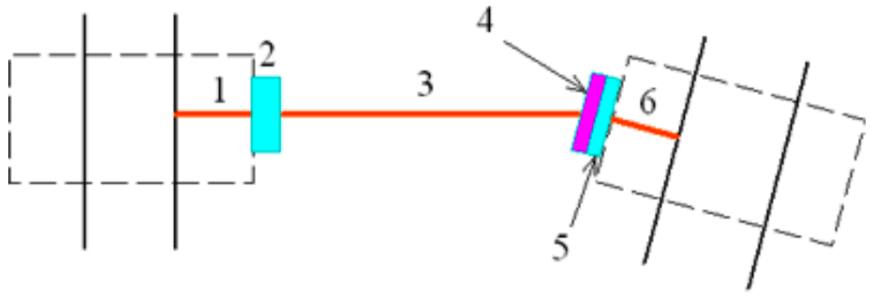


Figure 8.9: Pseudo-Euclidean maps.

The drifts (1,3,6) are in red. Some of the drifts (1,6) are part of the integration scheme. The misalignments are made of our six pseudo-Euclidean operators; they are in cyan (2,4). Finally a patch (4) is needed; it is in magenta.

The blue and cyan operators contain our six pseudo-Euclidean maps. Therefore to go from 1 to 6 in the figure may involve over 20 maps. By using the group properties, this can be reduced to six maps!

NINE

Symplectic Integration and Splitting

PTC generally attempts to integrate the maps of each magnet using a symplectic integrator. The current release of PTC supports two exceptions to explicit symplectic integration: the traveling wave cavity used in linear accelerators and the pancake type for fitted B-fields. Support for other exceptions could be added to be PTC: exact wigglers and exact helical dipoles, exact solenoids, etc.

This chapter discusses the elements of PTC amenable to explicit symplectic integration.

9.1 Philosophy

PTC's philosophy for symplectic integration, which is based on the work of Richard Talman,[1] involves six steps:

1. Split the elements in the lattice into integration nodes[2] using one of PTC's integration methods. For a list of PTC's integration methods, see Section 9.1.
2. Fit all the stuff you would normally fit using your matching routines.
3. Examine the resulting lattice functions and perhaps some short-term dynamic aperture.
4. If your results are satisfactory, reduce the number of integration nodes, the sophistication of the integration method, or both. Then go back to step 1.
5. If your results are not satisfactory, increase the number of integration nodes, the sophistication of the integration method, or both. Then go back to step 1.
6. After oscillating between steps 4 and 5, make up your mind and call that the lattice.

For your particular lattice, store all the information at step 6 so that you do not have to repeat the process!

In this chapter, we describe step 1 in detail. The other steps depend on the results you want for your simulation.

Example 3 in Section ?? gives an example—the "Talman algorithm"—of PTC code that performs these steps.

Integration Methods

PTC has six integration methods.

In the drift-kick-drift (D-K-D) case, PTC has three methods of integration:

- method 2, the naïve second-order method, which has one kick per integration step,

[1] In the early days of "kick" codes, physicists believed that the drift-kick-drift (D-K-D) kick model was massively inadequate because it requires a large number of steps to achieve decent convergence and thus the correct tunes. Such codes were restricted to special applications such as radiation and spin calculations—Chao's code SLIM.

The reason for this state of affairs was two-fold. First, a technical reason. We did not have high-order symplectic integrators. Second, a philosophical reason. People did not understand their own approaches, which contradicted the inadequacy of kick codes.

The technical issue was resolved by Ruth and later by a cabal of authors including Neri, Forest, and finally Yoshida. We can use high-order symplectic integrators on our usual "MAD8" Hamiltonian for the body of the magnet.

Almost simultaneously, the code TEAPOT emerged from the suffocating entrails of the SSC-CDG. TEAPOT integrated exactly using a D-K-D method—or more accurately, a PROT-KICK-PROT method—the combined function S-bend. TEAPOT was a second-

- method 4, the Ruth-Neri-Yoshida fourth-order method, which has three kicks per integration step,
- method 6, the Yoshida sixth-order method, which has seven kicks per integration step.

In the matrix-kick-matrix (M-K-M) case, PTC also has three methods of integration:

- method 1,
- method 3,
- method 5.

For more information about integration methods, see Section 9.4.

9.2 Splitting Tutorial Source File

The example code in this chapter is from the tutorial source file `ptc_splitting.f90` in *PTC Splitting Tutorial Source File: `ptc_splitting.f90`*, appendix D. The line numbers of the code in the examples refer to the line numbers of the code in the appendix.

9.3 Splitting the Lattice

This section describes the routines, global parameters, and arguments involved in splitting elements in the lattice.

Global Parameters

The default values for the global parameters are in *red italic* type.

Four global parameters are involved in splitting the lattice:

- `resplit_cutting` (*0*, 1, or 2) For more information, see Section 9.3.
- `sixtrack_compatible` (true or *false*) This global parameter enforces a second-order integrator for all magnets.
- `radiation_bend_split` (true or *false*) This global parameter splits bends with integration method 2 to improve radiation or spin results. For more information about PTC's methods of integration, see Section 9.1.
- `fuzzy_split` (default to *1.0*) If this global parameter is greater than 1.0, PTC lets some magnets with slightly too-long integration steps go through, i.e., if `ds<lmax0*fuzzy_split`.

Splitting Routines

Mandatory arguments for the routines are in `regular black` type. Optional arguments are in *`red italic`* type.

`THIN_LENS_RESTART(R1,`*`FIB,USEKNOB`*`)`

This routine resets all magnets to second-order integration with one step.

`THIN_LENS_RESPLIT(R,`*`THIN,EVEN,LIM(1:2),LMAX0,XBEND,FIB,USEKNOB`*`)`

This routine splits all the magnets in the lattice into integration nodes, or *thin lenses*.

To restrict the action of `THIN_LENS_RESTART` and `THIN_LENS_RESPLIT` to specific fibres or groups of magnets, use the optional arguments `FIB` and `USEKNOB`. For more information, see Section 9.3.

Arguments

This section describes the arguments for the THIN_LENS_RESTART and THIN_LENS_RESPLIT routines.

R1 or R Argument

The name of the layout to be split.

Optional Argument THIN

The optional argument THIN describes an approximate integrated quadrupole strength for which a single integration node, or *thin lens*, in the body of an element should be used. For example, the quadrupoles QF and QD in Example 1 below have an integrated strength of

```
KF L = 0.279309637578521      KD L=-0.197159744173073
```

Example 1

For our build-psr layout (see *build_PSR*, page 26), we make following calls:

```
CALL MOVE_TO(R1,QF,"QF",POS)
CALL MOVE_TO(R1,QD,"QD",POS)
THIN=0.01D0
LIMITS(1:2)=100000

CALL THIN_LENS_RESPLIT(R1,THIN,LIM=LIMITS)
WRITE(6,*) QF%MAG%NAME,QF%MAG%P%METHOD,QF%MAG%P%NST
WRITE(6,*) QD%MAG%NAME,QD%MAG%P%METHOD,QD%MAG%P%NST
```

NST is the number of integration steps.
Table 9.1 shows the results.

Element	Method	Steps	Kicks/Step	Total Kicks
QF	2	27	1	27
QD	2	19	1	19
B	2	15	1	15

Table 9.1: Results of Example 1.

KF/THIN is about 27.9, which is close to the 27 integration steps that result from Example 1, and KD/THIN is about 19.7, which is close to the 19 integration steps that result from Example 1. Example 1 behaves as expected: it splits according to quadrupole strength. The method stayed 2, i.e., second-order integrator. This is not efficient if the number of steps must be large. Therefore we must teach the splitting algorithm to switch to the fourth-order or sixth-order integrator. We discuss how to do this in Section 9.3.

In the bend, we also look at the amount of natural horizontal focusing, vertical focusing, or both. In this example, a small ring with bends of 36 degrees, the bends are unusually strong. The global parameter radiation_bend_split and the optional argument xbend can be used to affect the bends.

LIM Optional Argument

The optional array `lim(2)` determines when PTC should choose the second-order, fourth-order, or sixth-order integration method.

- `lim(1)` > `KL/THIN`: second-order method,
- `lim(2)` > `KL/THIN` > `lim(1)`: fourth-order method,
- `KL/THIN` > `lim(2)`: sixth-order method.

Example 2

Let us rerun the previous case with a different limit array:

```
THIN = 0.01D0
LIMITS(1) = 8
LIMITS(2) = 24

CALL THIN_LENS_RESPLIT(R1,THIN,LIM=LIMITS)
WRITE(6,*) QF%MAG%NAME, QF%MAG%P%METHOD, QF%MAG%P%NST
WRITE(6,*) QD%MAG%NAME, QD%MAG%P%METHOD, QD%MAG%P%NST
```

Note that `KF/THIN`, which is about 27.9, is greater than `lim(2)`, which is 24. `KD/THIN`, which is about 19.7, is less than `lim(2)`. Both are greater than `lim(1)`.

Table 9.2 shows the results.

Element	Method	Steps	Kicks/Step	Total Kicks
QF	6	3	7	21
QD	4	6	3	18
B	4	5	3	15

Table 9.2: Results of Example 2.

PTC uses integration method 6 for element `QF`, which results in three integration steps with seven kicks per step: a total of 21 kicks. PTC uses integration method 4 for element `QD`, which results in six integration steps with three kicks per step: a total of 18 kicks. As we can see, the number of kicks has remained more or less constant while the integration method has increased in order.

Example 3

This PTC code is an example of the six-step PTC philosophy for splitting (see § 9.1), which we are calling the "Talman algorithm". The example reuses Example 2's limit array.

```
!!!! TALMAN ALGORITHM !!!!! !!!!!  EXAMPLE 3
WRITE(6,*) "  "
WRITE(6,*) "!!!! TALMAN ALGORITHM !!!!! !!!!!  EXAMPLE 3"
WRITE(6,*) "  "

! PART 1A
```

```
THIN=0.001D0  ~! CUT LIKE CRAZY
CALL THIN_LENS_RESPLIT(R1,THIN,LIM=LIMITS)
!!! PTC COMMAND FILE: COULD BE A MAD-X COMMAND OR WHATEVER
! COMPUTING TUNE AND BETA AROUND THE RING

! PART 1B
CALL READ_PTC_COMMAND77("FILL_BETA0.TXT")
WRITE(6,*) "  "
WRITE(6,*) " NOW REDUCING THE NUMBER OF STEPS AND REFITTING "
WRITE(6,*) "  "
! PART 2
DO I=0,2
THIN=0.01D0+I*0.03

! PART 2A
CALL THIN_LENS_RESPLIT(R1,THIN,LIM=LIMITS) ! REDUCING NUMBER OF CUTS
! FITTING TO PREVIOUS TUNES

! PART 2B
CALL READ_PTC_COMMAND77("FIT_TO_BETA0_RESULTS.TXT")
! COMPUTING DBETA/BETA AROUND THE RING

! PART 2C
CALL READ_PTC_COMMAND77("FILL_BETA1.TXT")
ENDDO
```

Example 3 uses the limits from Example 2: `lim(1)` of 8 and `lim(2)` of 24 with `THIN=0.001D0`. `KF/THIN` is now about 279, and `KD/THIN` is about 197. Both are much greater than `lim(2)`. PTC will use integration method 6, which has seven kicks per integration step.

Table 9.3 and the list below show the results for Part 1A of Example 3.

Method	?	?
2	40	40
4	0	0
6	30	6230

Table 9.3: Results of Example 3 for Part 1A.

- Number of slices: 6270.
- Total NST: 930.
- Total NST due to Bend Closed Orbit: 0.
- Biggest ds: 0.115885454545455.

The quadrupoles and the dipoles are split using integration method 6 with a grand total of 6240 slices.

In Part 1B, compute the tunes and betas around the ring. The fractional tunes are:

```
0.142192063715077      0.854078356314425
```

And the betas are stored for future use.

In Part 2A, the magnet is resplit less and less finely, the tune is refitted, and finally the delta beta around the ring is estimated as a measure of the splitting adequacy.

The results for `THIN = 0.01` are:

- `<DBETA/BETA> = 4.025629639896395E-006`
- `MAXIMUM OF DBETA/BETA = 6.034509389788590E-006`

The results for `THIN = 0.04` are:

- `<DBETA/BETA> = 2.065365900468319E-003`
- `MAXIMUM OF DBETA/BETA = 4.014532098810251E-003`

The results for `THIN = 0.07` are:

- `<DBETA/BETA> = 5.779446473894797E-003`
- `MAXIMUM OF DBETA/BETA = 1.068563516341058E-002`

One can also look at the chromaticities. For example, the chromaticities at the beginning were:

```
-2.87009384223620       -2.03916389491785
```

At the end, the chromaticities are:

```
-2.86244921970493       -2.03671376872069
```

At this point, we can freeze the lattice for good. Of course refitting the tune was only an example: in other lattices we may want to rematch a more complete set of properties: dispersion, alphas, phase advances, etc.

XBEND Optional Argument

This section discusses usage of the `XBEND` optional argument when `exact=.true.`

The lattice uses sector bends. Because this is an ideal lattice, the tune should be the same regardless of whether with `exact_model=true` or `false`. Therefore we will fit to the same tune as before, passing it through the same algorithm.

Example 4

```
WRITE(6,*) "  "
WRITE(6,*) "!!!! SBEND ORBIT SMALL PROBLEM !!!!! !!!!!  EXAMPLE 4"
WRITE(6,*) "  "

CALL APPEND_EMPTY_LAYOUT(M_U)  ! NUMBER 2
CALL SET_UP(M_U%END)
R1=>M_U%END

EXACT=.TRUE.
METHOD=DRIFT_KICK_DRIFT
CALL RUN_PSR(R1,EXACT,METHOD)

WRITE(6,*) "  "
WRITE(6,*) " NOW REDUCING THE NUMBER OF STEPS AND REFITTING "
WRITE(6,*) "  "
DO I=0,2
 THIN=0.01D0+I*0.03
```

```
 CALL THIN_LENS_RESPLIT(R1,THIN,LIM=LIMITS) ! REDUCING NUMBER OF CUTS
 ! FITTING TO PREVIOUS TUNES
CALL READ_PTC_COMMAND77("FIT_TO_BETA0_RESULTS_2.TXT")
  ! COMPUTING DBETA/BETA AROUND THE RING
CALL READ_PTC_COMMAND77("FILL_BETA1_2.TXT")
ENDDO
```

The results of the last iteration for the closed orbit are:

```
 -3.129618316516444E-002  3.357101188909470E-003  0.000000000000000E+000
  0.000000000000000E+000  0.000000000000000E+000  0.000000000000000E+000
```

The results of the last iteration for the stability is T.
The results of the last iteration for the tunes are:

```
0.142192063715077       0.854078356314425
```

In addition:

- <DBETA/BETA> = 1.184251326377263E-002
- MAXIMUM OF DBETA/BETA = 2.032348481520253E-002

We notice that the closed orbit is wild and that the fluctuation of the beta functions is twice as big as before.

This problem can be alleviated by enforcing a finer split of the bend. This is done by setting the XBEND argument to some small value that corresponds approximately to the norm of the residual orbit.

Example 5

In example 5, the splitting line is replaced by this one:

```
CALL THIN_LENS_RESPLIT(R1,THIN,LIM=LIMITS,XBEND=1.D-4)
```

The results of the final iteration steps for the closed orbit are:

```
 -3.377915412576128E-003  3.670253104757456E-004  0.000000000000000E+000
  0.000000000000000E+000  0.000000000000000E+000  0.000000000000000E+000
```

The results of the final iteration for the stability is T.
The results of the final iteration for the tunes are:

```
0.142192063715077       0.854078356314425
```

In addition:

- <DBETA/BETA> = 3.542348342695508E-003
- MAXIMUM OF DBETA/BETA = 5.551255781973659E-003

EVEN Optional Argument

There are three possibilities for the EVEN optional argument:

- EVEN=true enforces an even split,
- EVEN=false enforces an odd split,
- EVEN is not in the call statement: an even or odd split is acceptable.

Example 6

`EVEN=true` is important if the motion must be observed at the center of a magnet. This is useful during matching procedures in particular.

The data in table 9.4 show the "indifferent" splitting. `T1` is a pointer to the integration node at the beginning of the fibre. `TM` points to the center if it exists. `T2` points to the end of the fibre. `TM` and `T1` are identical when the number of steps is odd; there is no center of the magnet, and by default `TM` points to `T1`.

Name	Method	Steps	T1%pos	TM%pos	T2%pos
QF	6	3	35	35	41
QD	4	6	52	57	61
B	4	5	67	67	75

Table 9.4: Results of Example 6 when even is not specified.

The data in table 9.5 shows that when we use `EVEN=true`, `TM` is always different from `T1`. `TM` points directly at the center of the magnet.

Name	Method	Steps	T1%pos	TM%pos	T2%pos
QF	6	4	39	43	46
QD	4	6	59	64	68
B	4	6	75	80	84

Table 9.5: Results of Example 6 when even=.true.

LMAX0 Optional Argument

The `lmax0` optional argument must be used in conjunction with the `resplit_cutting` global parameter set to `1` or `2`. The `resplit_cutting` global parameter is normally defaulted to zero.

If `resplit_cutting=1`, the ordinary magnets are left as is. However, the drifts are split so that the maximum length of an integration node cannot exceed `lmax0`.

Example 7

```
call move_to(r1,d1,"D1",pos)
call move_to(r1,d1_next,"D1",pos)
call move_to(r1,d2,1) !!! Put the pointer for search back at position 1
call move_to(r1,d2,"D2",pos)
call move_to(r1,qf,"QF",pos)
call move_to(r1,qd,"QD",pos)
call move_to(r1,b,"B",pos)
resplit_cutting=1
call THIN_LENS_restart(r1)
thin=0.01d0
CALL THIN_LENS_resplit(R1,THIN,even=.true.,lmax0=0.05d0,xbend=1.d-4)
```

Table 9.6 shows the results of the above run.

Name	Method	Steps	T1%pos	TM%pos	T2%pos
D1	2	46	1	26	102
D1	2	46	103	26	152
D2	2	10	57	64	70
QF	6	4	95	99	102
QD	6	2	203	206	208
B	4	6	223	228	232

Table 9.6: Results of Example 7 for `resplit_cutting=1`.

Notice that only the drifts `D1` and `D2` were split.

If `resplit_cutting=2`, then all the magnets and the drifts are split to achieve a maximum length of `lmax0`. This is useful in space-charge calculations. See table 9.7.

Name	Method	Steps	T1%pos	TM%pos	T2%pos
D1	2	46	1	26	166
D1	2	46	167	26	216
D2	2	10	67	74	80
QF	6	12	151	159	166
QD	6	12	267	275	282
B	4	52	297	325	352

Table 9.7: Results of Example 7 for `resplit_cutting=2`.

Now all the magnets are split.

FIB Optional Argument

Continuing with the previous example, we make the following call.

Example 8

```
WRITE(6,*) "!!!! FIB KEYWORD  !!!!! !!!!!  EXAMPLE 8"
CALL THIN_LENS_RESTART(R1)
CALL THIN_LENS_RESPLIT(R1,THIN,EVEN=.TRUE.,LMAX0=0.005D0,XBEND=1.D-4,FIB=D1_NEXT)
```

Table 9.8 shows the results.

Name	Method	Steps	T1%pos	TM%pos	T2%pos
D1	2	46	1	26	50
D1	*2*	*458*	*167*	*26*	*628*
D2	2	10	67	74	80
QF	6	12	151	159	166
QD	6	12	679	687	694
B	4	52	709	737	764

Table 9.8: Results of Example 8.

Only the fibre D1_NEXT is affected. All the others are left intact. This allows the splitting of a single fibre.

USEKNOB Optional Argument

We can use the type pol_block to produce a splitting of the lattice.

Example 9

```
FAM=0
FAM%NAME="D1"
R1=FAM
CALL THIN_LENS_RESTART(R1) ! PUTS BACK METHOD =2 AND NST=1 EVERYWHERE
CALL THIN_LENS_RESPLIT(R1,THIN,EVEN=.TRUE.,LMAX0=0.005D0,XBEND=1.D-4,
  USEKNOB=.TRUE.)
```

If useknob is true, then the magnets with the knob flag "on" are split, assuming no other flags prevent it. In this particular case, resplit_cutting=2, therefore, the drifts will be split on the basis of LMAX0=0.005D0.

Table 9.9 shows the results.

Name	Method	Steps	T1%pos	TM%pos	T2%pos
D1	2	*458*	*1*	*232*	*462*
D1	2	*458*	*488*	*232*	*949*
D2	2	1	468	468	472
QF	2	1	483	483	487
QD	2	1	1412	1412	1416
B	2	1	1422	1422	1426

Table 9.9: Results of Example 9 when useknob=.true.

If we perform the same call with useknob=.false., then the magnets with knob=true are masked and the other magnets are split.

```
CALL THIN_LENS_RESPLIT(R1,THIN,EVEN=.TRUE.,LMAX0=0.005D0,XBEND=1.D-4,
  USEKNOB=.FALSE.)
```

Table 9.10 shows the results.

Name	Method	Steps	T1%pos	TM%pos	T2%pos
D1	2	1	1	1	5
D1	2	1	924	1	928
D2	2	*92*	*112*	*160*	*207*
QF	6	*102*	*818*	*871*	*923*
QD	6	*102*	*934*	*987*	*1039*
B	4	*510*	*1136*	*1393*	*1649*

Table 9.10: Results of Example 9 when useknob=.false.

9.4 Other Splitting Routines

This section discusses PTC integration methods 1, 3, and 5.

Splitting a Single Fibre

This routine splits one fibre.

```
RECUT_KIND7_ONE(FIBRE,LMAX0,DRIFT)
```

In the presence of space charge or for other reasons, it may be more important to observe the beam at many locations "approximately" than to use a high-order integration method.

PTC provides integration method 1 for certain magnets.

Drift-Kick-Drift: Strict Talman Interpretation

For this type of algorithm, the `exact_model = true` and the `exact_model = false` can be switched into method 1 if and only if the original method was method 2, i.e., second-order integrator. Basically, negative propagators (drifts) are not acceptable: this is the strict Talman interpretation to which PTC does not adhere except in the case of switching to integration method 1.

Figure 9.1 shows a pictorial example.

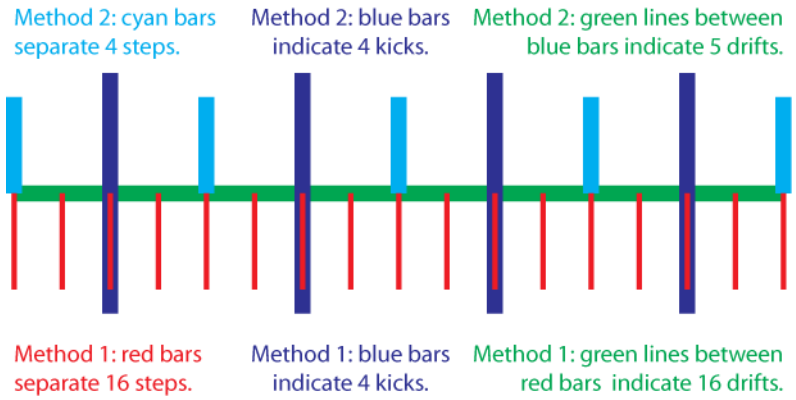


Figure 9.1: Drift-kick-drift for integration methods 1 and 2.

The blue bars and green lines represent the original drift-kick-drift of integration method 2. The figure shows four kicks and five drifts. There are four steps, separated by the cyan bars. The new integration method 1 split is simply a splitting of the drifts. There are now 16 drifts (but still four kicks). Obviously, the results should be the same to machine precision.

Example with the 36-degree bend of the PSR (Los Alamos):

```
BE  = SBEND("B", 2.54948D0, TWOPI*36.D0/360.D0);
  X=0.001D0
  CALL TRACK(BE,X,DEFAULT)
  WRITE(6,*) BE%MAG%P%METHOD,BE%MAG%P%NST
  WRITE(6,*) X
    CALL RECUT_KIND7_ONE(BE,2.54948D0/16,.FALSE.)
X=0.001D0
  CALL TRACK(BE,X,DEFAULT)
```

```
  WRITE(6,*) BE%MAG%P%METHOD,BE%MAG%P%NST
  WRITE(6,*) X
X=0.001D0
  CALL ALLOC(Y)
  Y=X
  CALL TRACK(BE,Y,DEFAULT)
  X=Y
  WRITE(6,*) BE%MAG%P%METHOD,BE%MAG%P%NST
  WRITE(6,*) X
  PAUSE 888
```

Integration results are:

```
     2      4      ------ ! ORIGINAL METHOD AND NUMBER OF STEPS
 3.962104445927857E-003  1.253543855340728E-003  3.546933066933068E-003
 1.000000000000000E-003  1.000000000000000E-003  2.523695791724136E-003
     1     16
 3.962104445927856E-003  1.253543855340728E-003  3.546933066933066E-003
 1.000000000000000E-003  1.000000000000000E-003  2.523695791724136E-003
     1     16
 3.962104445927856E-003  1.253543855340728E-003  3.546933066933066E-003
 1.000000000000000E-003  1.000000000000000E-003  2.523695791724136E-003
```

Note that method 2 has four integration steps, and method 1 has 16 integration steps. A larger number of integration steps for method 1 does not change the accuracy.

Matrix-Kick-Matrix

Integration methods 2, 4, and 6 switching to methods 1, 3, and 5.

PTC has a matrix-kick-matrix method where the matrix is energy independent. The delta dependence is buried in the kick.

Our comments here apply to `exact_model=.false.`

Integration methods 2, 4, and 6—shown in figure 9.2—always split the integration step using matrices of equal length. These methods are the so-called biased integration methods. Although they have existed in accelerators since the days of SSC, recently they have been discussed at length by McLachlan and also by Laskar and Robutel.

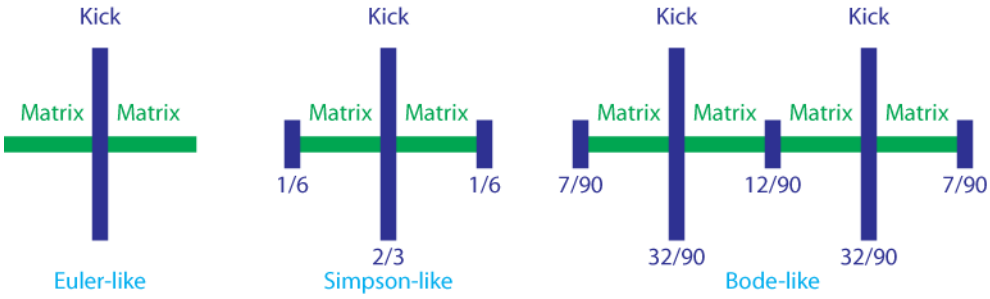


Figure 9.2: Matrix-kick-matrix for integration methods 2, 4, and 6.

In accelerators, integration methods 2, 4, and 6 have the advantage of producing the "exact" tune for a typical ideal machine. The effect of the matrix on the kicks is of order 2, 4, or 6, although all three methods are truly second-order integrators strictly speaking—the word *biased* refers to this uneven way of ordering the perturbation due to the kicks Hamiltonian that produced the matrices.

Let us retry the above example with the Bode-like integrator:

```
      6      4     ------ ! ORIGINAL METHOD AND NUMBER OF STEPS
 3.966179536791447E-003  1.252157150179758E-003  3.546933066933068E-003
 1.000000000000000E-003  1.000000000000000E-003  2.529326133576445E-003
      5     16
 3.966179536791447E-003  1.252157150179758E-003  3.546933066933068E-003
 1.000000000000000E-003  1.000000000000000E-003  2.529326133576445E-003
      5     16
 3.966179536791447E-003  1.252157150179758E-003  3.546933066933068E-003
 1.000000000000000E-003  1.000000000000000E-003  2.529326133576445E-003
```

Please note that `exact_model=true` works only with straight elements if the matrix-kick-matrix method (kind7) is selected. Bends should use the drift-kick-drift method (integration method 2) if `exact_model=true`.

Drifts

Drifts can be split using:

```
RECUT_KIND7_ONE(D1,0.45D0/16.D0,.TRUE.)
```

Splitting an Entire Lattice

This routine applies `RECUT_KIND7_ONE` over the entire layout.

```
RECUT_KIND7(LAYOUT,LMAX0,DRIFT)
```

Appendices

A

Internal States

PTC contains a series of flags held in the global variable `DEFAULT`. These *internal-state* flags allow you to control certain aspects of PTC's behavior based on the needs of your simulation. For example, you could run one simulation with the `RADIATION` flag off and another simulation with the flag on. By comparing the results, you would answer the question: "How important is radiation to my simulation?"

Here are six internal-state flags and the PTC behavior each flag controls:

1. TOTALPATH ensures a computation of the total path length or total time of flight.
2. TIME selects time of flight rather than path length (cT to be precise).
3. RADIATION turns on classical radiation.
4. NOCAVITY forces the code to ignore RF cavities. It has also implications on the normal form if performed in three degrees of freedom.
5. FRINGE turns on quadrupole fringe fields based on the b_2 and a_2 components in the element.
6. EXACTMIS forces the misalignments to be treated exactly.

PTC has a special internal state called `DEFAULT` that selects path length rather than time of flight, does not ensure a computation of the total path length, turns off classical radiation, includes RF cavities, turns off quadrupole fringe fields, and does not treat misalignments exactly.

This example shows how to set the default internal-state environment described above:

```
CALL MAKE_STATES(.FALSE.)
EXACT_MODEL=.TRUE.
DEFAULT=DEFAULT
CALL UPDATE_STATES
```

`MAKE_STATES` is `TRUE` for electrons and `FALSE` for protons. When `EXACT_MODEL` is `TRUE`, PTC uses the full square-root Hamiltonian.

The next example shows how to set the default internal-state environment with the NOCAVITY and EXACTMIS flags:

```
CALL MAKE_STATES(.FALSE.)
EXACT_MODEL=.TRUE.
```

```
DEFAULT=DEFAULT+NOCAVITY+EXACTMIS
CALL UPDATE_STATES
```

The following six flags are strictly related to TPSA calculations:

1. If PARA_IN is specified, TPSA knobs are included in the calculation. It is activated by a unary + on a state, for example, `TRACK(PSR,Y,1,+DEFAULT)`.
2. If ONLY_4D is specified, then neither path length nor time is a TPSA variable. This means that the phase-space dimension in the normal form will be 4 (X, P_x, Y, P_y). Also X(5) will not be TPSA unless DELTA is also specified (see next the item). This flag has no effect on PTC tracking. PTC always tracks six phase-space variables.
3. If DELTA is specified, then ONLY_4D is also true. However, in this case, X(5) is the fifth TPSA variable: X, P_x, Y, P_y plus Energy as the fifth variable. The phase-space dimension in the normal form will also be 4; momentum compaction cannot be computed.
4. SPIN
5. SPIN_ONLY
6. SPIN_DIM

This example shows how to set the default internal-state environment with a four-dimensional phase space in the normal form:

```
DEFAULT=DEFAULT+ONLY_4D
```

The next example shows how to set the default internal-state environment with a four-dimensional phase space in the normal form and with energy as the fifth-dimension TPSA variable:

```
DEFAULT=DEFAULT+DELTA
```

B

Data Types

This appendix contains descriptions and examples of the FORTRAN90 data that PTC uses for *s*-based tracking and time-based tracking.

B.1 S-BASED TRACKING

This section discusses the following PTC data types for *s*-based tracking and gives the FORTRAN90 definitions for several important types:

- layout,
- fibre,
- chart, including magnet frame,
- patch.

Layout

The `LAYOUT` type is a doubly linked list whose nodes, of data type `FIBRE`, contain the magnet, the local charts, and the patches.

```
TYPE LAYOUT
  CHARACTER(120), POINTER ::  NAME  ! IDENTIFICATION
  INTEGER, POINTER ::  INDEX,HARMONIC_NUMBER  ! IDENTIFICATION, CHARGE SIGN
  LOGICAL(LP),POINTER :: CLOSED
  INTEGER,  POINTER :: N  ! TOTAL ELEMENT IN THE CHAIN
  INTEGER,POINTER :: NTHIN
  ! NUMBER IF THIN LENSES IN COLLECTION  (FOR SPEED ESTIMATES)
  REAL(DP),  POINTER :: THIN
  ! PARAMETER USED FOR AUTOMATIC CUTTING INTO THIN LENS
  ! POINTERS OF LINK LAYOUT
  INTEGER, POINTER :: LASTPOS  ! POSITION OF LAST VISITED
  TYPE (FIBRE), POINTER :: LAST  ! LAST VISITED
  !
  TYPE (FIBRE), POINTER :: END
  TYPE (FIBRE), POINTER :: START
  TYPE (FIBRE), POINTER :: START_GROUND
  ! STORE THE GROUNDED VALUE OF START DURING CIRCULAR SCANNING
  TYPE (FIBRE), POINTER :: END_GROUND
  ! STORE THE GROUNDED VALUE OF END DURING CIRCULAR SCANNING
```

```
  TYPE (LAYOUT), POINTER :: NEXT
  TYPE (LAYOUT), POINTER :: PREVIOUS
  TYPE(NODE_LAYOUT), POINTER :: T  !  ASSOCIATED  CHILD THIN LENS LAYOUT
  TYPE (MAD_UNIVERSE), POINTER :: PARENT_UNIVERSE
  TYPE(LAYOUT_ARRAY), POINTER :: DNA(:)
  !   TYPE(SIAMESE_ARRAY), POINTER :: GIRDER(:)
  !   TYPE(JUNCTION_ARRAY), POINTER :: CON2(:)
END TYPE LAYOUT
```

The first lines of the TYPE definition contain data specific to the layout itself: data concerning steps of integration statistics.

Besides the fibres, the two most important quantities in this layout are N and CLOSED. The variable N is the number of fibres in the layout. N is updated each time magnets are inserted or deleted. The boolean CLOSED refers to the topology of the so-called "base space" or, in accelerator parlance, the s-variable. Is the variable s periodic (a ring), or is the variable s defining an interval (a straight beam line)? In the case of a ring, CLOSED is set to true, otherwise it is set to false. This does not happen automatically: the user must set it to the desired value.

The pointer T points to the layout's its associated NODE_LAYOUT. This pointer is normally grounded because PTC does not systematically create node layouts or *thin layouts*. When not grounded, the pointer contains the expanded representation displayed on the left side of figure B.5.

Figure B.1 illustrates the roles of the pointer variables. The figure shows a layout with four elements. Obviously, the number of elements in an actual beam line could be enormous.

First of all, a linked list must have at least one pointer; this allows the creation of a simple linked list rather than a doubly linked list. In our case this role is played by the pointer PREVIOUS. In a simple linked list, each node has the red PREVIOUS pointer. The pointer END points to the end of the list, in our case fibre 4. Then the list is traversed backwards using the pointer PREVIOUS.

A more complex structure is required to handle two-way traversing. To do this, we use the pointer START. At each node we add the green pointer NEXT. The green pointer points to the next element until it finally ends on the START pointer. The START pointer as well as the END pointer are nullified or grounded. Thus one can perform an "ASSOCIATED" check on either NODE%NEXT or NODE%PREVIOUS to determine if the extremities of the list have been reached.

We want to have a *circular* linked list for tracking a ring. When circular, the magenta link pointing to the grounded START is cut. The last fibre's green pointer now points to fibre 1 (blue link). Thus the list is made circular in the forward direction. The purpose of the START GROUND pointer is to remember the location of the grounded START pointer to re-establish the ordinary two-way terminated list if necessary. The same type of operation is performed on the magenta link pointing to the END pointer. The list is the fully circular. PTC routines permit a programmer to toggle back and fourth between terminated and circular lists.

The LAST pointer remembers the last fibre which was accessed by any of the maintenance routines. This allows the routine MOVE TO, which locates an actual fibre, to find its target faster. In our case speed is not of great importance. We tend to traverse a LAYOUT in the order of tracking.

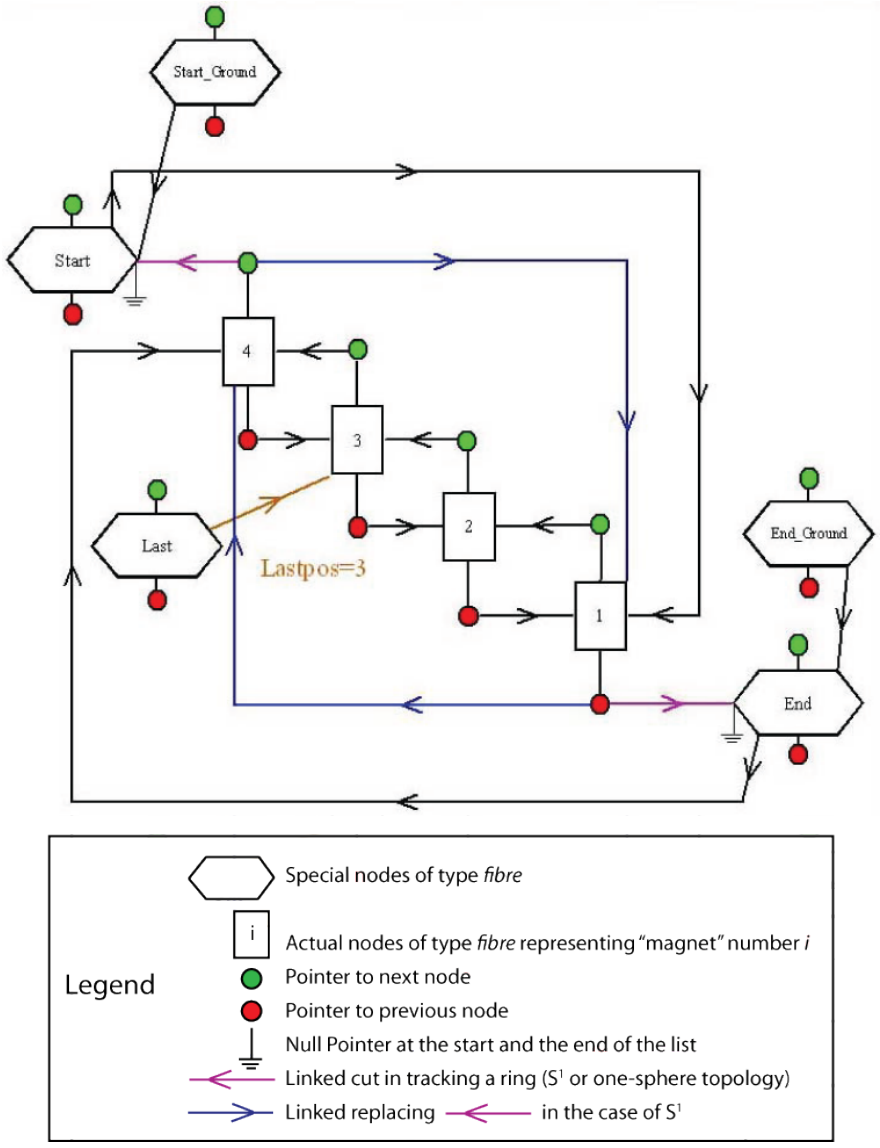


Figure B.1: A layout is a linked list of fibres.

Fibre

The type FIBRE is recursively defined. This allows the creation of a linked list. One can think of a linked list as a chain; data potentially hangs on each link. In our case, the fundamental datum is the object CHART of data type CHART. This contains three actual charts (affine frames of reference): one at each end of the fibre and one in the middle.

```
TYPE FIBRE
  !  BELOW ARE THE DATA CARRIED BY THE NODE
  INTEGER,POINTER ::DIR
  TYPE(PATCH),POINTER ::PATCH
```

```
  TYPE(CHART),POINTER ::CHART
  TYPE (ELEMENT), POINTER ::  MAG
  TYPE (ELEMENTP),POINTER ::  MAGP
  !  END OF DATA
  !  POINTER TO THE MAGNETS ON EACH SIDE OF THIS NODE
  TYPE (FIBRE),POINTER :: PREVIOUS
  TYPE (FIBRE),POINTER :: NEXT
  !  POINTING TO PARENT LAYOUT AND PARENT FIBRE DATA
  TYPE (LAYOUT),POINTER :: PARENT_LAYOUT
  TYPE(INFO),POINTER ::I
  TYPE(INTEGRATION_NODE),POINTER :: T1,T2
  ! FIRST AND LAST INTEGRATION_NODE CHILDREN CORRESPOUNDING TO PATCHES
  TYPE(INTEGRATION_NODE),POINTER :: TM  ! MIDDLE INTEGRATION_NODE
  INTEGER,POINTER ::POS     ! POSITION IN LAYOUT
  ! NEW STUFF....
  REAL(DP), POINTER :: BETA0,GAMMA0I,GAMBET,MASS  !,P0C
  INTEGER, POINTER :: CHARGE
  REAL(dDP), POINTER :: AG
  ! TO TIE LAYOUTS
  TYPE (FIBRE),POINTER :: P
  TYPE (FIBRE),POINTER :: N
  INTEGER,POINTER :: LOC
END TYPE FIBRE
```

The type definition includes the beam element `MAG` and its polymorphic version `MAGP`. `MAG` is the generic magnet to which we attach a single particle propagator. `MAGP` is almost a carbon copy of `MAG`. The integer `DIR` defines the direction of propagation through the fibre. We can enter the magnet either from the front or from the back. `P0C` and `BETA0` define a preferential frame of reference for the energy of this fibre. It is usually the same as the energy of the element `MAG`.

`T1` and `T2` are pointers to the first and last integration-node children. The integer `POS` locates the fibre in the node layout. See figure B.5.

Chart

Type `CHART` contains the information that locates an element in three-dimensional space at the position where we want it to be.

```
TYPE CHART
  TYPE(MAGNET_FRAME), POINTER :: F
  !  FIBRE MISALIGNMENTS
  REAL(DP),DIMENSION(:),  POINTER::   D_IN,ANG_IN
  REAL(DP),DIMENSION(:),  POINTER::   D_OUT,ANG_OUT
END TYPE CHART
```

`CHART` refers to the data type `MAGNET_FRAME`.

```
TYPE MAGNET_FRAME
  REAL(DP), POINTER,DIMENSION(:)  ::   A
  REAL(DP), POINTER,DIMENSION(:,:)::   ENT
```

```
  REAL(DP), POINTER,DIMENSION(:)  ::   O
  REAL(DP), POINTER,DIMENSION(:,:)::   MID
  REAL(DP), POINTER,DIMENSION(:)  ::   B
  REAL(DP), POINTER,DIMENSION(:,:)::   EXI
END TYPE MAGNET_FRAME
```

Figure B.2 shows the three charts (affine frames of reference) attached to an element.

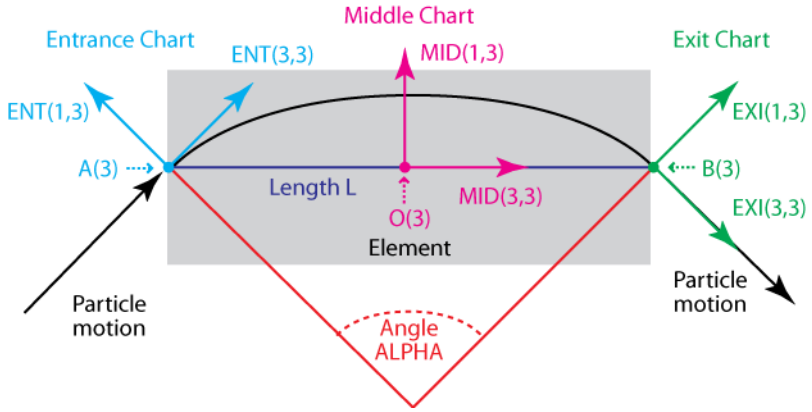


Figure B.2: Three charts attached to an element.

The variables `(A,ENT)`, `(O,MID)`, and `(B,EXI)` define the three charts. The variables `L` and `ALPHA` characterize the gray plane of the magnet. At the center of this plane lies the chart at Ω. This is where a magnet is compressed for misalignment purposes.

The `TRACK` routine sends a ray from the cyan entrance chart `(A,ENT)` through the middle chart `(O,MIS)` to the green exit chart `(B,EXI)`.

Misalignment

The `CHART` contains the misalignment information for a fibre (which, of course, points to an element).

The variables `D_IN,ANG_IN` and `D_OUT,ANG_OUT` are the displacements and rotations at the entrance and exit of the fibre. In figure B.3, the purple arrows represent `D_IN` and `D_OUT`. The dotted purple arcs represent the angles `ANG_IN` and `ANG_OUT`.

Figure B.4 provides a three-dimensional view of a misaligned planar fibre. The position `FIBRE%CHART%F%A(3)` and the vector triad basis `FIBRE%CHART%F%ENT(3,3)` specify the fibre's desired location.

Patch

The data type `PATCH` contains variables used in the main tracking loop to perform certain adjustments if the exit chart of one element does not connect smoothly with the entrance chart of the element that follows it, for example, when transferring from one beam line to another.

```
TYPE PATCH
  INTEGER(2), POINTER:: PATCH
```

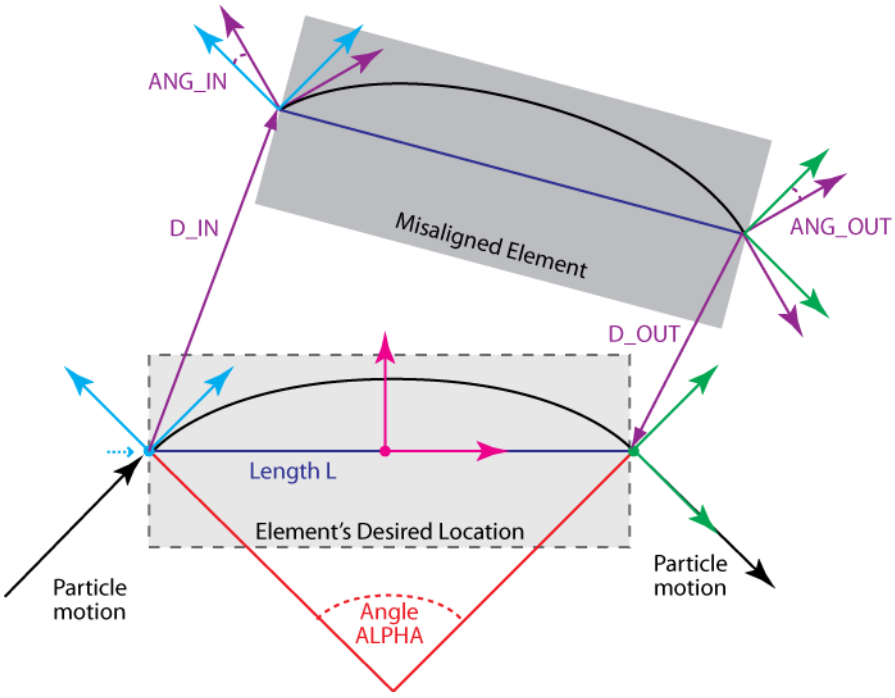


Figure B.3: Misalignments for a element.

```
  ! IF TRUE, SPACIAL PATCHES NEEDED
  INTEGER, POINTER :: A_X1,A_X2
  ! FOR ROTATION OF PI AT ENTRANCE = -1, DEFAULT = 1 ,
  INTEGER, POINTER :: B_X1,B_X2
  ! FOR ROTATION OF PI AT EXIT = -1    , DEFAULT = 1
  REAL(DP),DIMENSION(:), POINTER:: A_D,B_D
  ! ENTRANCE AND EXIT TRANSLATIONS  A_D(3)
  REAL(DP),DIMENSION(:), POINTER:: A_ANG,B_ANG
  ! ENTRANCE AND EXIT ROTATIONS    A_ANG(3)
  INTEGER(2), POINTER:: ENERGY
  ! IF TRUE, ENERGY PATCHES NEEDED
  INTEGER(2), POINTER:: TIME
  ! IF TRUE, TIME PATCHES NEEDED
  REAL(DP), POINTER:: A_T,B_T
  ! TIME SHIFT NEEDED SOMETIMES WHEN RELATIVE TIME IS USED
END TYPE PATCH
```

B.2 Time-based Tracking

This section discusses the following PTC data types for time-based tracking and gives the FORTRAN90 code:

- integration node, including probe, temporal probe, and temporal beam,
- node layout, including beam.

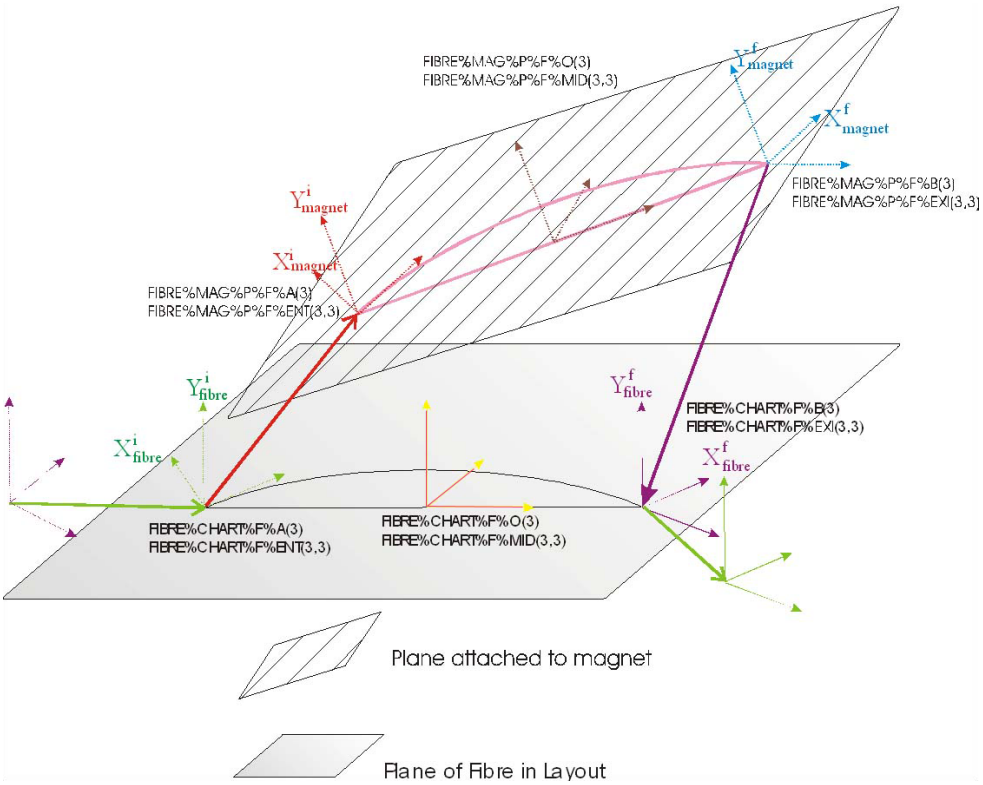


Figure B.4: Misaligned planar fibre in three dimensions.

Integration Node

Integration nodes allow us to look at any point in the layout. One of the most important applications is to permit collective forces. Moreover, we can extend the coordinates of a particle to permit first-order time-based tracking within the higher order s framework. To do so, we add to two coordinates to the particle:

$$(\vec{z} = x,\ p\_x,\ y,\ p\_y,\ t,\ p\_t,\ \delta s,\ p\_n)$$

The coordinate δs is the distance from the beginning of the integration node measured in the coordinate s used by the integrator.

The variable p_n is a pointer to the integration node in which the particle finds itself at time t.

During normal s-based tracking, δs is always zero. In a time-tracking mode, a drift is assumed between nodes to estimate the time of a particle; the result δs is computed. Since inverse drifts are exactly known, this reduces to the normal s-based tracking when collective effects are absent.

One immediate application is the tracking of several macro-particles in a recirculator with important wake field effects between the macro-particles: time ordering of the bunches is crucial and painlessly done in our framework.

The data type INTEGRATION_NODE is defined as follows:

```
TYPE INTEGRATION_NODE
```

```
  INTEGER, POINTER :: pos_in_fibre, CAS
  INTEGER, POINTER ::  pos
  real(dp), POINTER :: S(:)
  real(dp), POINTER :: ref(:)
  real(dp), pointer :: ent(:,:),a(:)
  real(dp), pointer :: exi(:,:),b(:)
  real(dp), POINTER :: delta_rad_in
  real(dp), POINTER :: delta_rad_out
  INTEGER, POINTER :: TEAPOT_LIKE
  TYPE (INTEGRATION_NODE), POINTER :: NEXT
  TYPE (INTEGRATION_NODE), POINTER :: PREVIOUS
  TYPE (NODE_LAYOUT), POINTER :: PARENT_NODE_LAYOUT
  TYPE(FIBRE), POINTER :: PARENT_FIBRE
  !   TYPE(EXTRA_WORK), POINTER :: WORK
  TYPE(BEAM_BEAM_NODE), POINTER :: BB
END TYPE INTEGRATION_NODE
```

Each integration node is either

1. the entrance patch/misalignment/tilt of its parent fibre (integration_node%cas=-1);
2. the entrance fringe field (integration_node%cas=1);
3. one of N steps in the body of the element (integration_node%cas=0);
4. the exit fringe field (integration_node%cas=2);
5. the exit patch/misalignment/tilt of its parent fibre (integration_node%cas=-2).

See figure B.5.

`NEXT` and `PREVIOUS` are the pointers necessary to create the `NODE_LAYOUT`, which is patterned closely on the `LAYOUT`.

`PARENT_FIBRE` points to the parent fibre which engendered the `INTEGRATION_NODE`.

`POS` is the position in the `NODE_LAYOUT`.

`POS_IN_FIBRE` is the position in the `PARENT_FIBRE`.

`TEAPOT_LIKE` indicates whether the internal frame of reference is curved or straight. This is useful to move approximately inside an integration-node step.

For `S`:

- `S(1)` contains the ideal arc length position `LD`,
- `S(2)` contains the local integration position `0<S(2)<L`,
- `S(3)` contains the total `L` up to that integration node,[1]
- `S(4)` contains the length of the integration step `DL : FIBRE%MAG%L/FIBRE%MAG%P%NST`.

[1] This is similar to the survey L of MAD8 in the sense that for a "real" Cartesian bend the distance between the two parallel faces is used. In other words, it is the sum around the ring of the integration variables, whatever they may be. For a MAD-type RBEND it is still LD because these bends are really sectors with wedges.

Probe

The data type `probe` holds information about the location of a particle.

```
TYPE PROBE
  REAL(DP) X(6)
  TYPE(SPINOR) S
  LOGICAL U
  TYPE(INTEGRATION_NODE),POINTER :: LOST_NODE
END TYPE PROBE
```

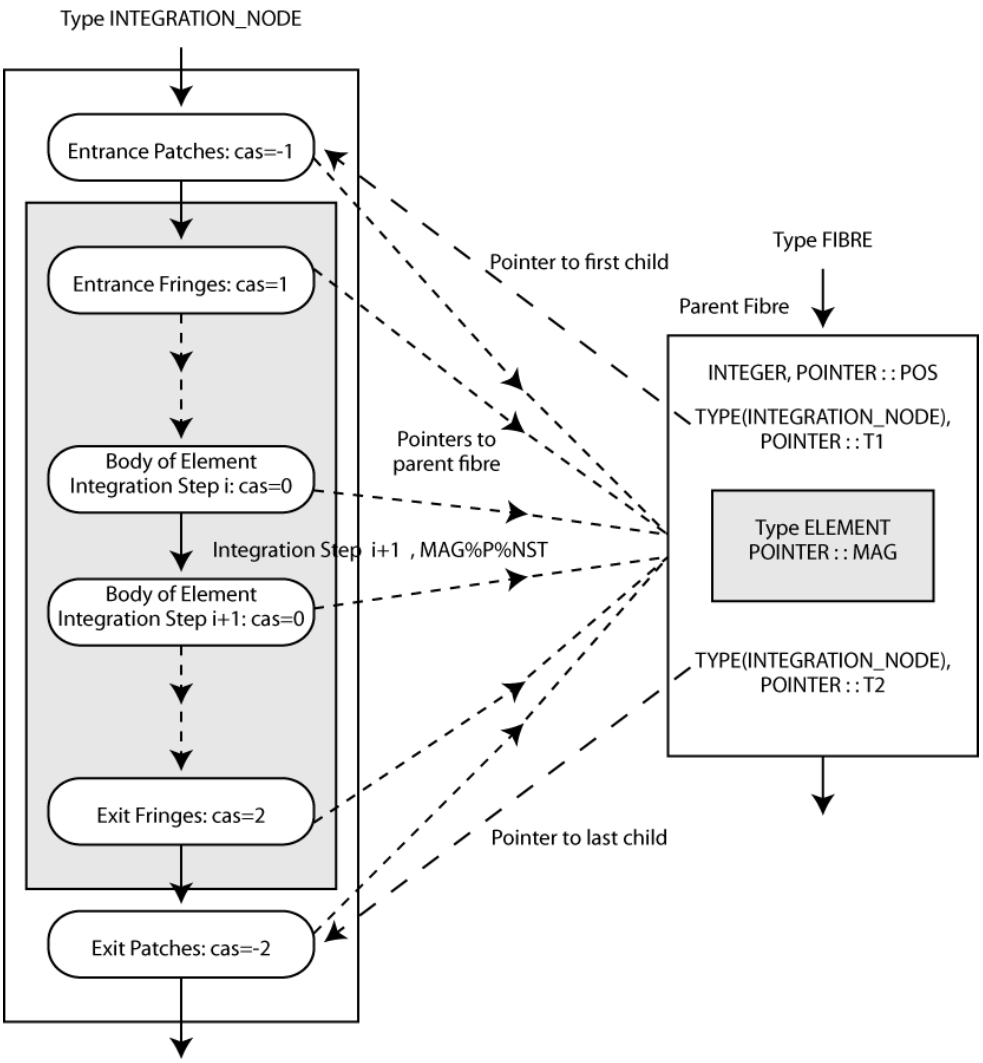


Figure B.5: Type `integration_node` and type `fibre`.

The probe tracks position `X(6)` and spin `S`. The logical `U` is true if the particle is unstable.

Here is the definition of the data type `PROBE_8`:

```
TYPE PROBE_8
  TYPE(REAL_8) X(6)
  TYPE(SPINOR_8) S
  REAL(DP) E_IJ(NDIM2,NDIM2)
  LOGICAL U
  TYPE(INTEGRATION_NODE),POINTER :: LOST_NODE
END TYPE PROBE_8
```

Temporal Probe

The data type `temporal probe` holds a probe and information relevant to time-based tracking.

```
TYPE TEMPORAL_PROBE
  TYPE(PROBE) XS
  TYPE(INTEGRATION_NODE), POINTER :: NODE
  REAL(DP) DS,POS(6)
  TYPE(INTERNAL_STATE) STATE
END TYPE TEMPORAL_PROBE
```

The probe `XS` contains the coordinates.
The `NODE` points to the integration node that the particle is in.
The distance `DS` is the approximate distance that the particle traveled inside the node, assuming a drift.
`POS(6)` is the position of the particle in an absolute 3D frame.
`STATE` is the tracking state of that beam/machine.

Temporal Beam

The data type `temporal beam` is a collection of particles (probes).

```
TYPE TEMPORAL_BEAM
  TYPE(TEMPORAL_PROBE), POINTER :: TP(:)
  REAL(DP) A(3),ENT(3,3),P0C,TOTAL_TIME
  INTEGER N
  TYPE(INTEGRATION_NODE),POINTER :: C  ! POINTER CLOSE TO A(3)
  TYPE(INTERNAL_STATE) STATE
END TYPE TEMPORAL_BEAM
```

A temporal beam must be allocated:

```
CALL ALLOC(TB,N,P0C)
```

`P0C` is the reference momentum of the beam.
We must set the initial conditions of the layout:

```
CALL POSITION_TEMPORAL_BEAM(LAYOUT,TB,STATE)
```

The subroutine `POSITION_TEMPORAL_BEAM`

- locates the beam in three dimensions and stores `TB%TP(i)%POS(1:3)=(X,Y,Z)`, which is PTC's global frame;
- calls the `LOCATE_TEMPORAL_BEAM` routine, which in turn calls the `ORIGINAL_P_TO_PTC` routine, which adjusts the momenta.

The beam's values of `TB%TP(i)%XS%X(1:6)` are now in PTC's local coordinates.

Node Layout

The data type NODE_LAYOUT is a linked list of nodes, which represents an expanded beam line. The nodes do not contain new data; their fibres contain their data.

The type NODE_LAYOUT is defined as follows:

```
TYPE NODE_LAYOUT
  CHARACTER(120), POINTER ::  NAME ! IDENTIFICATION
  INTEGER, POINTER ::  INDEX ! IDENTIFICATION
  LOGICAL(LP),POINTER ::CLOSED
  INTEGER,  POINTER :: N  ! TOTAL ELEMENT IN THE CHAIN
  ! POINTERS OF LINK LAYOUT
  INTEGER, POINTER :: LASTPOS   ! POSITION OF LAST VISITED
  TYPE (INTEGRATION_NODE), POINTER :: LAST ! LAST VISITED
  !
  TYPE (INTEGRATION_NODE), POINTER :: END
  TYPE (INTEGRATION_NODE), POINTER :: START
  TYPE (INTEGRATION_NODE), POINTER :: START_GROUND
  ! STORE THE GROUNDED VALUE OF START DURING CIRCULAR SCANNING
  TYPE (INTEGRATION_NODE), POINTER :: END_GROUND
  ! STORE THE GROUNDED VALUE OF END DURING CIRCULAR SCANNING
  TYPE (LAYOUT), POINTER :: PARENT_LAYOUT
  TYPE(ORBIT_LATTICE), POINTER :: ORBIT_LATTICE
END TYPE NODE_LAYOUT
```

Beam

The purpose of the node layout is to track a beam. We now define the data type BEAM.

```
TYPE BEAM
  !   TYPE(REAL_8), POINTER :: Y(:)
  REAL(DP), POINTER :: X(:,:)
  !   REAL(DP), POINTER :: SIGMA(:),DX(:),BBPAR,ORBIT(:)
  LOGICAL(LP), POINTER :: U(:)
  TYPE(BEAM_LOCATION), POINTER::POS(:)
  INTEGER, POINTER :: N,LOST
  !   INTEGER, POINTER :: CHARGE
  !   LOGICAL(LP),POINTER :: TIME_INSTEAD_OF_S
  !   LOGICAL(LP),POINTER :: BEAM_BEAM,BBORBIT
END TYPE BEAM
```

We also define the data type BEAM_LOCATION:

```
TYPE BEAM_LOCATION
  TYPE (INTEGRATION_NODE), POINTER :: NODE
END TYPE BEAM_LOCATION
```

LOST is the number of lost particles.

In a normal mode, the BEAM is pushed integration node to integration node. It can also be pushed from a location S1 to S2.

Additionally, time tracking is possible with TOTALPATH=true. In that case, the position of the particle is given by specifying the actual integration node immediately proceeding the particle and the distance from that integration node measured in the integration variable L. The node is located using the pointer POS(:)%Node, and the distance is located in X(:,:). Drifting back and forth to that position is done approximately with a drift either in Cartesian or Polar coordinates depending on the variable TEAPOT_LIKE of the type INTEGRATION_NODE.

```
TYPE BEAM
  REAL(DP), POINTER~:: X(N,1:7)
  LOGICAL(LP), POINTER :: U(N)
  TYPE(BEAM_LOCATION), POINTER::POS(N)
  INTEGER, POINTER :: N,LOST
  INTEGER, POINTER :: CHARGE
  LOGICAL(LP),POINTER :: TIME_INSTEAD_OF_S
END TYPE BEAM
```

The array X(N,1:7) contains N (macro)particles whose PTC coördinates are located in X(N,1:6). The array U(N) contains a stability flag: true is unstable. LOST is the number of lost particles.

In the "additionally" paragraph, the third sentence originally read as follows:

"The node is located using the pointer POS(N)%Node and the distance is located in X(N,7)."

C

PTC Geometry Tutorial Source File: `ptc_geometry.f90`

This appendix contains the complete FORTRAN90 listing for the PTC example ***ptc_geometry***. Most of this example is discussed in *Modeling an Accelerator with PTC*, chapter 3. More advanced aspects of this example are discussed in *Linking Magnets Together and Moving Them as a Group*, chapter 4.

ptc_geometry.f90

```
program ptc_geometry
use madx_ptc_module
use pointer_lattice
implicit none

character*48 :: command_gino
logical(lp) :: doit
integer :: i, j, mf, pos, example
real(dp) :: b0
real(dp), dimension(3) :: a, d
real(dp), dimension(6) :: fix1, fix2, mis, x
type(real_8), dimension(6) :: y1, y2
type(layout), pointer :: L1, L2, L3, L4, L5, L6
type(layout), pointer :: PSR1, PSR2, Fig8, Col1, Col2
type(fibre), pointer :: p1, p2, b, f
type(internal_state) :: state

type(pol_block) :: qf(2), qd(2)
type(normalform) :: n1, n2
type(damap) :: id
type(taylor) :: eq(4)
type(gmap) :: g
!---------------------------------

Lmax = 100.d0
use_info = .true.

!== user stuff : one layout necessary before starting GUI
```

```
   call ptc_ini_no_append
30
   !thin=-1
   !call thin_lens_resplit(m_u%end, thin, xbend = 1.d-10)

   !==========================!
35 !== set up DNA sequences ==!
   !==========================!
   call append_empty_layout(m_u) ! DNA sequence 1
   call set_up(m_u%end)
   L1 => m_u%end
40 call build_PSR(L1)

   call append_empty_layout(m_u) ! DNA sequence 2
   call set_up(m_u%end)
   L2 => m_u%end
45 call build_Quad_for_Bend(L2)

   call append_empty_layout(m_u) ! DNA sequence 3
   call set_up(m_u%end)
   L3 => m_u%end
50 call build_PSR_minus(L3)

   call append_empty_layout(m_u) ! DNA sequence 4
   call set_up(m_u%end)
   L4 => m_u%end
55 call build_PSR(L4)

   call append_empty_layout(m_u) ! DNA sequence 5
   call set_up(m_u%end)
   L5 => m_u%end
60 call build_PSR_minus(L5)

   call append_empty_layout(m_u) ! DNA sequence 6
   call set_up(m_u%end)
   L6 => m_u%end
65 call build_PSR(L6)


   !================================!
   !== create "trackable" layouts ==!
70 !================================!

   !== PSR1 : forward ring  (layout 7)
   call append_empty_layout(m_u)
   PSR1 => m_u%end
75
   p1 => L1%start
   p2 => L2%start
   do i = 1, L1%n
     if(p1%mag%name == "B") then
```

```
    ! read bends from L2
    call append_point(PSR1, p2)
    f => PSR1%end
    d = p1%chart%f%o - f%chart%f%o
    call translate(f, d)
    call compute_entrance_angle(f%chart%f%mid, p1%chart%f%mid, a)
    call rotate(f, f%chart%f%o, a, basis = f%chart%f%mid)
    p2 => p2%next
  else
    call append_point(PSR1, p1)
  end if
  p1 => p1%next
end do ! elements in PSR1 now in correct locations

f => PSR1%start
do i = 1, PSR1%n
  if(f%mag%name == "B_QUAD") then
    call find_patch(f%previous, f, next = .true.)
    call find_patch(f, f%next, next = .false.)
  end if
  f => f%next
end do ! PSR1 now patched

PSR1%name = "PSR 1"
PSR1%closed = .true.
call ring_L(PSR1, .true.) ! make it a ring topologically


!== PSR2 : backward ring (layout 8)
call append_empty_layout(m_u)
PSR2 => m_u%end

p1 => L1%end
p2 => L2%end
do i = 1, L1%n
  if(p1%mag%name == "B") then
    call append_point(PSR2, p2)
    p2 => p2%previous
  else
    call append_point(PSR2, p1)
  end if
  f => PSR2%end
  f%dir = -1
  f%charge = -1
  p1 => p1%previous
end do

f => PSR2%start
do i = 1, PSR2%n
  if(f%mag%name == "B_QUAD") then
    call find_patch(f%previous, f, next = .true.)
```

```
    call find_patch(f, f%next, next = .false.)
  end if
  f => f%next
end do

PSR2%name = "PSR 2"
PSR2%closed = .true.
call ring_l(PSR2, .true.) ! make it a ring topologically


!== Fig8 : figure-eight lattice (layout 9)
d = zero
d(3) = -40.d0
call translate(L4, d)
a = zero
a(2) = pi
call rotate(L3, L3%start%chart%f%a, a)
call move_to(L4, p1, "B", pos)
d = p1%chart%f%a - L3%end%chart%f%b
call translate(L3, d)

call append_empty_layout(m_u)
Fig8 => m_u%end
p1 => L4%start
do i = 1, L4%n
  call append_point(Fig8, p1)
  p1 => p1%next
end do

write(6,*) p1%mag%name
call append_point(Fig8, p1)
p1 => p1%next
write(6,*) p1%mag%name
call append_point(Fig8, p1)
p1 => p1%next
write(6,*) p1%mag%name
call append_point(Fig8, p1)

p1 => L3%end
do i = 1, L3%n
  call append_point(Fig8, p1)
  Fig8%end%dir = -1
  if(p1%mag%name == "B") p1%mag%bn(1) = -p1%mag%bn(1)
  p1 => p1%previous
end do

p1 => L4%end%previous%previous
write(6,*) p1%mag%name
call append_point(Fig8, p1)
p1 => p1%next
write(6,*) p1%mag%name
```

```
call append_point(Fig8, p1)
p1 => p1%next
write(6,*) p1%mag%name
call append_point(Fig8, p1)

write(6,*) "Fig8 has ", Fig8%n, " fibres"
Fig8%name = "Figure-Eight"
Fig8%closed = .true.
call ring_l(Fig8, .true.) ! make it topologically closed

p1 => Fig8%start
do i = 1, Fig8%n
  call check_need_patch(p1, p1%next, 1.d-10, pos)
  if(pos /= 0) call find_patch(p1, p1%next, next = .false.)
  p1 => p1%next
end do


!== Col1 : lower collider ring (layout 10)
!== Col2 : upper collider ring (layout 11)
d = zero
d(3) = 40.d0
call translate(L6, d)
a = zero
a(2) = pi
call rotate(L5, L5%start%chart%f%a, a)
call move_to(L6, p1, "B", pos)
d = p1%chart%f%a - L5%end%chart%f%b
call translate(L5, d)

call append_empty_layout(m_u)
Col1 => m_u%end
p1 => L6%start
do i = 1, L6%n
  call append_point(Col1, p1)
  p1 => p1%next
end do

write(6,*) "Collider 1 has ", Col1%n, " fibres"
Col1%name = "Collider 1"
Col1%closed = .true.
call ring_l(Col1, .true.) ! make it a ring topologically

call append_empty_layout(m_u)
Col2 => m_u%end
p1 => L6%start%next%next
do i = 1, 6
  write(6,*) p1%mag%name
  call append_point(Col2, p1)
  Col2%end%dir = -1
  p1 => p1%previous
```

```
  end do
  p1 => L5%start
do i = 1, L3%n
    call append_point(Col2, p1)
    p1 => p1%next
  end do

write(6,*) "Collider 2 has ", Col2%n, " fibres"
  Col2%name = "Collider 2"
  Col2%closed = .true.
  call ring_l(Col2, .true.) ! make it a ring topologically

p1 => Col2%start
  do i = 1, Col2%n
    call check_need_patch(p1, p1%next, 1.d-10, pos)
    if(pos /= 0) call find_patch(p1, p1%next, next = .false.)
    p1 => p1%next
end do


  !======================!
  !== set up DNA arrays ==!
!======================!

  allocate(PSR1%DNA(2))
  PSR1%DNA(1)%L => L1
  do i = 2, 2
    PSR1%DNA(i)%L => PSR1%DNA(i-1)%L%next ! L2
  end do

  allocate(PSR2%DNA(2))
  PSR2%DNA(1)%L => L1
do i = 2, 2
    PSR2%DNA(i)%L => PSR2%DNA(i-1)%L%next ! L2
  end do

  allocate(Fig8%DNA(2))
Fig8%DNA(1)%L => L3
  do i = 2, 2
    Fig8%DNA(i)%L => Fig8%DNA(i-1)%L%next ! L4
  end do

allocate(Col1%DNA(2))
  Col1%DNA(1)%L => L5
  do i = 2, 2
    Col1%DNA(i)%L => Col1%DNA(i-1)%L%next ! L6
  end do

  allocate(Col2%DNA(2))
  Col2%DNA(1)%L => L5
  do i = 2, 2
```

```
  Col2%DNA(i)%L => Col2%DNA(i-1)%L%next ! L6
end do


!== maps with polymorphs
!== simple fit

qf(1) = 0
qf(1)%name = "qf"
qf(1)%ibn(2) = 1
qf(2) = 0
qf(2)%name = "qf"
qf(2)%ibn(2) = 3
qd(1) = 0
qd(1)%name = "qd"
qd(1)%ibn(2) = 2
qd(2) = 0
qd(2)%name = "qd"
qd(2)%ibn(2) = 4
Col1%dna(1)%L = qf(1)
Col1%dna(1)%L = qd(1)
CoL1%dna(2)%L = qf(2)
CoL1%dna(2)%L = qd(2)

101 continue
state = default0 + only_4d0

fix1 = 0.d0
fix2 = 0.d0;
call init(state, 2, c_%np_pol) ! c_%np_pol is automatically computed
call find_orbit(CoL1, fix1, 1, state, 1.d-6)
call find_orbit(Col2, fix2, 1, state, 1.d-6)
call alloc(y1)
call alloc(y2)
call alloc(id)
call alloc(n1)
call alloc(n2)
call alloc(eq);
id=1 ! identity damap
y1 = id + fix1 ! this is permitted in ptc only (not fpp)
y2 = id + fix2 ! closed orbit added to map
call track(Col1, y1, 1, +state) ! unary + activates knobs
call track(Col2, y2, 1, +state)
n1 = y1 ! normal forms: abused of language permitted by ptc
n2 = y2 ! normally one should do => damap=y; normalform=damap
write(6,*) " tunes 1 "
write(6,*) n1%tune(1:2)
write(6,*) " tunes 2 "
write(6,*) n2%tune(1:2)
eq(1) = n1%dhdj%v(1) - 0.254d0
eq(2) = n1%dhdj%v(2) - 0.255d0
```

```
eq(3) = n2%dhdj%v(1) - 0.130d0
eq(4) = n2%dhdj%v(2) - 0.360d0
do i = 1, 4
  eq(i) = eq(i) <= c_%npara
end do

call kanalnummer(mf,"eq.txt")
do i=1,4
  call daprint(eq(i), mf)
end do
close(mf)

call kill(y1)
call kill(y2)
call kill(id)
call kill(n1)
call kill(n2)
call kill(eq)
call init(1,4)
call alloc(g,4)
call kanalnummer(mf,"eq.txt")
do i = 1, 4
  call read(g%v(i), mf)
end do
close(mf)

g = g.oo.(-1)
tpsafit(1:4) = g
set_tpsafit = .true.
set_element = .true.
Col1%dna(1)%L = qf(1)
Col1%dna(1)%L = qd(1)
Col1%dna(2)%L = qf(2)
Col1%dna(2)%L = qd(2)
set_tpsafit = .false.
set_element = .false.
call kill(g)
write(6,*) " more "
read(5,*) i
if(i == 1) goto 101
call kill_para(Col1%dna(1)%l)
call kill_para(Col1%dna(2)%l)


!================================!
!== set up Siamese and Girder ==!
!================================!

call move_to(Col1, p1, 67)
call move_to(Col2, p2, 7)
p1%mag%siamese => p2%mag
```

```
   p2%mag%siamese => p1%mag
   call move_to(Col1, p1, 4)
   call move_to(Col2, p2, 70)
   p1%mag%siamese => p2%mag
   p2%mag%siamese => p1%mag

   call move_to(Col1, p1, 68)
   f => p1  ! remember start of girder linked-list
   do i = 2, 7
     p2 => p1%next
     p1%mag%girders => p2%mag
     p1 => p1%next
   end do
   call move_to(Col2, p2,  7)
   p1%mag%girders => p1%mag%siamese
   p1%mag%siamese%girders => p2%mag
   p2%mag%girders => p2%mag%siamese
   call move_to(Col1, p1, 67)
   call move_to(Col2, p2, 14)
   p1%mag%girders => p2%mag
   p2%mag%girders => f%mag

   call move_to(Col1, p1, 1)
   call alloc_af(p1%mag%girder_frame, girder = .true.)
   p1%mag%girder_frame%ent = p1%mag%parent_fibre%chart%f%ent
   p1%mag%girder_frame%a   = p1%mag%parent_fibre%chart%f%a
   p1%mag%girder_frame%exi = p1%mag%parent_fibre%chart%f%ent
   p1%mag%girder_frame%b   = p1%mag%parent_fibre%chart%f%a

   a = p1%mag%girder_frame%a
   a(3) = a(3) - 5.d0
   call move_to(Col1, b, 67)
   call alloc_af(b%mag%siamese_frame)
   call find_patch(b%mag%p%f%a, b%mag%p%f%ent, &
                   a, p1%mag%girder_frame%ent, &
                   b%mag%siamese_frame%d, b%mag%siamese_frame%angle)
   a = p1%mag%girder_frame%a
   a(3) = a(3) + 5.d0
   call move_to(Col1, b, 4)
   call alloc_af(b%mag%siamese_frame)
   call find_patch(b%mag%p%f%a, b%mag%p%f%ent, &
                   a, p1%mag%girder_frame%ent, &
                   b%mag%siamese_frame%d, b%mag%siamese_frame%angle)


   !============================!
   !== example misalignments ==!
   !============================!

   write(6,*) "Example # (from the manual) 1--11 ?"
   read(5,*) example
```

```
    call move_to(Col2, p2, 7)
    if(example == 1) then
      mis = 0.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
    elseif(example == 2) then
      mis = 0.d0
      mis(1) = 2.0d0
      call misalign_girder(p2, mis)
    elseif(example == 3) then
      mis = 0.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_girder(p2, mis, add = .false.)
    elseif(example == 4) then
      mis = 0.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_girder(p2, mis, add = .true.)
    elseif(example == 5) then
      mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis)
    elseif(example == 6) then
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_siamese(p2, mis)
    elseif(example == 7) then
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_siamese(p2, mis)
      mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis, add = .false.)
    elseif(example == 8) then
      mis = 0.d0
      mis(1) = 2.d0
      call misalign_siamese(p2, mis)
      mis = 0.d0
      mis(1) = 2.d0
      mis(5) = pi / 8.d0
      call misalign_girder(p2, mis, add = .true.)
    elseif(example == 9) then
      mis = 0.d0
```

```
  mis(1) = 2.d0
  mis(5) = pi / 8.d0
  call misalign_girder(p2, mis)
  mis = 0.d0
  mis(1) = 2.d0
  call misalign_siamese(p2, mis, add = .false.)
elseif(example == 10) then
  mis = 0.d0
  mis(1) = 2.d0
  mis(5) = pi / 8.d0
  call misalign_girder(p2, mis)
  mis = 0.d0
  mis(1) = 2.d0
  call misalign_siamese(p2, mis, add = .true.)
elseif(example == 11) then
  mis = 0.d0
  mis(1) = 2.d0
  mis(5) = pi / 8.d0
  call misalign_girder(p2, mis)
  mis = 0.d0
  mis(1) = 2.d0
  call misalign_siamese(p2, mis, add = .false., &
                        preserve_girder = .true.)
end if

999 command_gino = "opengino"
call context(command_gino)   ! context makes them capital
call call_gino(command_gino)
111 command_gino = "mini"
call context(command_gino)   ! context makes them capital
call call_gino(command_gino)

!== vaguely necessary baloney ==
command_gino = "closegino"
call call_gino(command_gino)
call ptc_end
end program ptc_geometry


!================================================================
subroutine  build_PSR(PSR)
use madx_ptc_module
use pointer_lattice
implicit none

type(layout), target :: PSR

real(dp) :: ang, brho, kd, kf, Larc
type(fibre) :: b, d1, d2, qd, qf
type(layout) :: cell
!----------------------------------
```

```
call make_states(.false.)
exact_model = .true.
default = default + nocavity + exactmis
call update_states
madlength = .false.

ang = (twopi * 36.d0 / 360.d0)
Larc = 2.54948d0
brho = 1.2d0 * (Larc / ang)
call set_mad(brho = brho, method = 2, step = 10)
madkind2 = drift_kick_drift

kf =  2.72d0 / brho
kd = -1.92d0 / brho

d1 = drift("D1", 2.28646d0)
d2 = drift("D2", 0.45d0)
qf = quadrupole("QF", 0.5d0, kf)
qd = quadrupole("QD", 0.5d0, kd)
b  = rbend("B", Larc, ang)
cell = d1 + qd + d2 + b + d2 + qf + d1

PSR = 10 * cell
PSR = .ring.PSR

call survey(PSR)
end subroutine build_PSR


!=====================================================================
subroutine  build_PSR_minus(PSR)
use madx_ptc_module
use pointer_lattice
implicit none

type(layout), target :: PSR

real(dp) :: ang, brho, kd, kf, Larc
type(fibre) :: b, d1, d2, qd, qf
type(layout) :: cell
!---------------------------------

call make_states(.false.)
exact_model = .true.
default = default + nocavity + exactmis
call update_states
madlength = .false.

ang = (twopi * 36.d0 / 360.d0)
Larc = 2.54948d0
```

```
590 brho = 1.2d0 * (Larc / ang)
    call set_mad(brho = brho, method = 6, step = 10)
    madkind2 = drift_kick_drift

    kf =  2.72d0 / brho
595 kd = -1.92d0 / brho

    d1 = drift("D1", 2.28646d0)
    d2 = drift("D2", 0.45d0)
    qf = quadrupole("QF", 0.5d0, kf)
600 qd = quadrupole("QD", 0.5d0, kd)
    b  = rbend("B", Larc, ang)
    cell = d1 + qd + d2 + b + d2 + qf + d1

    PSR = b + d2 + qf + d1 + 8 * cell + d1 + qd + d2 + b
605 PSR = .ring.PSR

    call survey(PSR)
    end subroutine build_PSR_minus

610

    !=====================================================================
    subroutine  build_Quad_for_Bend(PSR)
    use madx_ptc_module
    use pointer_lattice
615 implicit none

    type(layout),target :: PSR

    real(dp) :: ang, ang2, brho, b1, Larc, Lq
620 type(fibre) :: b
    !----------------------------------

    call make_states(.false.)
    exact_model = .true.
625 default = default + nocavity + exactmis
    call update_states
    madlength = .false.

    ang = (twopi * 36.d0 / 360.d0)
630 Larc = 2.54948d0
    brho = 1.2d0 * (Larc / ang)
    call set_mad(brho = brho, method = 6, step = 10)
    madkind2 = drift_kick_drift

635 ang2 = ang / two
    b1 = ang / Larc
    Lq = Larc * sin(ang2) / ang2

    b = quadrupole("B_QUAD", Lq, 0.d0);
640 call add(b, 1, 0, b1)
```

```
    b%mag%permfringe = .true.
    b%magp%permfringe = .true.
    b%mag%p%bend_fringe = .true.
    b%magp%p%bend_fringe = .true.

    PSR = 10 * b
    PSR = .ring.PSR

    call survey(PSR)
end subroutine build_Quad_for_Bend
```

D

PTC Splitting Tutorial Source File: `ptc_splitting.f90`

This appendix contains the complete FORTRAN90 listing for the PTC example ***ptc_splitting***. This example is discussed in *Symplectic Integration and Splitting*, chapter 9.

ptc_splitting.f90

```
program ptc_splitting
use run_madx
use pointer_lattice
implicit none

character*48 :: command_gino
type(layout), pointer :: r1, r2, r3, r4, r5, r6
type(layout), pointer :: psr1, psr2, fig8, col1, col2
type(fibre),pointer :: p, b, f, qf, qd, d1, d2
integer i,pos,j,mf,example
logical(lp) doneit
real(dp) a(3),d(3),x(6),b0,mis(6),thin
type(real_8) y1(6),y2(6)
real(dp) fix1(6),fix2(6)
type(normalform) n1,n2
type(damap) id
type(internal_state) state
type(taylor) eq(4)
type(gmap) g
integer method,limits(2)
logical exact
!----------------------------------

lmax=100.d0
use_info=.true.

!== user stuff : one layout necessary before starting GUI
call ptc_ini_no_append
```

```
use_info=.true.

!!!!!!!!! producing the dna !!!!!!!!!!

call append_empty_layout(m_u) ! number 1
call set_up(m_u%end)
r1 => m_u%end

exact = .false.
method = drift_kick_drift
call build_PSR(r1, exact, method)

call move_to(r1, qf, "qf", pos)
call move_to(r1, qd, "qd", pos)
call move_to(r1, b,  "b",  pos)

!!!! first respliting !!!!! example 1
thin = 0.01d0
limits(1:2) = 100000

call thin_lens_resplit(r1, thin, lim = limits)
write(6,*) qf%mag%name, qf%mag%p%method, qf%mag%p%nst
write(6,*) qd%mag%name, qd%mag%p%method, qd%mag%p%nst
write(6,*) b%mag%name,  b%mag%p%method,  b%mag%p%nst

pause 1
!!!! second respliting !!!!! !!!!! example 2
thin = 0.01d0
limits(1) = 8
limits(2) = 24

call thin_lens_resplit(r1, thin, lim = limits)
write(6,*) qf%mag%name, qf%mag%p%method, qf%mag%p%nst
write(6,*) qd%mag%name, qd%mag%p%method, qd%mag%p%nst
write(6,*) b%mag%name,  b%mag%p%method,  b%mag%p%nst

pause 2

!!!! Talman algorithm !!!!! !!!!! example 3
write(6,*) " "
write(6,*) "!!!! Talman algorithm !!!!! !!!!! example 3"
write(6,*) " "

thin = 0.001d0 ! cut like crazy
call thin_lens_resplit(r1, thin,l im = limits)
!!! ptc command file: could be a mad-x command or whatever
call read_ptc_command77("fill_beta0.txt") ! computing tune and beta around the rin

write(6,*) " "
write(6,*) " now reducing the number of steps and refitting "
write(6,*) " "
```

```
do i = 0, 2
  thin = 0.01d0 + i * 0.03
  call thin_lens_resplit(r1, thin, lim = limits) ! reducing number of cuts
  call read_ptc_command77("fit_to_beta0_results.txt") ! fitting to previous tunes
  call read_ptc_command77("fill_beta1.txt") ! computing dbeta/beta around the ring
end do

pause 3

write(6,*) " "
write(6,*) "!!!! sbend orbit small problem !!!!! !!!!! example 4"
write(6,*) " "

call append_empty_layout(m_u) ! number 2
call set_up(m_u%end)
r1 => m_u%end

exact = .true.
method = drift_kick_drift
call build_PSR(r1, exact, method)

write(6,*) " "
write(6,*) " now reducing the number of steps and refitting "
write(6,*) " "
do i = 0, 2
  thin = 0.01d0 + i * 0.03
  call thin_lens_resplit(r1, thin) ! reducing number of cuts
  call read_ptc_command77("fit_to_beta0_results_2.txt") ! fitting to previous tunes
  call read_ptc_command77("fill_beta1_2.txt") ! computing dbeta/beta around the ring
end do

pause 4
write(6,*) " "
write(6,*) "!!!! sbend orbit small problem !!!!! !!!!! example 5"
write(6,*) " "
write(6,*) " "
write(6,*) " now reducing the number of steps and refitting with xbend=1.d-4 "
write(6,*) " "
do i=0,2
  thin = 0.01d0+i*0.03
  call thin_lens_resplit(r1, thin, xbend = 1.d-4) ! reducing number of cuts
  call read_ptc_command77("fit_to_beta0_results_2.txt") ! fitting to previous tunes
  call read_ptc_command77("fill_beta1_2.txt") ! computing dbeta/beta around the ring
end do
pause 5

write(6,*) "!!!! even !!!!! !!!!! example 6"
call move_to(r1, d1, "d1", pos)
call move_to(r1, d2, "d2", pos)
call move_to(r1, qf, "qf", pos)
call move_to(r1, qd, "qd", pos)
```

```
call move_to(r1, b,  "b",  pos)

call thin_lens_restart(r1) ! puts back method =2 and nst=1 everywhere
thin = 0.01d0
call thin_lens_resplit(r1, thin, even = .true., xbend = 1.d-4) ! reducing number o
call make_node_layout(r1)
write(6,*) qf%mag%name, qf%mag%p%method, qf%mag%p%nst, qf%t1%pos, qf%tm%pos, qf%t2
write(6,*) qd%mag%name, qd%mag%p%method, qd%mag%p%nst, qd%t1%pos, qd%tm%pos, qd%t2
write(6,*) b%mag%name,  b%mag%p%method,  b%mag%p%nst,  b%t1%pos,  b%tm%pos,  b%t2%

call thin_lens_restart(r1) ! puts back method =2 and nst=1 everywhere
thin = 0.01d0
call thin_lens_resplit(r1, thin, lim = limits, xbend = 1.d-4) ! reducing number of
call make_node_layout(r1)
write(6,*) qf%mag%name, qf%mag%p%method, qf%mag%p%nst, qf%t1%pos, qf%tm%pos, qf%t2
write(6,*) qd%mag%name, qd%mag%p%method, qd%mag%p%nst, qd%t1%pos, qd%tm%pos, qd%t2
write(6,*) b%mag%name,  b%mag%p%method,  b%mag%p%nst,  b%t1%pos,  b%tm%pos,  b%t2%

write(6,*) "!!!! lmax0 keyword !!!!! !!!!! example 7"
resplit_cutting = 1
call thin_lens_restart(r1) ! puts back method =2 and nst=1 everywhere
thin = 0.01d0
call thin_lens_resplit(r1, thin, even = .true., lmax0 = 0.05d0, xbend = 1.d-4) ! r
call make_node_layout(r1)
write(6,'(a8,2x,5(i4,8x))') d1%mag%name(1:8), d1%mag%p%method, d1%mag%p%nst, d1%t1
write(6,'(a8,2x,5(i4,8x))') d2%mag%name(1:8), d2%mag%p%method, d2%mag%p%nst, d2%t1
write(6,'(a8,2x,5(i4,8x))') qf%mag%name(1:8), qf%mag%p%method, qf%mag%p%nst, qf%t1
write(6,'(a8,2x,5(i4,8x))') qd%mag%name(1:8), qd%mag%p%method, qd%mag%p%nst, qd%t1
write(6,'(a8,2x,5(i4,8x))') b%mag%name(1:8),  b%mag%p%method,  b%mag%p%nst,  b%t1%
resplit_cutting = 2
call thin_lens_restart(r1) ! puts back method =2 and nst=1 everywhere
thin = 0.01d0
call thin_lens_resplit(r1, thin, even = .true., lmax0 = 0.05d0, xbend = 1.d-4) ! r
call make_node_layout(r1)
write(6,'(a8,2x,5(i4,8x))') d1%mag%name(1:8), d1%mag%p%method, d1%mag%p%nst, d1%t1
write(6,'(a8,2x,5(i4,8x))') d2%mag%name(1:8), d2%mag%p%method, d2%mag%p%nst, d2%t1
write(6,'(a8,2x,5(i4,8x))') qf%mag%name(1:8), qf%mag%p%method, qf%mag%p%nst, qf%t1
write(6,'(a8,2x,5(i4,8x))') qd%mag%name(1:8), qd%mag%p%method, qd%mag%p%nst, qd%t1
write(6,'(a8,2x,5(i4,8x))') b%mag%name(1:8),  b%mag%p%method,  b%mag%p%nst,  b%t1%

999 command_gino = "opengino"
call context(command_gino)  ! context makes them capital
call call_gino(command_gino)
111 command_gino = "mini"
call context(command_gino)  ! context makes them capital
call call_gino(command_gino)

!!!!!!!!! vaguelynecessary baloney
command_gino = "closegino"
call call_gino(command_gino)
```

```
call ptc_end
end program ptc_splitting

!========================================================================
subroutine build_PSR(PSR, exactTF, imethod)
use run_madx
use pointer_lattice
implicit none

type(layout), target :: PSR
logical(lp) :: exactTF
integer :: imethod

real(dp) :: ang, brho, kd, kf, larc, lq
type(fibre) :: b, d1, d2, qd, qf
type(layout) :: cell
!----------------------------------

call make_states(.false.)
default = default + nocavity + exactmis
call update_states
madlength = .false.

exact_model = exactTF

ang = (twopi * 36.d0 / 360.d0)
larc = 2.54948d0
brho = 1.2d0 * (larc / ang)
call set_mad(brho = brho, method = 6, step = 100)
madkind2 = imethod

kf =  2.72d0 / brho
kd = -1.92d0 / brho
lq = 0.5d0
write(6,'(a)') "kf * lq, kd * lq :"
write(6,*) kf * lq, kd * lq

d1 = drift("D1", 2.28646d0)
d2 = drift("D2", 0.45d0)
qf = quadrupole("QF", lq, kf)
qd = quadrupole("QD", lq, kd)
!b = rbend("B", larc, ang)
b = sbend("B", larc, ang)
cell = d1 + qd + d2 + b + d2 + qf + d1

PSR = 10 * cell
PSR = .ring.PSR

call survey(PSR)
call clean_up
```

```
    end subroutine build_PSR
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```

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